*.) What is credit risk ?
→ Risk of someone not paying back what they owe
→ Risk of reduced returns due to credit events eg: ratings downgrade, delay in coupon payments

Eg

Bank
↓ Borrowers' default on loans
↓ Counterparty default on derivatives
↓ Losses on treasury portfolio (Investments of the bank)

- Risk of vendor or outsourced party failing to meet obligations

Eg: Airline company
- Derivatives (for hedging FX risk & commodity risk & interest rate risk) [Risk of the counterparty defaulting]
- Supplier fails to deliver ATF

*.) Mitigation
- Borrowers' default

- Borrowers' default

Before loan

- Credit assessment
- Scoring models
- Rating models
- External rating

After giving loan

- Regularly monitor
- Appropriately model expected & unexpected losses

2) Counterparty default on derivatives

- Enter into comprehensive legal agreement with counterparty

ISDA allows netting

Oct 2020

The Bilateral Netting of QFC Act

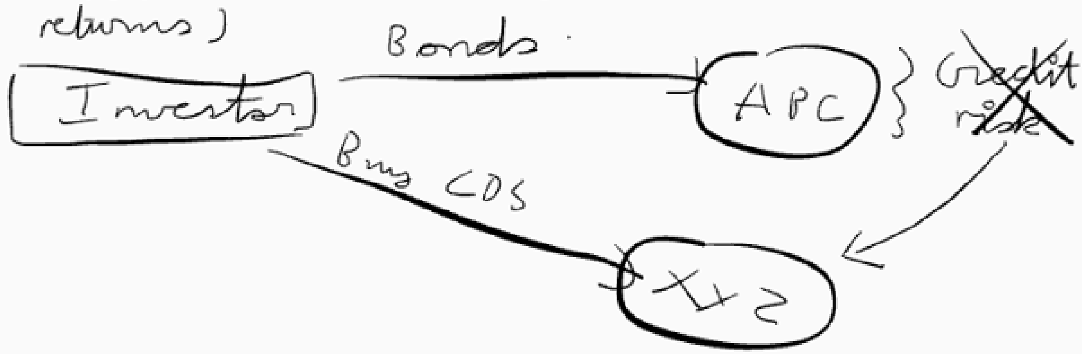
Eg:

Der	MTM
A	100
B	50
C	-80
D	-60
E	20

I owe 140 to the counterparty
I am owed 170 by " "
On a net basis, I am owed 30 by the counterparty
If counterparty defaults / goes bankrupt – I will have to pay 140 first but rank with all other creditors for receivable of 170.

- For exchange traded derivatives, margining virtually settles every contract on a daily basis
- For OTC derivatives, enter into agreements which stipulate collateral requirements
  eg: CSA – Credit Support Annex

3) losses on treasury portfolio
- Thorough investment analysis
- Credit ratings as an indicator
- Buy credit default swap
  (Gives rise to new credit risk & reduced returns)



4) Risk of vendors failing to meet obligations
- Contract to stipulate penalties if either party fails
- Buy cyber insurance to protect against system failure

* Credit risk evaluation

Retail
- Salary / income
- Last 6 months statement
- Area of residence
- Own house / rented
- No. of dependants
- CIBIL score
- Collateral

Corporate
- Past profitability
- Assessment of business
- Management discussions
- Market outlook for industry
- Audit report
- External credit rating
- Collateral

Five C's for measuring credit risk
- Credit history
- capacity to repay
- capital
- collateral
- Conditions of the loan

Credit Risk

Expected loss   Unexpected

Prob. of Default × Loss given Default × Exposure at default = EL

Structural Models   Reduced form models   Intensity based models

* . Structural model

→ Based on cap structure of the company (Amount of debt & equity in the company)

→ Merton model

→ Altman's Z score

* . Reduced form models

→ Uses statistical data to predict prob. of default

→ Market related, macroeconomic, industry related data

* . Intensity based models

→ P(default) : Force of default

→ Consider intensity of company to move to default state

→ Two-state model & Multiple state model

* . Merton Model

→ Black-Scholes options framework

→ considers the availability & value of total assets the company has

F(t) : Underlying asset i.e. assets of the company

D : Value of debt issued (Amt to be repaid)

r : risk-free rate

σ : Volatility of assets

t : Time to maturity for debt

Eg : Issued debt of 5m maturing in 10 y
Equity of 10m

$$\text{Assets} = \text{Liab} + \text{Equity}$$
$$\text{Assets} = 10 + 5 = 15m$$

After 10 years —

Bonds will be repaid if $F(10) > 5m$

" " not " " " " $< 5m$

$$\text{Value of debt} = D \;;\; F(t) \geq D$$
$$= F(t) \;;\; F(t) < D$$

$$\text{Payoffs for bondholders} = \min\{D, F(t)\}$$
$$= D - \max\{D - F(t), 0\}$$

