INSTITUTE OF ACTUARIAL & QUANTITATIVE STUDIES

Class: SY BSc

Subject : Introduction to Derivatives and Financial Markets

Chapter: Unit 3 Chapter 2

Chapter Name: Hedging Strategies using Futures

Precap of Previous Chapter

- Futures are standardized, exchange-traded contract to buy and sell an asset at a future time for a certain price.
- A futures contract has various specifications which include the asset, the contract size, delivery place, delivery month and price limits.
- Margins are good faith deposit made with exchange while striking a contract. There is Initial, maintenance and variation margins involved.
- Clearing house ensures the performance of each parties.

Today's Agenda

1. Introduction to Hedging using Futures
    1. Perfect Hedge
    2. Dynamic and Static Hedge
    3. Short Hedge
    4. Long Hedge

2. Arguments For & Against Hedging

1. Understanding Futures Payoff
    1. Payoff for Long position
    2. Payoff for Short position

2. Parameters to consider before a Hedge

5. Basis
    1. Example – Strengthening and Weakening
    2. Basis Risk
    3. Basis Risk – Example
    4. Effective price for the Asset
    5. Choice of a Contract

6. Cross Hedge

5. Hedge Ratio

5. Minimum Variance Hedge Ratio

5. Optimal Number of Contracts

Today's Agenda

1 Introduction to Hedging using Futures

Many of the participants in futures markets are hedgers. Their aim is to use futures markets to reduce a particular risk that they face.

What type of Risk?
This risk might relate to fluctuations in the price of oil, a foreign exchange rate, the level of the stock market, or some other variable.

There could be interest-rate risk, currency risk, credit risk, counter-party risk, etc to which the investor will have an exposure. Futures could be used to hedge such exposures.

1.1 Perfect Hedge

A perfect hedge is one that completely eliminates the risk. Perfect hedges are rare.

For the most part, therefore, a study of hedging using futures contracts is a study of the ways in which hedges can be constructed so that they perform as close to perfect as possible.

What parameters of your asset holding do you need to match with the underlying of the futures contract to have a perfect or close to perfect hedge?

1.2 Dynamic and Static Hedge

Static Hedge

A static hedge is one that does not need to be re-balanced as the price of other characteristics (such as volatility) of the securities it hedges change.

A simple example of a static hedge is a future that is used to hedge a position in a foreign currency.

Dynamic Hedge

A dynamic hedge is one that requires constant re-balancing based on changes in different parameters.

Delta Hedging can be an example of dynamic hedging.

Here we restrict our attention to what might be termed hedge-and-forget strategies.

We assume that no attempt is made to adjust the hedge once it has been put in place.

Question

An investor holds shares in TCS worth Rs. 1,50,000. He feels that in the coming two months time the price of the share will see a fall. As a result, the investor wishes to hedge his exposure to this downfall.

What position should the investor take in a futures contract with a suitable underlying?

What will be the effect of the hedge?

1.3 Short Hedge

A short hedge is appropriate when the hedger already owns an asset and expects to sell it at some time in the future.

A short hedge can also be used when an asset is not owned right now but will be owned at some time in the future.

For example, a short hedge could be used by a farmer who owns some hogs and knows that they will be ready for sale at the local market in two months.

The previous question is also an example of a short hedge.

1.4 Long Hedge

Hedges that involve taking a long position in a futures contract are known as long hedges.

A long hedge is appropriate when a company knows it will have to purchase a certain asset in the future and wants to lock in a price now

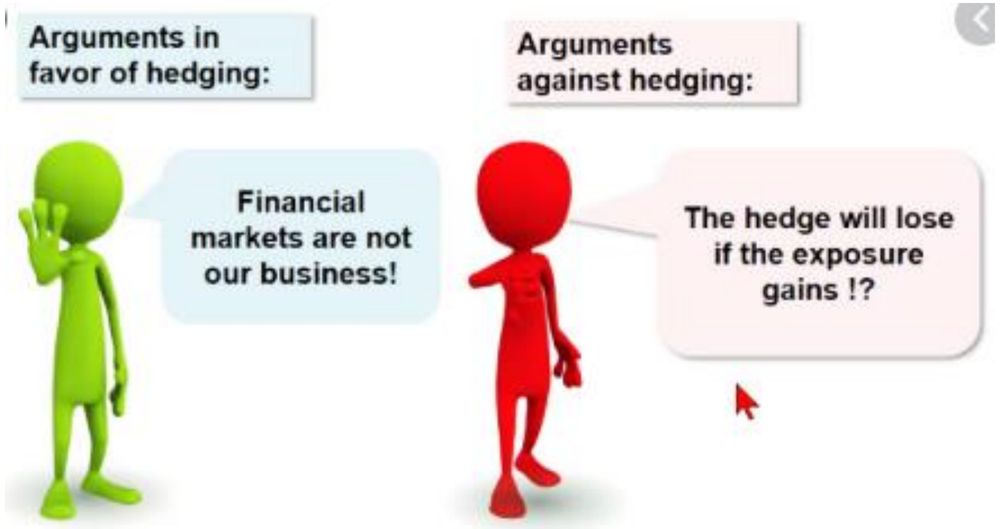
For example, suppose that it is now January 15. A copper fabricator knows it will require 100,000 pounds of copper on May 15 to meet a certain contract. The spot price of copper is 340 cents per pound, and the futures price for May delivery is 320 cents per pound. The fabricator can hedge its position by taking a long position in four futures contracts offered by the COMEX division of the CME Group and closing its position on May 15.

