INSTITUTE OF ACTUARIAL & QUANTITATIVE STUDIES

Subject: IDFM

Chapter: UNIT 2

Category: Practice Questions

1. What is a lower bound for the price of a 4-month call option on a non-dividend-paying stock when the stock price is $28, the strike price is $25, and the risk-free interest rate is 8% per annum?

Ans –

The lower bound is

$$28 - 25e^{0.08\times 0.3333} = \$3.66$$

2. What is a lower bound for the price of a 1-month European put option on a nondividend-paying stock when the stock price is $12, the strike price is $15, and the riskfree interest rate is 6% per annum?

Ans –

The lower bound is

$$15e^{-0.06\times 0.08333} - 12 = S2.93$$

3. Give two reasons why the early exercise of an American call option on a non-dividendpaying stock is not optimal. The first reason should involve the time value of money. The second should apply even if interest rates are zero.

Ans –

Delaying exercise delays the payment of the strike price. This means that the option holder is able to earn interest on the strike price for a longer period of time. Delaying exercise also provides insurance against the stock price falling below the strike price by the expiration date. Assume that the option holder has an amount of cash K and that interest rates are zero. When the option is exercised early it is worth S_T at expiration. Delaying exercise means that it will be worth $\max(K, S_T)$ at expiration.

4. Why is an American call option on a dividend-paying stock always worth at least as much as its intrinsic value. Is the same true of a European call option? Explain your answer.

Ans –

An American call option can be exercised at any time. If it is exercised its holder gets the intrinsic value. It follows that an American call option must be worth at least its intrinsic value. A European call option can be worth less than its intrinsic value. Consider, for example, the situation where a stock is expected to provide a very high dividend during the life of an option. The price of the stock will decline as a result of the dividend. Because the European option can be exercised only after the dividend has been paid, its value may be less than the intrinsic value today.

5. The price of a non-dividend-paying stock is $19 and the price of a 3-month European call option on the stock with a strike price of $20 is $1. The risk-free rate is 4% per annum. What is the price of a 3-month European put option with a strike price of $20?

Ans –

The price of a non-dividend paying stock is $19 and the price of a three-month European call option on the stock with a strike price of $20 is $1. The risk-free rate is 4% per annum. What is the price of a three-month European put option with a strike price of $20?

In this case, $c = 1$, $T = 0.25$, $S_0 = 19$, $K = 20$, and $r = 0.04$. From put–call parity

$$p = c + Ke^{-rT} - S_0$$

or

$$p = 1 + 20e^{-0.04\times0.25} - 19 = 1.80$$

so that the European put price is $1.80.

6. Explain why the arguments leading to put–call parity for European options cannot be used to give a similar result for American options.

Ans –

When early exercise is not possible, we can argue that two portfolios that are worth the same at time T must be worth the same at earlier times. When early exercise is possible, the argument falls down. Suppose that $P + S > C + Ke^{-rT}$. This situation does not lead to an arbitrage opportunity. If we buy the call, short the put, and short the stock, we cannot be sure of the result because we do not know when the put will be exercised.

7. i) The price of a European call that expires in 6 months and has a strike price of $30 is $2. The underlying stock price is $29, and a dividend of $0.50 is expected in 2 months and again in 5 months. Risk-free interest rates (all maturities) are 10%. What is the price of a European put option that expires in 6 months and has a strike price of $30?

Ans –

Using the notation in the chapter, put-call parity [equation (10.10)] gives

$$c + Ke^{-rT} + D = p + S_0$$

or

$$p = c + Ke^{-rT} + D - S_0$$

In this case

$$p = 2 + 30e^{-0.1\times6/12} + (0.5e^{-0.1\times2/12} + 0.5e^{-0.1\times5/12}) - 29 = 2.51$$

In other words the put price is $2.51.

ii) Explain the arbitrage opportunities in part i) if the European put price is \$3.

If the put price is \$3.00, it is too high relative to the call price. An arbitrageur should buy the call, short the put and short the stock. This generates $-2+3+29=\$30$ in cash which is invested at 10%. Regardless of what happens a profit with a present value of $3.00-2.51=\$0.49$ is locked in.