If $F(t) = 3m$,

$$\text{Payoff} = 5 - \max\{5-3, 0\}$$
$$= 5 - 2 = \underline{\underline{3}}$$

If $F(t) = 10m$,

$$\text{Payoff} = 5 - \max\{5-10, 0\} = 5 - 0 = 5$$

$$\text{Payoff} = \underbrace{D}_{\text{Debt to be repaid}} - \underbrace{\max\{D - F(t), 0\}}_{\text{Put option on the assets of the company}}$$

For equity holders —
Hold a call option on company assets with strike = D

$Max\{F(t) - D, 0\}$

If $F(t) = 3$, Payoff = 0

If $F(t) = 10m$, Payoff = 5

<u>Limitations</u>

→ How to derive volatility? Company assets are not tradeable
→ σ is usually not constant
→ Cannot evaluate coupon paying debt
→ Asset values or prices are not continuous

<u>Eg:</u> $F(0) = £15m$

$\sigma = 25\%$ pa

$t = 5$ years

Price of ZCB = £77.88

$$\text{Price of ZCB} = 100e^{-\delta t}$$

$$\therefore 77.88 = 100e^{-5\delta}$$

$$\therefore \delta = \frac{-1}{5}\ln\left(\frac{77.88}{100}\right)$$

$$\therefore \underline{\delta = 5\% \text{ pa}}$$

$\therefore$ Value of debt —

$$= 77.88 - [77.88\,\Phi(-d_2) - 15m\,\Phi(-d_1)]$$

$$d_1 = \frac{\ln(F(0)/k) + (r + \sigma^2/2)t}{\sigma\sqrt{t}}$$

$$= \frac{\ln(15/10) + (0.05 + 0.25^2/2) \times 5}{0.25\sqrt{5}}$$

$$= 1.45204$$

$\therefore d_2 = d_1 - \sigma\sqrt{T} = \underline{0.893023}$

$\therefore \Phi(-d_2) = 0.185922 \quad , \quad \Phi(-d_1) = 0.073245$

$\therefore$ Value of debt —

$= 77.88 - (77.88 \times 0.185922 - 150 \times 0.073245)$

$= \underline{\underline{£74.38715}}$

Credit spread : Extra return earned due to investing in a non-risk free asset

Credit spread = Return on asset − Risk-free return

Price of Govt. bond $= 77.88$

$\therefore$ Risk-free rate $= 5\%$

Price of company's bonds $= 74.387$

$\therefore$ Return on company's bonds

$= -\frac{1}{5} \ln\left(\frac{74.387}{100}\right) = \underline{5.9177\%}$

$\therefore$ Credit spread $= 5.9177 - 5$

$= \boxed{0.9177\%}$

* Junior ZCB $= L_2$

Senior ZCB $= L_1$

$T = 3$

Senior ZCB paid before Junior ZCB

$$\text{Senior ZCB} = \begin{cases} L_1 & ; F(t) \geq L_1 \\ F(t) & ; F(t) < L_1 \end{cases}$$

$$\text{Senior ZCB} = L_1 - \text{Max}\{L_1 - F(t), 0\}$$

$$\text{Equity} = F(t) - L_1 - L_2 \quad ; \quad F(t) \geq (L_1 + L_2)$$
$$= 0 \quad ; \quad F(t) < L_1 + L_2$$
$$\therefore \text{Equity} = \text{Max}\{F(t) - \underbrace{(L_1 + L_2)}_{K}, 0\}$$

$$\text{Junior debt} = 0 \quad ; \quad F(t) \leq L_1$$
$$= L_2 \quad ; \quad F(t) \geq (L_1 + L_2)$$
$$= F(t) - L_1 \quad ; \quad L_1 < F_t < L_1 + L_2$$

$$\text{Junior debt} = \min\{\max\{F(t) - L_1, 0\}, L_2\}$$

$F(0) = £3.2m$
$L_1 = £1.2m$
$L_2 = £2m$
$T = 3$ years
$r = 4\%$ pa
$\sigma = 30\%$ pa