2 Arguments for Hedging

- Most companies are in the business of manufacturing, or retailing or wholesaling, or providing a service. They have no particular skills or expertise in predicting variables such as interest rates, exchange rates, and commodity prices.

- It makes sense for them to hedge the risks associated with these variables as they become aware of them. The companies can then focus on their main activities—for which presumably they do have particular skills and expertise.

- By hedging, they avoid unpleasant surprises such as sharp rises in the price of a commodity that is being purchased.



2 Arguments against Hedging

- **Hedging and Shareholders** – One argument sometimes put forward is that the shareholders can, if they wish, do the hedging themselves. They do not need the company to do it for them. A shareholder with a well-diversified portfolio may be immune to many of the risks faced by a corporation.

- **Hedging and Competitors –** If hedging is not the norm in a certain industry, it may not make sense for one particular company to choose to be different from all others.

- **Hedging can lead to a Worse Outcome** – It is difficult to explain situation where there is a loss on the hedge or there is a gain on the underlying. Hence it is important to realize that a hedge using futures contracts can result in a decrease or an increase in a company's profits relative to the position it would be in with no hedging

3 Understanding Forward/Futures Payoffs

Before we move on to the understanding of different hedging strategies, we consider the payoff from a forward or a futures contract.

We will look through a numerical example and the payoff graph for:

a. Long position
b. Short position

Example

We will consider the following numerical illustration

Forward contracts can be used to hedge foreign currency risk. Suppose that, on May 24, 2010, the treasurer of a US corporation knows that the corporation will pay £1 million in 6 months (i.e., on November 24, 2010) and wants to hedge against exchange rate moves. Using the quotes in Table 1.1, the treasurer can agree to buy £1 million 6 months forward at an exchange rate of 1.4422. The corporation then has a long forward contract on GBP. It has agreed that on November 24, 2010, it will buy £1 million from the bank for $1.4422 million. The bank has a short forward contract on GBP. It has agreed that on November 24, 2010, it will sell £1 million for $1.4422 million. Both sides have made a binding commitment.

Table 1.1 Spot and forward quotes for the USD/GBP exchange rate, May 24, 2010 (GBP = British pound; USD = US dollar; quote is number of USD per GBP).

	Bid	*Offer*
Spot	1.4407	1.4411
1-month forward	1.4408	1.4413
3-month forward	1.4410	1.4415
6-month forward	1.4416	1.4422

3.1 Long Position

Consider the position of the corporation in the trade we have just described.
What are the possible outcomes?

The forward contract obligates the corporation to buy £1 million for $1,442,200.

- **Increase ↑**

If the spot exchange rate rose to, say, 1.5000, at the end of the 6 months, the forward contract would be worth $57,800 (= $1,500,000 –$1,442,200) to the corporation. It would enable £1 million to be purchased at an exchange rate of 1.4422 rather than 1.5000.

- **Decrease ↓**

Similarly, if the spot exchange rate fell to 1.3500 at the end of the 6 months, the forward contract would have a negative value to the corporation of $92,200 (= $1,350,000 – $1,442,200) because it would lead to the corporation paying $92,200 more than the market price for the sterling.

3.1 Long Position

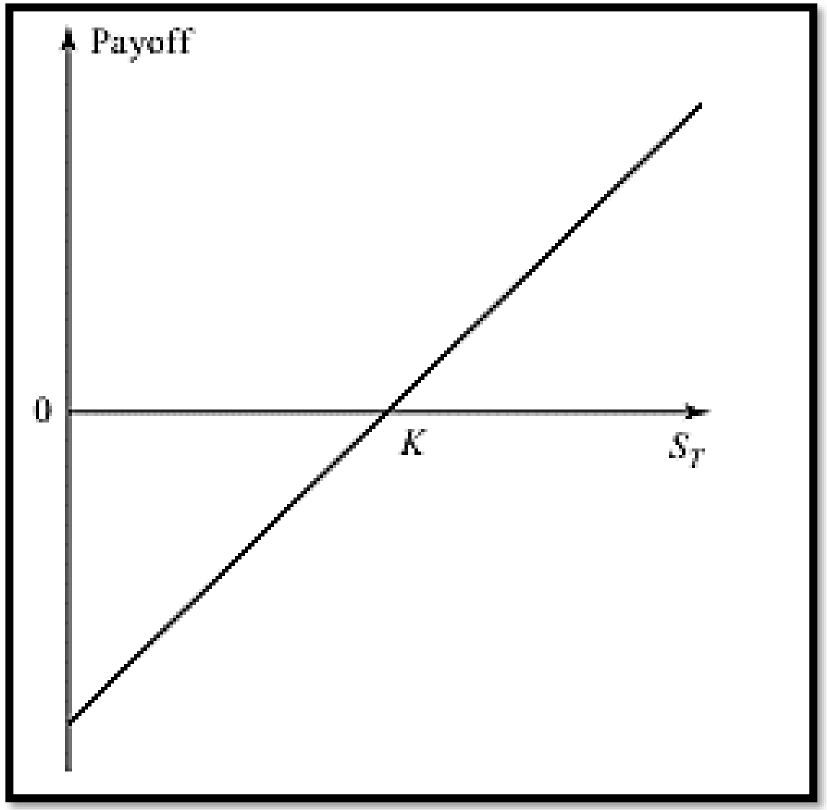
In general, the payoff from a long position in a forward contract on one unit of an asset is

$$S_T - K$$

where K is the delivery price and S_T is the spot price of the asset at maturity of the contract.

This is because the holder of the contract is obligated to buy an asset worth S_T for K.

The graph for a long forward position is as follows:-



3.2 Short Position

Consider the position of the bank in the trade we have just described.
What are the possible outcomes?

