$$= \exp\left[\frac{\theta\left(-\frac{1}{\mu}\right) - \ln\mu}{1/\alpha} + \left[(\alpha-1)\ln x - \ln\Gamma(\alpha) + \alpha\ln\right.\right.$$

$$\theta = \frac{-1}{\mu} \Rightarrow \mu = \frac{-1}{\theta}; \quad a(\phi) = \frac{1}{\alpha}, \; c(y,\phi) =$$

$$b(\theta) = \ln\mu = \ln\left(\frac{-1}{\theta}\right.$$

$$E(Y) = b'(\theta)$$

$$= \frac{1}{\frac{-1}{\theta}} \times -\frac{1}{\theta^2} = \frac{-\theta}{\theta^2} = \frac{-1}{\theta} = \underline{\underline{\mu}}$$

$$V(Y) = a(\phi) \cdot b''(\theta)$$

$$= \frac{1}{\alpha} \times \frac{1}{\theta^2} = \underline{\underline{\frac{1}{\alpha\theta^2}}}$$

* Components of a GLM

1) Appropriate distⁿ

Prob → Binomial

No. of claims → Poisson

Continuous scale → Exp/Normal/Gamma

2) Linear predictor (η)

→ fⁿ of independent variables (covariates)

→ Linear in parameters

3) Link function $[g(\mu)]$

→ Links mean response to the predictor

→ Links mean response to linear predictor

→ Canonical link functions in Tables

Bin → $g(\mu) = \ln\left(\frac{\mu}{1-\mu}\right)$ $\qquad\qquad \ln\left(\frac{\mu}{1-\mu}\right) =$

Poisson → $g(\mu) = \ln \mu$

Gama/Exp → $g(\mu) = \frac{1}{\mu}$

Normal → $g(\mu) = \mu$

* **Linear predictor**

Variables ⇓ Continuous

Factors ⇓ Categorical

$\alpha_i = 0$
$= 0.3$
$= -0.5$

1.) Numerical variable → $\alpha + \beta_1 x$ → No. of par ⇒ α,

2.) Categorical " → α_i → No. of par = No.

3.) Num + Cat → $\underline{\beta_0} + \beta_1 x + \underline{\alpha_i} = \alpha_i + \beta_1 x$ ⇒ β

4.) Num . Cat (interaction) → $\beta_i x$

$\beta_1 = 0.3$ $\qquad x \Rightarrow$ Age
$\beta_2 = 0.5$ $\qquad$ 25 M → 25 × 0.

5.) Num * Cat = num + cat + num.cat

$= \beta_0 + \beta_1 x + \underline{\alpha_i} + \gamma_i x$

cont eff of cont + eff of cat ⇒ $\alpha_i + (\beta_1 + \gamma_i) x = \alpha_i + \gamma_i x$

$i = 1, 2, \ldots, h$; $j = 1, 2, \ldots$

$$\frac{\alpha + \beta x}{LP}$$

; Other

; M

F

$\beta_1 \Rightarrow 2$

of cat - 1

, $\alpha_i \Rightarrow$ No. of cat

+ 1

β = 7.5

n

+ eff of cat $\Rightarrow \alpha_i + (\beta_i \cdots)$

6.) cat + cat $= \alpha_i + \gamma_j$; $i = 1, 2, \ldots, n$; $j = 1, 2, \ldots, r$

7.) cat . cat $= \alpha_{ij}$

8.) cat * cat $=$ cat 1 + cat 2 + cat 1 . cat 2

$= \alpha_i + \beta_j + \gamma_{ij}$

$= \gamma_{ij}$

Num $\to \alpha + \beta_1 x$

Cat $\to \gamma_i$

Num . cat $\to (\alpha + \beta_1 x)\gamma_i = \underline{\cancel{\alpha\gamma_i}} + \underline{\beta\gamma_i x}$

$i \Rightarrow$ Gender ; $i = M, F$

$j \Rightarrow$ Source of emp ; $j =$ SE, Salaried

	M	F
SE	0.02	0.05
S	-0.05	-0.01

$\Leftarrow \alpha_{F, SE}$

Eg : Age $\Rightarrow$ Cont var

Gender $\Rightarrow$ Cat var $\Rightarrow$ 2 cat

Vehicle group $\Rightarrow$ Cat var $\Rightarrow$ 20 cat

Age * (Gender + VG)

Age $\Rightarrow \beta_0 + \beta_1 x$

$G \Rightarrow \alpha_i$

$VG \Rightarrow \beta_j$

$G + VG \Rightarrow \alpha_i + \beta_j$

$Age * (Gender + VG)$

$= Age + (Gender + VG) + Age.(Gender + VG)$

$= \beta_0 + \beta_1 x + (\alpha_i + \beta_j) + (\beta_0 + \beta_1 x)(\alpha_i + \beta_j)$

$= \cancel{\beta_0} + \cancel{\beta_1 x} + \alpha_i + \beta_j + \alpha_i \beta_0 + \beta_0 \beta_j + \alpha_i \beta_1 x + \beta_j \beta_1$

$= \alpha_i + \beta_j + (\alpha_i x + \beta_j x)$

$= \alpha_i + \beta_j + \alpha_i x + \beta_j x$

Eg: Bin

Link function $g(\mu) = \ln\left(\frac{\mu}{1-\mu}\right)$

Linear predictor $\eta = \alpha_i + \beta_1 N + \beta_2 S$

$g(\mu) = \eta$

$\therefore \ln\left(\frac{\mu}{1-\mu}\right) = \eta$

$\therefore \frac{\mu}{1-\mu} = e^{\eta}$

$\therefore \mu = e^{\eta} - \mu e^{\eta}$

$\therefore \mu = \frac{e^{\eta}}{1+e^{\eta}}$ —— (1)