Price of L_1 per £100 nominal $= £88.26$
$= £1.05912m$

Assets = Equity + Debt

$\therefore F(0) = \text{Equity} + \text{Price}(L_1) + \text{Price}(L_2)$

$\therefore 3.2 = \text{Equity} + 1.05912 + \text{Price}(L_2)$

Value of Equity $\Rightarrow$ Call option with $K = L_1 + L_2$
$\Rightarrow$ Call with $K = 1.2 + 2 = 3.2m$

$\therefore$ Price of equity $= F(0)\,\Phi(d_1) - (L_1 + L_2)\,e^{-rt}\,\Phi(d_2)$

$$d_1 = \frac{\ln(3.2/3.2) + (0.04 + 0.3^2/2) \times 3}{0.3\sqrt{3}}$$

$0.3\sqrt{3}$

$= 0.490748$ $\qquad \Phi(d_1) = 0.688198$

$d_2 = -0.02887$ $\qquad \Phi(d_2) = 0.488485$

$E = 3.2 \times 0.688198 - 3.2 \times e^{-0.04 \times 3} \times 0.488485$

$= £0.81584m$

$\therefore \text{Price}(L_2) = F(0) - \text{Price}(L_1) - \text{Price}(E)$

$= 3.2 - 1.05912 - 0.81584$

$= £1.32504m$

$= £66.25198$ per £100 nominal

$\Downarrow$

$\frac{1.32504}{2} \times 100$

* Two - state Model

→ Intensity based model

$P[X(t+dt) = D \mid X(t) = ND] = \lambda dt + o(dt)$

$P[X(t+dt) = ND \mid X(t) = ND] = 1 - \lambda dt + o(dt)$

${}_tP_x = e^{-\int_0^t \mu_s ds} \Longrightarrow {}_tq_x = 1 - e^{-\int_0^t \mu_s ds}$

$\therefore P(ND) = e^{-\int_0^t \lambda_s ds} \Rightarrow P(D) = 1 - e^{-\int_0^t \lambda_s ds}$

Σ intensities $= 0$

Σ prob $= 1$

$\delta \rightarrow$ recovery rate

$1 - \delta \Rightarrow$ Loss given default

P(Bond) = EPV of future payoff

$\therefore P(\text{Bond} @ 0) = e^{-rT}\left[(\text{Payoff } ND) \times P(ND) + (\text{Payoff } D) \times P(D)\right]$

Eg: $\lambda(t) = 0.002t$

5 year ZCB

5 year spot yield $= 5.25\%$

i.) 5-year P(Default)

$= 1 - \exp\left[-\int_0^5 0.002t \, dt\right]$

$= 1 - \exp\left[-\frac{0.002}{2}\left(t^2\right]_0^5\right]$

$= 1 - \exp(-0.001 \times 25)$

$= 1 - e^{-0.025}$

$= \boxed{0.02469}$

ii.) $\delta(t) = 1 - 0.05t$

$\delta(5) = 1 - 0.25 = 0.75$

$\therefore$ Loss given default $= 0.25$

$\therefore P(\text{Bond}) = e^{-0.0525\times 5}\left[0.02469\times 75 + 0.97531\times 100\right]$

$= \boxed{£76.43789}$

8. $\delta_A = 60\%$ $\quad T = 9\text{months}$ $\quad P_A = \$82$

$\delta_B = 50\%$ $\quad r = 1.5\%\text{pa}$ $\quad P_B = \$79$

i) $P(\text{Bond}) = e^{-rT}\left[P(D)\times\delta + P(ND)\times 100\right]$

$\therefore P_A = e^{-0.015\times 0.75}\left[P(D)\times 60 + (1-P(D))\times 100\right]$

$\therefore \dfrac{82}{e^{-0.015\times 0.75}} = 60P(D) + 100 - 100P(D)$

$\therefore 40P(D) = 100 - 82.92771$

$\therefore P(D) = \underline{0.426807}$

$P(D) = 1 - e^{-\int_0^t \lambda_s\, ds}$

$\therefore 1 - e^{-0.75\lambda_A} = 0.426807$

$\therefore e^{-0.75\lambda_A} = 0.573193$

$\therefore \lambda_A = -\dfrac{1}{0.75}\ln(0.573193)$

$\therefore \boxed{\lambda_A = 0.742044}$

For Bond B —

$79 = e^{-0.015\times 0.75}\left[50P(D) + 100 - 100P(D)\right]$

$\therefore \quad 100 - 79.89377 = 50P(D)$

$\therefore \quad P(D) = \underline{0.402125}$

$\therefore \quad 1 - e^{-0.75\lambda_B} = 0.402125$

$\therefore \quad \lambda_B = \frac{-1}{0.75} \ln(1 - 0.402125)$

$\therefore \quad \boxed{\lambda_B = 0.685831}$

ii) Derivative on company bonds which will pay £100,000 if both bonds default within 9 months