The forward contract obligates the Bank will sell £1 million for $1,442,200.

- **Increase ↑**

If the spot exchange rate rose to, say, 1.5000, at the end of the 6 months, the forward contract would be worth negative value of $57,800 (= $1,442,200 – $1,500,000) to the bank.

- **Decrease ↓**

Similarly, if the spot exchange rate fell to 1.3500 at the end of the 6 months, the forward contract would have a value of $92,200 (= $1,442,200 – $1,350,000) because it would lead to the bank receiving $92,200 more than the market price for the sterling.

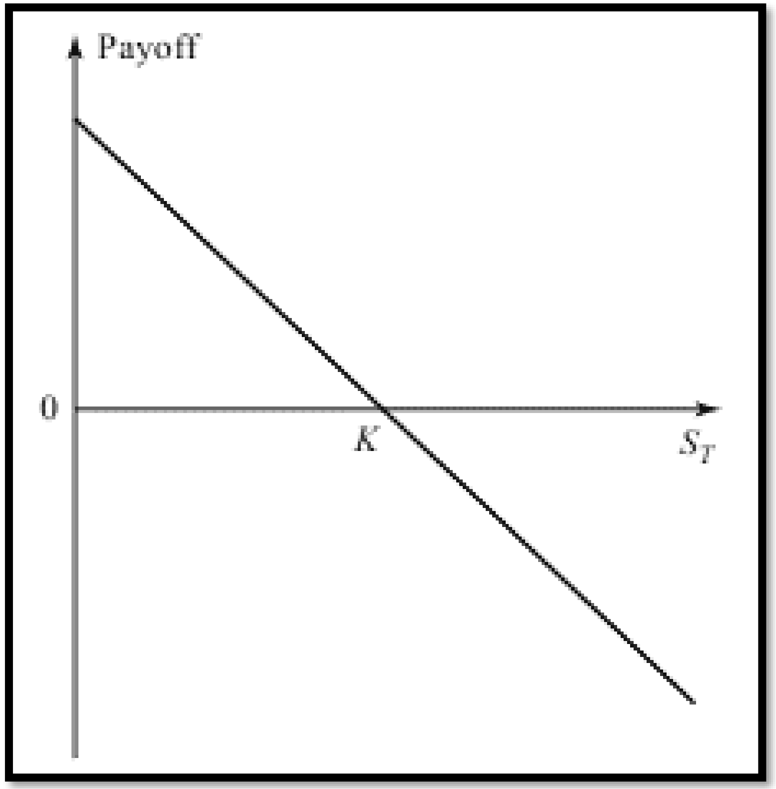
3.2 Short Position

In general, the payoff from a short position in a forward contract on one unit of an asset is

$$K - S_T$$

where K is the delivery price and S_T is the spot price of the asset at maturity of the contract.

The graph for a short forward position is as follows:-



4 Parameters to Consider before a Hedge

Hedged Item

- The asset/item already existing in the portfolio which has an exposure to risk.
- It is hedged using certain instrument.

Hedging Instrument

- The derivative instrument used to make a hedge.
- Includes Futures, Options, Customized Forwards etc.

What parameters do we need to match between the two to make a hedge?

1. The Asset
2. Contract Size
3. Maturity dates etc.

4 Parameters to Consider before a Hedge

Discuss the consequences of mismatch in various parameters we discussed before.

5 Basis Risk

Basis is the difference between spot and futures.

Basis = Spot price of asset to be hedged – Futures price of contract used

Basis risk arises because of the uncertainty about the basis when the hedge is closed out.

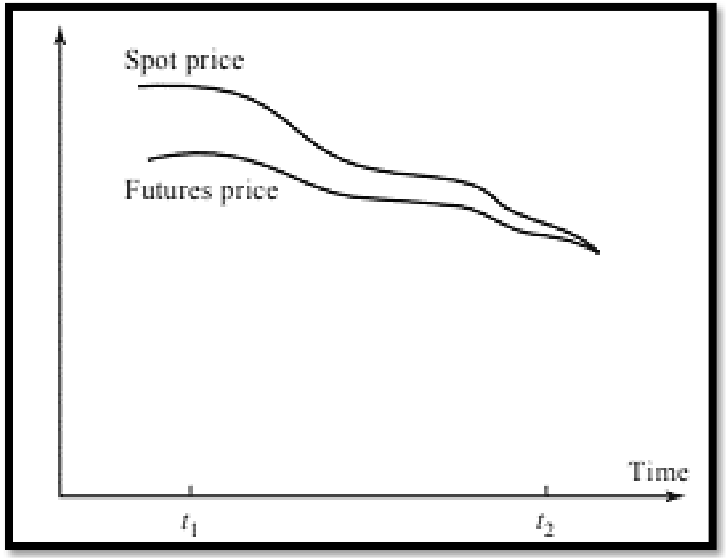
As time passes, the spot price and the futures price for a particular month do not necessarily change by the same amount. As a result, the basis changes.

An increase in the basis is referred to as a strengthening of the basis; a decrease in the basis is referred to as a weakening of the basis.

5 Basis

The basis in a hedging situation is as follows:
Basis = Spot price of asset to be hedged – Futures price of contract used

- As time passes, the spot price and the futures price for a particular month do not necessarily change by the same amount.
- As a result, the basis changes. An increase in the basis is referred to as a strengthening of the basis; a decrease in the basis is referred to as a weakening of the basis.
- Figure beside illustrates how a basis might change over time in a situation where the basis is positive prior to expiration of the futures contract.