If the stock price is above \$30 in six months, the call option is exercised, and the put option expires worthless. The call option enables the stock to be bought for \$30, or $30e^{-0.10\times 6/12}=\$28.54$ in present value terms. The dividends on the short position cost $0.5e^{-0.1\times 2/12}+0.5e^{-0.1\times 5/12}=\$0.97$ in present value terms so that there is a profit with a present value of $30-28.54-0.97=\$0.49$.

If the stock price is below \$30 in six months, the put option is exercised and the call option expires worthless. The short put option leads to the stock being bought for \$30, or $30e^{-0.10\times 6/12}=\$28.54$ in present value terms. The dividends on the short position cost $0.5e^{\ 0.1\times 2/12}+0.5e^{-0.1\times 5/12}=\$0.97$ in present value terms so that there is a profit with a present value of $30-28.54-0.97=\$0.49$.

8. A 4-month European call option on a dividend-paying stock is currently selling for \$5. The stock price is \$64, the strike price is \$60, and a dividend of \$0.80 is expected in 1 month. The risk-free interest rate is 12% per annum for all maturities. What opportunities are there for an arbitrageur?

Ans –

The present value of the strike price is $60e^{-0.12\times 4/12}=\$57.65$. The present value of the dividend is $0.80e^{\ 0.12\times 1/12}=0.79$. Because

$$5<64-57.65-0.79$$

the condition in equation (10.8) is violated. An arbitrageur should buy the option and short the stock. This generates $64-5=\$59$. The arbitrageur invests \$0.79 of this at 12% for one month to pay the dividend of \$0.80 in one month. The remaining \$58.21 is invested for four months at 12%. Regardless of what happens a profit will materialize.

If the stock price declines below \$60 in four months, the arbitrageur loses the \$5 spent on the option but gains on the short position. The arbitrageur shorts when the stock price is \$64, has to pay dividends with a present value of \$0.79, and closes out the short position when the stock price is \$60 or less. Because \$57.65 is the present value of \$60, the short position generates at least $64-57.65-0.79=\$5.56$ in present value terms. The present value of the arbitrageur's gain is therefore at least $5.56-5.00=\$0.56$.

If the stock price is above \$60 at the expiration of the option, the option is exercised. The arbitrageur buys the stock for \$60 in four months and closes out the short position. The present value of the \$60 paid for the stock is \$57.65 and as before the dividend has a present value of \$0.79. The gain from the short position and the exercise of the option is therefore exactly $64-57.65-0.79=\$5.56$. The arbitrageur's gain in present value terms is exactly $5.56-5.00=\$0.56$.

PRACTICE QUESTIONS

UNIT 2

9. Give an intuitive explanation of why the early exercise of an American put becomes more attractive as the risk-free rate increases and volatility decreases.

Ans –

The early exercise of an American put is attractive when the interest earned on the strike price is greater than the insurance element lost. When interest rates increase, the value of the interest earned on the strike price increases making early exercise more attractive. When volatility decreases, the insurance element is less valuable. Again this makes early exercise more attractive.

10. The price of a European call that expires in 6 months and has a strike price of $30 is $2. The underlying stock price is $29, and a dividend of $0.50 is expected in 2 months and again in 5 months. Risk-free interest rates (all maturities) are 10%. What is the price of a European put option that expires in 6 months and has a strike price of $30?

Ans –

Using the notation in the chapter, put-call parity [equation (10.10)] gives

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In this case

$$p = 2 + 30e^{-0.1\times 6/12} + (0.5e^{-0.1\times 2/12} + 0.5e^{-0.1\times 5/12}) - 29 = 2.51$$

In other words the put price is $2.51.

11. Explain two ways in which a bear spread can be created.

Ans –

A bear spread can be created using two call options with the same maturity and different strike prices. The investor shorts the call option with the lower strike price and buys the call option with the higher strike price. A bear spread can also be created using two put options with the same maturity and different strike prices. In this case, the investor shorts the put option with the lower strike price and buys the put option with the higher strike price.

12.When is it appropriate for an investor to purchase a butterfly spread?