Price of derivative = £7,900

$\therefore P(Der) = $ EPV of future payoff

$\therefore \quad 7900 = e^{-0.015 \times 0.75}\left[P(\text{Both } ND) \times 0 + 100000 \times P(DD)\right]$

$\therefore \quad 7900 = e^{-0.015 \times 0.75} \times 100{,}000 \times P(DD)$

$\therefore \quad \underline{P(DD) = 0.079894}$

$\therefore \quad 1 - e^{-0.75\lambda_{DD}} = 0.079894$

$\therefore \quad \boxed{\lambda_{DD} = 0.111022}$

iii) Max price of the derivative by considering max. possible double default rate

$\Rightarrow$ Max price will arise if there is perfect correlation between default intensities i.e. λ_A & λ_B are perfectly correlated & $\rho_{AB} = 1$

$P(Der) = e^{-0.015 \times 0.75}\left[100{,}000 \times \left[1 - e^{-0.75\lambda}\right]\right]$

$\lambda_A = 0.742044$ & $\lambda_B = 0.685831$

$\Rightarrow \lambda_B = 0.685831$

Lower the λ, higher is $(1 - e^{-0.75\lambda})$

$\therefore$ Select $\min(\lambda_A, \lambda_B) = \lambda_{DD}$

$\therefore \lambda_{DD} = \min(\lambda_A, \lambda_B) = \lambda_B = 0.685831$

$\therefore P(D_{er}) = e^{-0.015 \times 0.75} \times 100000 \times (1 - e^{-0.75 \times 0.685831})$

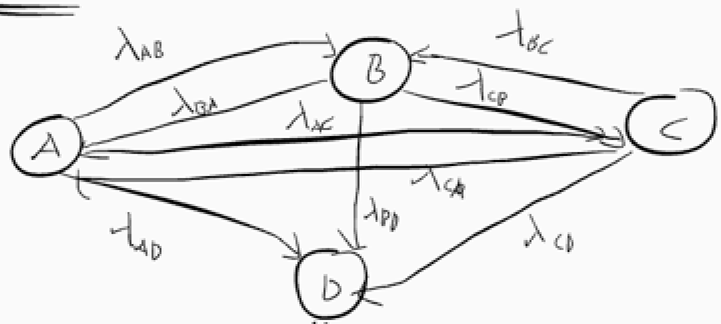
$= \boxed{£39{,}465.5}$

* <u>Multiple state model</u>

→ Based on credit rating grades

→ One type is Jarrow - Lando - Turnbull (JLT) model

JLT



⇑ Absorbing state

$$P[X(t+dt) = j \mid X(t) = i] = \lambda_{ij} + o(dt)$$

$$P[X(t+dt) = i \mid X(t) = i] = 1 - \sum_{i \neq j} \lambda_{ij} + o(dt)$$

$$\therefore P(\text{moving out}) = 1 - \exp\left[-\int_0^t \lambda_s \, ds\right]$$

Eg: 3 - State credit model with

H ⇓ Healthy, U ⇓ Unhealthy, D ⇓ Defaulted

$P_{HU} = 0.1$

$P_{UH} = 0.05$

$P_{HD} = 0.02$

$P_{UD} = 0.3$

$P_{Dj} = 0$

$$\begin{array}{c} \\ H \\ U \\ D \end{array} \begin{array}{ccc} H & U & D \end{array}$$

$$\begin{array}{c} H \\ U \\ D \end{array} \begin{bmatrix} \underline{\underline{0.88}} & 0.1 & 0.02 \\ 0.05 & \underline{\underline{0.65}} & 0.3 \\ 0 & 0 & 1 \end{bmatrix}$$

i.) Company in healthy state at time 0
Calculate company is in default state at time 2.