5.1 Basis - Example

Strengthening and Weakening of the basis

As an example, we will consider the case where the spot and futures prices at the time the hedge is initiated are $3.00 and $2.95, respectively. Over the time the spot price and futures price are as follows:

Time	Spot Price	Futures price	Basis
0	3.00	2.95	3.00-2.95 = 0.05
1	2.90	2.80	2.90-2.80 = 0.10
2	2.86	2.75	2.85-2.75 = 0.11
3	2.70	2.69	2.70-2.69 = 0.01
4	2.65	2.65	0

Strengthening of the basis

Weakening of the basis

5.2 Basis Risk

Basis risk arises because of the uncertainty about the basis when the hedge is closed out.

What do you think are the factors the affect basis risk.? What should you take care off while choosing a hedge contract?

5.3 Basis Risk - Example

Suppose a rice farmer wants to hedge against possible price fluctuations in the market. For example, in December, he decides to enter into a short-sell position in a futures contract in order to limit his exposure to a possible decline in the cash price prior to the time when he will sell his crop in the cash market.

Assume that:
Spot price of rice = $50
Futures price for a March futures contract = $55

Then: Basis = ?

Ans: The basis, then, is $5.00 (the futures price minus the spot price).

Now let's look at the scenario few months later.....

5.3 Basis Risk - Example

Basis Narrowing

Suppose the farmer decides to lift the hedge in February, due to falling prices.

At the time he decides to close out his market positions, the spot price is $47 and the March futures price is $49. He sells his rice crop at $47 per unit and lifts his hedge by buying futures to close out his short sell position at $49. In this case, his $3 per unit loss in the cash market is more than offset by his $6 gain from short selling futures ($55 – $49).

Therefore, his net sales revenue becomes $53 ($47 cash price + $6 futures profit).

The farmer has enjoyed extra profits as a result of the basis narrowing from $5 to $2.

5.3 Basis Risk - Example

Basis Constant

If the basis remained constant, then the farmer would not gain any extra profit, nor incur any additional loss.

His $3.00 profit in futures would have exactly offset the $3.00 loss in the cash market. It's important to note, however, that while his short sell hedge in futures didn't generate any additional profit, it did successfully protect him from the price decline in the cash market.

If he had not taken the futures position, then he would have suffered a $3.00 per unit loss.

5.3 Basis Risk - Example

Basis Widening

Suppose the cash market price declined while the futures price increased. Suppose when the farmer closed out his short sell futures hedge, the cash price was $47 but the futures price was $57. Then he would have lost $3.00 per unit in the cash market and lost an additional $2.00 in his short futures trade ($57 – $55).

His net sales revenue would be only $45 per unit. Why the extra loss? Because in this instance the basis widened, as opposed to narrowing or remaining constant. It was the opposite of the basis pattern the farmer was looking for to successfully hedge his cash crop.

In this case, the farmer took the basis risk and lost.

5.4 Effective Price for the asset

We will assume that a hedge is put in place at time t_1 and closed out at time t_2. As an example, we will consider the case where the spot and futures prices at the time the hedge is initiated are \$2.50 and \$2.20, respectively, and that at the time the hedge is closed out they are \$2.00 and \$1.90, respectively. This means that S_1 = 2:50, F_1 = 2:20, S_2 = 2:00, and F_2 = 1:90.

From the definition of the basis, we have

$$b_1 = S_1 - F_1 \text{ and } b_2 = S_2 - F_2$$

so that, in our example, b_1 = 0:30 and b_2 = 0:10.

Consider first the situation of a hedger who knows that the asset will be sold at time t_2 and takes a short futures position at time t_1. The price realized for the asset is S_2 and the profit on the futures position is $F_1 - F_2$.

The effective price that is obtained for the asset with hedging is therefore

$$\boldsymbol{S_2 + F_1 - F_2 = F_1 + b_2}$$

In our example, this is \$2.30.
The value of F_1 is known at time t_1. If b_2 were also known at this time, a perfect hedge would result. The hedging risk is the uncertainty associated with b_2 and is known as basis risk.

5.5 Choice of a Contract

One key factor affecting basis risk is the choice of the futures contract to be used for hedging. This choice has two components:
1. The choice of the asset underlying the futures contract
2. The choice of the delivery month

- **For the asset** – If the asset being hedged exactly matches an asset underlying a futures contract, the choice is easy. In other circumstances, carry out a careful analysis to determine which of the available futures contracts has futures prices that are most closely correlated with the price of the asset being hedged.

- **Delivery month** – A good rule of thumb is therefore to choose a delivery month that is as close as possible to, but later than, the expiration of the hedge

Question

It is June 8 and a company knows that it will need to purchase 20,000 barrels of crude oil at some time in October or November. Oil futures contracts are currently traded for delivery every month on the NYMEX division of the CME Group and the contract size is 1,000 barrels. The company therefore decides to use the December contract for hedging and takes a long position in 20 December contracts.

The futures price on June 8 is $68.00 per barrel. The company finds that it is ready to purchase the crude oil on November 10. It therefore closes out its futures contract on that date. The spot price and futures price on November 10 are $70.00 per barrel and $69.10 per barrel.

Find the effective price paid by the company.

Solution

The gain on the futures contract is $69.10 - 68.00 = \$1.10$ per barrel. The basis when the contract is closed out is $70.00 - 69.10 = \$0.90$ per barrel. The effective price paid (in dollars per barrel) is the final spot price less the gain on the futures, or

$$70.00 - 1.10 = 68.90$$

This can also be calculated as the initial futures price plus the final basis,

$$68.00 + 0.90 = 68.90$$

The total price paid is $68.90 \times 20{,}000 = \$1{,}378{,}000$.