Ans –

A butterfly spread involves a position in options with three different strike prices (K_1, K_2, and K_3). A butterfly spread should be purchased when the investor considers that the price of the underlying stock is likely to stay close to the central strike price, K_2.

13. Call options on a stock are available with strike prices of \$15, \$17 ½ , and \$20, and expiration dates in 3 months. Their prices are \$4, \$2, and \$½ respectively.
Explain how the options can be used to create a butterfly spread.
Construct a table showing how profit varies with stock price for the butterfly spread.

Ans –

An investor can create a butterfly spread by buying call options with strike prices of \$15 and \$20 and selling two call options with strike prices of $\$17\frac{1}{2}$. The initial investment is $4+\frac{1}{2}-2\times 2=\$\frac{1}{2}$. The following table shows the variation of profit with the final stock price:

Stock Price, S_T	*Profit*
$S_T < 15$	$-\frac{1}{2}$
$15 < S_T < 17\frac{1}{2}$	$(S_T - 15) - \frac{1}{2}$
$17\frac{1}{2} < S_T < 20$	$(20 - S_T) - \frac{1}{2}$
$S_T > 20$	$-\frac{1}{2}$

14. A call option with a strike price of \$50 costs \$2. A put option with a strike price of \$45 costs \$3. Explain how a strangle can be created from these two options. What is the pattern of profits from the strangle?

Ans –

A strangle is created by buying both options. The pattern of profits is as follows:

Stock Price, S_T	*Profit*
$S_T < 45$	$(45 - S_T) - 5$
$45 < S_T < 50$	-5
$S_T > 50$	$(S_T - 50) - 5$

15. Suppose that put options on a stock with strike prices $30 and $35 cost $4 and $7, respectively. How can the options be used to create (a) a bull spread and (b) a bear spread? Construct a table that shows the profit and payoff for both spreads.

Ans –

A bull spread is created by buying the $30 put and selling the $35 put. This strategy gives rise to an initial cash inflow of S3. The outcome is as follows:

Stock Price	*Payoff*	*Profit*
$S_T \geq 35$	0	3
$30 \leq S_T < 35$	$S_T - 35$	$S_T - 32$
$S_T < 30$	-5	-2

A bear spread is created by selling the $30 put and buying the $35 put. This strategy costs $3 initially. The outcome is as follows:

Stock Price	*Payoff*	*Profit*
$S_T \geq 35$	0	-3
$30 \leq S_T < 35$	$35 - S_T$	$32 - S_T$
$S_T < 30$	5	2

16. A call with a strike price of $60 costs $6. A put with the same strike price and expiration date costs $4. Construct a table that shows the profit from a straddle. For what range of stock prices would the straddle lead to a loss?

Ans –

A straddle is created by buying both the call and the put. This strategy costs $10. The profit/loss is shown in the following table:

Stock Price	*Payoff*	*Profit*
$S_T > 60$	$S_T - 60$	$S_T - 70$
$S_T \leq 60$	$60 - S_T$	$50 - S_T$

This shows that the straddle will lead to a loss if the final stock price is between $50 and $70.

17. An investor believes that there will be a big jump in a stock price, but is uncertain as to the direction. Identify six different strategies the investor can follow and explain the differences among them.

Ans –

Possible strategies are:

- Strangle
- Straddle
- Strip
- Strap
- Reverse calendar spread
- Reverse butterfly spread

The strategies all provide positive profits when there are large stock price moves. A strangle is less expensive than a straddle, but requires a bigger move in the stock price in order to provide a positive profit. Strips and straps are more expensive than straddles but provide bigger profits in certain circumstances. A strip will provide a bigger profit when there is a large downward stock price move. A strap will provide a bigger profit when there is a large upward stock price move. In the case of strangles, straddles, strips and straps, the profit increases as the size of the stock price movement increases. By contrast in a reverse calendar spread and a reverse butterfly spread there is a maximum potential profit regardless of the size of the stock price movement.

18. What is the result if the strike price of the put is higher than the strike price of the call in a strangle?

Ans –

The result is shown in Figure S11.1. The profit pattern from a long position in a call and a put when the put has a higher strike price than a call is much the same as when the call has a higher strike price than the put. Both the initial investment and the final payoff are much higher in the first case