$P_D(2) = P_{HU} \times P_{UD} + P_{HH} \times P_{HD} + P_{HD} \times P_{DD}$

$= 0.1 \times 0.3 + 0.88 \times 0.02 + 0.02 \times 1$

$= 0.03 + 0.0176 + 0.02$

$= \boxed{0.0676}$

ii.) Company issues 2 Year ZCB

$r = 4\%$ pa

Payoff $= 100$; ND

$= x\%$ of 100 ; D

$\therefore$ Payoff $= 100$; ND

$= 100x$; D

Price of bond $= £87.63$

$\therefore$ P(Bond) = EPV of future payoff

$\therefore\ 87.63 = e^{-0.04\times 2}\left[100x . P(D) + 100 P(ND)\right]$

$\therefore\ 87.63 = e^{-0.08}\left[100x \times 0.0676 + 100(1-0.0676)\right]$

$\therefore\ 87.63e^{0.08} = 6.76x + 93.24$

$\therefore\ 1.688446 = 6.76x$

$\therefore\ \boxed{x = 24.98\%}$

iii) Calculate credit spread

$87.63 = 100e^{-2\delta}$

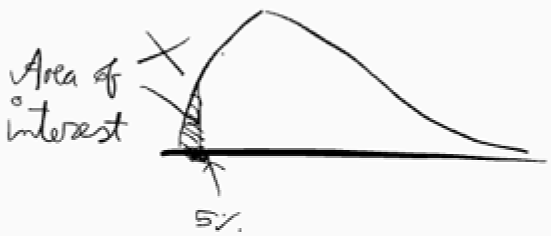
$\therefore\ \delta = -0.5\ln(0.8763)$

$\therefore\ \delta = 6.6023\%$

$r = 4\%$

$\therefore$ Credit spread $= 6.6023 - 4 = \boxed{2.6023\%}$

* Value at Risk



→ Losses at a certain confidence level over a given time period
→ Measure of underperformance

Limitations

→ Provides no indication of likely losses if VaR is breached
→

Variance - covariance Approach
⇓
Assumes normal distribution

Historical Simulation
⇓
Involves looking at historical movements

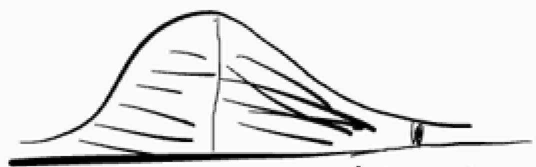
Monte Carlo
⇓
Specify risk factors & constraints
R generate simulations

* Unexpected loss
⇓
Expected loss

= PD × LGD × EAD

→ Need for determination of economic capital

EC = Capital to be held to cover risks being taken



Min (Expected losses provisions − Unexpected losses 5%, 0) = EC requirement

Basel Unexpected loss

→ PD
→ LGD
→ σ_{LGD} } ⇐ New parameters
→ σ_{PD}
→ EAD
→ Consider volatilities & correlations between PD & LGD parameters

*. Risk-adjusted return on risk-adjusted capital

RAROC ~ RORAC

RAROC

$$= \frac{\text{Revenue} - \text{expenses} - \text{expected losses}}{\text{Capital}}$$

RORAC

$$= \frac{\text{Revenue} - \text{Expenses}}{\text{Risk-adjusted capital (Regulatory capital)}}$$

RARORAC

$$= \frac{\text{Revenue} - \text{Expenses} - \text{expected losses}}{\text{Risk-adjusted capital}}$$

* Prepayment forecasting methodologies

→ If interest rates fall, prepayments tend to rise

→ Leads to lower interest income & compulsion to invest at lower rates of return

Expected interest rates

Market or economy factors

Individual characteristics

* Constant prepayment rate model

CPR = 5%

Interest rate = 10%

← 95

0 1 2

100 15 15

5 10 5.5 9.5

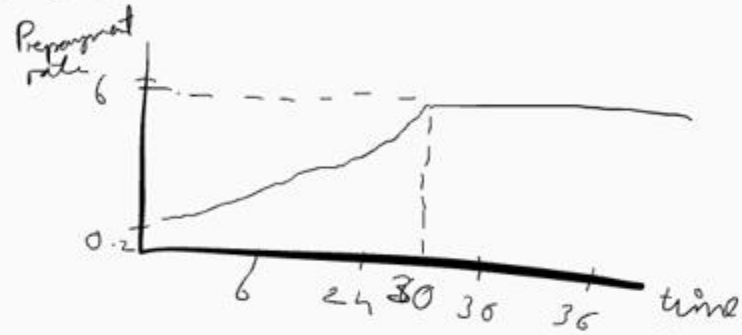
POS at start	Contractual EMI	Actual EMI
100	15	20

∴ Excess payment = 20 − 15 = 5

∴ Prepayment rate = $\frac{5}{100}$ = $\boxed{5\%}$

* PSA Model = $\frac{\text{Prepayment}}{\text{LOS @ start of period}}$

⇒ Public Securities Association model



0.2% increase in prepayment till 30 months ⇒ 100% PSA model

150% PSA model

⇒ 0.2 × 150% = 0.3% in prepayment till 30 months

Plateau @ 0.3 × 30 = 9%