Question

It is March 1. A Indian company expects to receive 50 million Japanese yen at the end of July. Yen futures contracts on the NSE have delivery months of March, June, September, and December. One contract is for the delivery of 12.5 million yen. The company therefore shorts four September yen futures contracts on March 1.

When the yen are received at the end of July, the company closes out its position. We suppose that the futures price on March 1 in cents per yen is 0.7800 and that the spot and futures prices when the contract is closed out are 0.7200 and 0.7250, respectively.

Find the effective price obtained and the total amount received by the Indian company.

Solution

The gain on the futures contract is 0.7800 – 0.7250 = 0.0550 cents per yen. The basis is 0.7200 – 0.7250 = –0.0050 cents per yen when the contract is closed out. The effective price obtained in cents per yen is the final spot price plus the gain on the futures:

$$0.7200 + 0.0550 = 0.7750$$

This can also be written as the initial futures price plus the final basis:

$$0.7800 + (-0.0050) = 0.7750$$

The total amount received by the company for the 50 million yen is 50×0.00775 million dollars, or $387,500.

6 Cross Hedge

Cross hedging occurs when the two assets are different, i.e. the hedged item and the asset underlying derivatives is different.

For example, an airline that is concerned about the future price of jet fuel. Because jet fuel futures are not actively traded, it might choose to use heating oil futures contracts to hedge its exposure. This is a cross hedge.

Points to Note in case of a Cross Hedge

1. The changes in the prices of the two asset should be as highly correlated as possible.
2. The delivery month should be the same as, or just after, the date the hedge will be lifted.

Discuss why the two assets in a cross hedge should have high correlation.

7 Hedge Ratio

Hedge Ratio – The hedge ratio is the ratio of the size of the position taken in futures contracts to the size of the exposure.

When the asset underlying the futures contract is the same as the asset being hedged, it is natural to use a hedge ratio of 1.0.
When cross hedging is used, setting the hedge ratio equal to 1.0 is not always optimal. The hedger should choose a value for the hedge ratio that minimizes the variance of the value of the hedged position.

We now consider how the hedger can do this.

$$\text{Hedge Ratio} = \frac{\text{Value of the Hedge Position}}{\text{Value of the Total Exposure}}$$

8 Minimum Variance Hedge Ratio

The minimum variance hedge ratio depends on the relationship between changes in the spot price and changes in the futures price.

Define:
ΔS: Change in spot price, S, during a period of time equal to the life of the hedge
ΔF: Change in futures price, F, during a period of time equal to the life of the hedge.
We will denote the minimum variance hedge ratio by h^*.

The formula for h^* is:

$$h^* = \rho \frac{\sigma_S}{\sigma_F}$$

where σ_S is the standard deviation of ΔS, σ_F is the standard deviation of ΔF, and ρ is the coefficient of correlation between the two.

Question

An airline expects to purchase 2 million gallons of jet fuel in 1 month and decides to use heating oil futures for hedging.

We suppose that Table 3.2 gives, for 15 successive months, data on the change, S, in the jet fuel price per gallon and the corresponding change, F, in the futures price for the contract on heating oil that would be used for hedging price changes during the month. Each heating oil contract traded on NYMEX is on 42,000 gallons of heating oil.

Calculate the Minimum Variance Hedge Ratio.

Table 3.2 Data to calculate minimum variance hedge ratio when heating oil futures contract is used to hedge purchase of jet fuel.

Month i	*Change in heating oil futures price per gallon* $(= \Delta F)$	*Change in jet fuel price per gallon* $(= \Delta S)$
1	0.021	0.029
2	0.035	0.020
3	−0.046	−0.044
4	0.001	0.008
5	0.044	0.026
6	−0.029	−0.019
7	−0.026	−0.010
8	−0.029	−0.007
9	0.048	0.043
10	−0.006	0.011
11	−0.036	−0.036
12	−0.011	−0.018
13	0.019	0.009
14	−0.027	−0.032
15	0.029	0.023

Solution

In this case, the usual formulas for calculating standard deviations and correlations give σ_F = 0:0313, σ_S = 0:0263, and ρ = 0:928

The formula for h^* is:

$$h^* = \rho \frac{\sigma_S}{\sigma_F}$$

Therefore;

$$0.928 \times \frac{0.0263}{0.0313} = 0.7777$$

9 Optimal Number of Contracts

To calculate the number of contracts that should be used in hedging,
Define:
Q_A: Size of position being hedged (units)
Q_F: Size of one futures contract (units)
N^* : Optimal number of futures contracts for hedging.

The futures contracts should be on $h^* Q_A$ units of the asset.

The number of futures contracts required is therefore given by:

$$N^* = \frac{h^* Q_A}{Q_F}$$

Question

An airline expects to purchase 2 million gallons of jet fuel in 1 month and decides to use heating oil futures for hedging.

We suppose that Table 3.2 gives, for 15 successive months, data on the change, S, in the jet fuel price per gallon and the corresponding change, F, in the futures price for the contract on heating oil that would be used for hedging price changes during the month. Each heating oil contract traded on NYMEX is on 42,000 gallons of heating oil.

Calculate the Minimum Variance Hedge Ratio.
Now Calculate the Optimal number of contracts the company should use for hedge.

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5	0.044	0.026
6	−0.029	−0.019
7	−0.026	−0.010
8	−0.029	−0.007
9	0.048	0.043
10	−0.006	0.011
11	−0.036	−0.036
12	−0.011	−0.018
13	0.019	0.009
14	−0.027	−0.032
15	0.029	0.023

Solution

In this case, the usual formulas for calculating standard deviations and correlations give σ_F = 0:0313, σ_S = 0:0263, and ρ = 0:928

The formula for h^* is:

$$h^* = \rho \frac{\sigma_S}{\sigma_F}$$

Therefore;

$$0.928 \times \frac{0.0263}{0.0313} = 0.7777$$

Each heating oil contract traded on NYMEX is on 42,000 gallons of heating oil. Therefore the optimal number of contracts is

$$\frac{0.7777 \times 2{,}000{,}000}{42{,}000} = 37.03$$

or, rounding to the nearest whole number, 37.

9 Tailing the Hedge

When futures are used for hedging, a small adjustment, known as tailing the hedge, can be made to allow for the impact of daily settlement.

In practice this means that the N^* now becomes:

$$N^* = \frac{h^* V_A}{V_F}$$

where V_A is the dollar value of the position being hedged and V_F is the dollar value of one futures contract (the futures price times Q_F).
i.e.
$V_A = Q_A$ x spot price
$V_F = Q_F$ x Futures price

Question

An airline expects to purchase 2 million gallons of jet fuel in 1 month and decides to use heating oil futures for hedging.

We suppose that Table 3.2 gives, for 15 successive months, data on the change, S, in the jet fuel price per gallon and the corresponding change, F, in the futures price for the contract on heating oil that would be used for hedging price changes during the month. Each heating oil contract traded on NYMEX is on 42,000 gallons of heating oil.

Calculate the Minimum Variance Hedge Ratio.
Now Calculate the Optimal number of contracts the company should use for hedge.

Suppose that now the spot price and the futures price are 1.94 and 1.99 dollars per gallon, respectively.
Calculate the optimal number of contracts by tailing the hedge.

Solution

Suppose that the spot price and the futures price are 1.94 and 1.99 dollars per gallon, respectively. Then V_A = 2,000,000 x 1:94 = 3,880,0000 while V_F = 42,000 x 1:99 = 83,580, so that the optimal number of contracts is:

$$\frac{0.7777 \times 3,880,000}{83,580} = 36.10$$

If we round this to the nearest whole number, the optimal number of contracts is now 36 rather than 37.

Theoretically, the futures position used for hedging should then be adjusted as the spot price and futures price change, but in practice this usually makes very little difference.

10 Stock Index

A stock index tracks changes in the value of a hypothetical portfolio of stocks. The weight of a stock in the portfolio at a particular time equals the proportion of the hypothetical portfolio invested in the stock at that time.

- The percentage increase in the stock index over a small interval of time is set equal to the percentage increase in the value of the hypothetical portfolio.
- If the hypothetical portfolio of stocks remains fixed, the weights assigned to individual stocks in the portfolio do not remain fixed. When the price of one particular stock in the portfolio rises more sharply than others, more weight is automatically given to that stock.

10.1 Stock Indices - Example

The Standard & Poor's 500 (S&P 500) Index is based on a portfolio of 500 different stocks: 400 industrials, 40 utilities, 20 transportation companies, and 40 financial institutions. The weights of the stocks in the portfolio at any given time are proportional to their market capitalizations. The stocks are those of large publicly held companies that trade on NYSE Euronext or Nasdaq OMX. The CME Group trades two futures contracts on the S&P 500. The one shown is on $250 times the index; the other (the Mini S&P 500 contract) is on $50 times the index.

In India, the first stock futures contracts were index-based, which were introduced in the year 2000. These futures are traded on stock exchanges like the Bombay Stock Exchange (BSE) and the National Stock Exchange (NSE). They're available for the BSE Sensex and the NSE Nifty 100. Other indices include:

Nifty 50: 50 underlying securities make up the NSE's Nifty index.

S&P BSE Bankex: These futures consist of banking stocks listed on the Sensex.

S&P BSE Bharat 22 Index: This index is made up of 22 central public sector enterprises.

10.1 Stock Indices - Example

Real-Time Stock Index Futures

Name	Month	Last	High	Low	Chg	Chg%	Time
Nifty 50	Jun 21	15,670.00	15,817.50	15,606.00	-98.60	-0.63%	17:43:04
Bank NIFTY	Jun 21	34,935.00	35,547.60	34,757.65	-330.30	-0.94%	15:30:00
US 30	Jun 21	34,580.5	34,616.0	34,518.0	-5.5	-0.02%	17:43:06
S&P 500	Jun 21	4,231.62	4,232.62	4,222.62	+5.87	+0.14%	17:43:01
Nasdaq	Jun 21	13,853.75	13,863.75	13,804.38	+42.25	+0.31%	17:43:02
Russel 2000	Jun 21	2,348.80	2,349.10	2,339.15	+7.10	+0.30%	17:43:05
S&P 500 VIX	Jun 21	17.77	18.05	17.70	-0.13	-0.73%	17:42:43

Check – Price and Performance of Stock Index Futures
https://in.investing.com/indices/indices-futures

11 Hedging an Equity Portfolio

Stock index futures can be used to hedge a well-diversified equity portfolio.

Define:
V_A: Current value of the portfolio
V_F : Current value of one futures contract (the futures price times the contract size).

If the portfolio mirrors the index, the optimal hedge ratio, equals 1.0 and the number of futures contracts that should be shorted is

$$N^* = \frac{V_A}{V_F}$$

Suppose, for example, that a portfolio worth $5,050,000 mirrors the S&P 500. The index futures price is 1,010 and each futures contract is on $250 times the index. In this case V_A = 5,050,000 and V_F = 1,010 x 250 = 252,500, so that 20 contracts should be shorted to hedge the portfolio.

11 Hedging an Equity Portfolio

What if the index does not replicate or mirror the portfolio?

When the portfolio does not exactly mirror the index, we can use the capital asset pricing model.

The parameter beta (β) from the capital asset pricing model is the slope of the best-fit line obtained when excess return on the portfolio over the risk-free rate is regressed against the excess return of the index over the risk-free rate.

11.1 Capital Asset Pricing Model

The capital asset pricing model (CAPM) is a model that can be used to calculate the expected return from an asset during a period in terms of the risk of the return.
The risk in the return from an asset is divided into two parts:

A. Systematic risk is risk related to the return from the market as a whole and cannot be diversified away.
B. Non-systematic risk is risk that is unique to the asset and can be diversified away by choosing a large portfolio of different assets.

CAPM argues that the return should depend only on systematic risk.

The CAPM formula is:

$$\text{Expected return on asset} = R_F + \beta(R_M - R_F)$$

where R_M is the return on the portfolio of all available investments, R_F is the return on a risk-free investment, and β (the Greek letter beta) is a parameter measuring systematic risk. It can be calculated as the weighted average of the betas of the stocks in the portfolio.

11.2 Inferences about Beta

- When β = 1:0, the return on the portfolio tends to mirror the return on the index; when β = 2:0, the excess return on the portfolio tends to be twice as great as the excess return on the index; when β = 0:5, it tends to be half as great; and so on.

- A portfolio with a β of 2.0 is twice as sensitive to movements in the index as a portfolio with a beta 1.0. It is therefore necessary to use twice as many contracts to hedge the portfolio.

- Similarly, a portfolio with a beta of 0.5 is half as sensitive to market movements as a portfolio with a beta of 1.0 and we should use half as many contracts to hedge it.

12 Hedging an Equity Portfolio with Beta

The Optimal number of contracts to make hedge is given by:

$$N^* = \beta \frac{V_A}{V_F}$$

This formula assumes that the maturity of the futures contract is close to the maturity of the hedge.

The hedge ratio h* is the slope of the best-fit line when changes in the portfolio are regressed against changes in the futures price of the index. Beta (β) is the slope of the best-fit line when the return from the portfolio is regressed against the return for the index.
Thus it implies that $h^* = \beta$.

12.1 Illustration

For example, that a portfolio worth $5,050,000 mirrors the S&P 500. The index futures price is 1,010 and each futures contract is on $250 times the index. Suppose that a futures contract with 4 months to maturity is used to hedge the value of a portfolio over the next 3 months in the following situation:

Value of S&P 500 index = 1,000
S&P 500 futures price = 1,010
Value of portfolio = $5,050,000
Risk-free interest rate = 4% per annum
Dividend yield on index = 1% per annum
Beta of portfolio = 1:5

Calculate the optimal number of contracts.

12.1 Illustration

One futures contract is for delivery of $250 times the index.
As before, V_F = 250 x 1,010 = 252,500.

The number of futures contracts that should be shorted to hedge the portfolio is

$$1.5 \times \frac{5{,}050{,}000}{252{,}500} = 30$$

Now suppose the index turns out to be 900 in 3 months and the futures price is 902. Calculate the gain on short futures position, also calculate the expected return on the portfolio.

12.1 Illustration

The gain from the short futures position is then
30 x (1010 - 902) x 250 = $810,000

The loss on the index is 10%. The index pays a dividend of 1% per annum, or 0.25% per 3 months. When dividends are taken into account, an investor in the index would therefore earn 9.75% over the 3-month period. Because the portfolio has a of 1.5, the capital asset pricing model gives:

Expected return on portfolio - Risk-free interest rate = 1:5 x (Return on index - Risk-free interest rate)

The risk-free interest rate is approximately 1% per 3 months. It follows that the expected return (%) on the portfolio during the 3 months when the 3-month return on the index is 9.75% is

1:0 + [1:5 x (9:75 - 1:0)] = - 15:125

12.1 Illustration

Finally calculate the expected value of the portfolio (inclusive of dividends) and thus the gain on the hedge.

12.1 Illustration

The expected value of the portfolio (inclusive of dividends) at the end of the 3 months is therefore

$5,050,000 x (1 – 0:15125) = $4,286,187

It follows that the expected value of the hedger's position, including the gain on the hedge, is

$4,286,187 + $810,000 = $5,096,187

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13 Reason to Hedge an Equity Portfolio

We saw the calculations and the gain through a hedge. But there remains a big question unanswered.

Why the hedger should go to the trouble of using futures contracts and why should he hedge?

Read Article – Hedging Futures
Try to solve the sum in the article on your own.
https://zerodha.com/varsity/chapter/hedging-futures/

Question

A fund manager has a portfolio worth $50 million with a beta of 0.87. The manager is concerned about the performance of the market over the next 2 months and plans to use 3-month futures contracts on the S&P 500 to hedge the risk.

The current level of the index is 1,250, one contract is on 250 times the index, the risk free rate is 6% per annum, and the dividend yield on the index is 3% per annum. The current 3-month futures price is 1,259.

(a) What position should the fund manager take to hedge all exposure to the market over the next 2 months?

Solution

(a) The number of contracts the fund manager should short is

$$0.87 \times \frac{50,000,000}{1259 \times 250} = 138.20$$

Rounding to the nearest whole number, 138 contracts should be shorted.

Question

A company has a $20 million portfolio with a beta of 1.2. It would like to use futures contracts on the S&P 500 to hedge its risk. The index futures price is currently standing at 1080, and each contract is for delivery of $250 times the index. What is the hedge that minimizes risk? What should the company do if it wants to reduce the beta of the portfolio to 0.6?

Solution

The formula for the number of contracts that should be shorted gives

$$1.2 \times \frac{20,000,000}{1080 \times 250} = 88.9$$

Rounding to the nearest whole number, 89 contracts should be shorted. To reduce the beta to 0.6, half of this position, or a short position in 44 contracts, is required.

13 Reasons To Hedge an Equity Portfolio

- A hedge using index futures removes the risk arising from market moves and leaves the hedger exposed only to the performance of the portfolio relative to the market.

- Hedging an equity portfolio reduces the systematic risk (appropriate when you feel that the stocks picked will outperform the market).

- Another reason for hedging may be that the hedger is planning to hold a portfolio for a long period of time and requires short-term protection in an uncertain market situation. The alternative strategy of selling the portfolio and buying it back later might involve unacceptably high transaction costs.

14 Changing the Beta of a Portfolio

Futures contracts are used to change the beta of a portfolio to some value other than zero.

1. Why do you think does one want to change the beta(increase or decrease) of a portfolio?

2. What positions should the investor take according to the change of beta requirements?

14 Changing the Beta of a Portfolio

In general, to change the beta of the portfolio from β to β^*,

1. When $\beta > \beta^*$, a short position in $N^* = (\beta - \beta^*)\frac{V_A}{V_F}$ contracts is required.

2. When $\beta^* < \beta$, a long position in $N^* = (\beta^* - \beta)\frac{V_A}{V_F}$ contracts is required.

Question

We continue the same illustration as earlier.

S&P 500 index = 1,000
S&P 500 futures price = 1,010
Value of portfolio = $5,050,000
Beta of portfolio = 1:5

As before, V_F = 250 x 1,010 = 252,500 and a complete hedge requires 1.5 x $\frac{5050000}{252500}$ contracts to be shorted.

Calculate the number of contracts required to short/long in order to:

1. Reduce the beta to 0.75
2. Increase the beta to 2.0

Note: Specify the position to be taken.

Solution

To reduce the beta of the portfolio from 1.5 to 0.75, the number of contracts shorted should be 15 rather than 30.

To increase the beta of the portfolio to 2.0, a long position in 10 contracts should be taken;

Question

It is July 16. A company has a portfolio of stocks worth \$100 million. The beta of the portfolio is 1.2. The company would like to use the CME December futures contract on the S&P 500 to change the beta of the portfolio to 0.5 during the period July 16 to November 16. The index futures price is currently 1,000 and each contract is on \$250 times the index.

(a) What position should the company take?

(b) Suppose that the company changes its mind and decides to increase the beta of the portfolio from 1.2 to 1.5. What position in futures contracts should it take?

Solution

(a) The company should short

$$\frac{(1.2-0.5)\times 100,000,000}{1000\times 250}$$

or 280 contracts.

(b) The company should take a long position in

$$\frac{(1.5-1.2)\times 100,000,000}{1000\times 250}$$

or 120 contracts.

Quick Recap

- Hedging is undertaken with basic aim to offsetting the risk faced in various positions taken in the market.
- Hedge could be of different types – perfect, dynamic or static hedge.
- If we sell or short a derivative instrument in order to hedge, it is called a short hedge.
- If we buy a derivative instrument in order to hedge, it is called a long hedge.
- Hedging has both its pros and cons.

Quick Recap

- The payoff from a long position in a forward contract on one unit of an asset is S_T - K.
- The payoff from a short position in a forward contract on one unit of an asset is K - S_T.
- Various parameters need to be matched well in order to keep a check on risks.
- Basis is the difference between spot and future prices at any point in time. Basis risk arises because of the uncertainty about the basis when the hedge is closed out.

Quick Recap

- Basis is the difference between the spot and the future price at any pint of time.

- Basis risk arises because of the uncertainty about the basis when the hedge is closed out.

- The effective price that is obtained for the asset with hedging is
  - $S_2 + F_1 - F_2 = F_1 + b_2$

- Choice of a contract has two key components:
  1. The choice of the asset underlying the futures contract
  2. The choice of the delivery month

Quick Recap

- Cross hedging occurs when the two assets are different, i.e. the hedged item and the asset underlying derivatives is different.

- The hedge ratio is the ratio of the size of the position taken in futures contracts to the size of the exposure.

- The minimum variance hedge ratio is $h^* = \rho \frac{\sigma_S}{\sigma_F}$

- The optimal number of futures contracts required is $N^* = \frac{h^* Q_A}{Q_F}$

- When futures are used for hedging, a small adjustment, known as tailing the hedge, can be made to allow for the impact of daily settlement.

Quick Recap

- A stock index tracks changes in the value of a hypothetical portfolio of stocks.
- Stock index futures can be used to hedge a well-diversified equity portfolio.
- If the portfolio mirrors the index, the optimal hedge ratio, equals 1.0 and the number of futures contracts that should be used is: N* = $\frac{V_A}{V_F}$.
- When portfolio does not mirror the index, Parameter Beta comes into the picture. The Optimal number of contracts to make hedge is given by: N* = $\beta \frac{V_A}{V_F}$

Quick Recap

- Reasons to Hedge an Equity Portfolio include:

- removes the risk arising from market moves
- reduces the systematic risk
- provides short-term protection

- In general, to change the beta of the portfolio from β to β^*,

1. When $\beta > \beta^*$, a short position in $N^* = (\beta - \beta^*)\frac{V_A}{V_F}$ contracts is required.
2. When $\beta^* < \beta$, a long position in $N^* = (\beta^* - \beta)\frac{V_A}{V_F}$ contracts is required.