



Subject: Introduction to Derivatives
& Financial Market

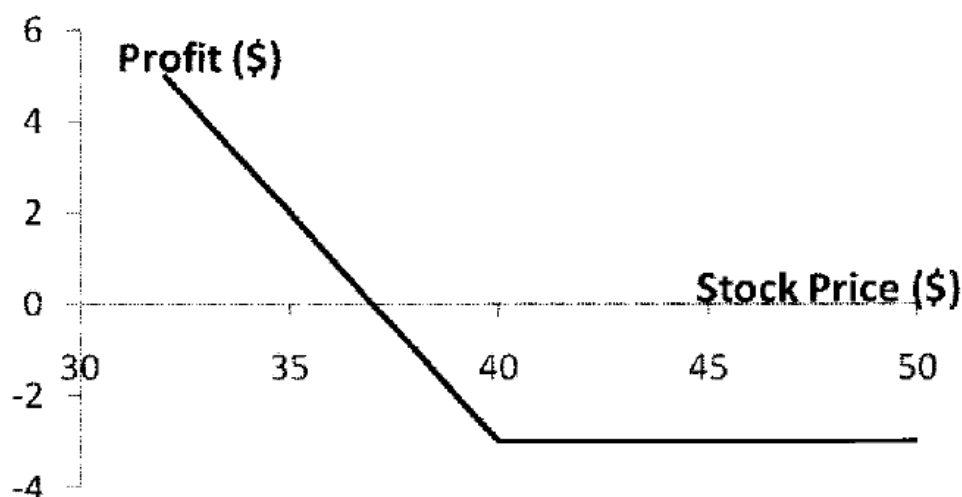
Chapter: Unit 3

Category: Practice Question - Solution

Q 1)

An investor buys a European put on a share for \$3. The stock price is \$42 and the strike price is \$40. Under what circumstances does the investor make a profit? Under what circumstances will the option be exercised? Draw a diagram showing the variation of the investor's profit with the stock price at the maturity of the option.

The investor makes a profit if the price of the stock on the expiration date is less than \$37. In these circumstances the gain from exercising the option is greater than \$3. The option will be exercised if the stock price is less than \$40 at the maturity of the option. The variation of the investor's profit with the stock price in Figure S9.1.

**Q 2)**

Explain why margins are required when clients write options but not when they buy options.

When an investor buys an option, cash must be paid up front. There is no possibility of future liabilities and therefore no need for a margin account. When an investor sells an option, there are potential future liabilities. To protect against the risk of a default, margins are required.

Q 3)

Suppose that a European call option to buy a share for \$100.00 costs \$5.00 and is held until maturity. Under what circumstances will the holder of the option make a profit? Under what circumstances will the option be exercised? Draw a diagram illustrating how the profit from a long position in the option depends on the stock price at maturity of the option.

Ignoring the time value of money, the holder of the option will make a profit if the stock price at maturity of the option is greater than \$105. This is because the payoff to the holder of the option is, in these circumstances, greater than the \$5 paid for the option. The option will be exercised if the stock price at maturity is greater than \$100. Note that if the stock price is between \$100 and \$105 the option is exercised, but the holder of the option takes a loss overall. The profit from a long position is as shown in Figure S9.3.

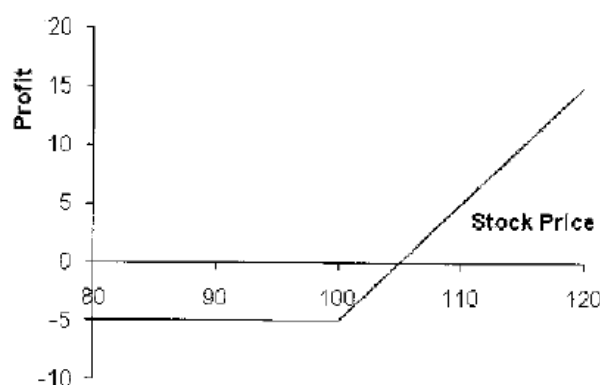


Figure S9.3 Profit from long position in Problem 9.9

Q 4)

A trader buys a call option with a strike price of \$45 and a put option with a strike price of \$40. Both options have the same maturity. The call costs \$3 and the put costs \$4. Draw a diagram showing the variation of the trader's profit with the asset price.

Figure S9.6 shows the variation of the trader's position with the asset price. We can divide the alternative asset prices into three ranges:

- When the asset price less than \$40, the put option provides a payoff of $40 - S_T$ and the call option provides no payoff. The options cost \$7 and so the total profit is $33 - S_T$.
- When the asset price is between \$40 and \$45, neither option provides a payoff. There is a net loss of \$7.
- When the asset price greater than \$45, the call option provides a payoff of $S_T - 45$ and the put option provides no payoff. Taking into account the \$7 cost of the options, the total profit is $S_T - 52$.

The trader makes a profit (ignoring the time value of money) if the stock price is less than

\$33 or greater than \$52. This type of trading strategy is known as a strangle and is discussed in Chapter 11.

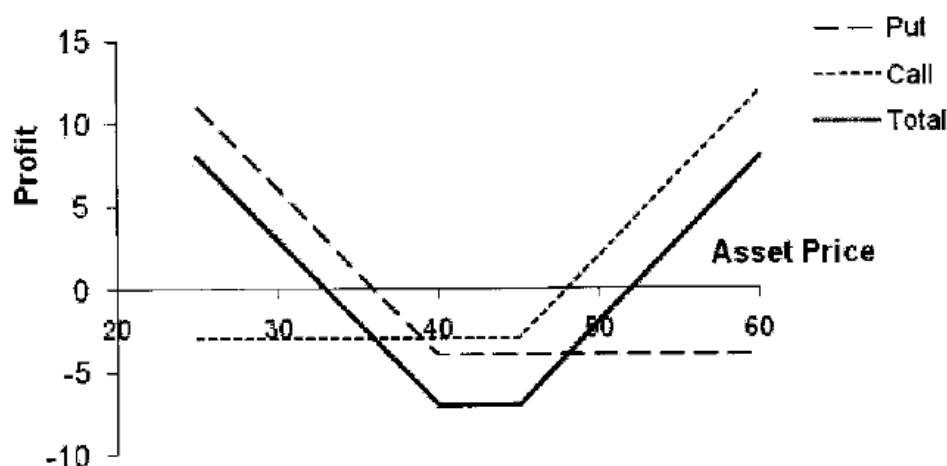


Figure S9.6 Profit from trading strategy in Problem 9.12

Q 5)

Explain why an American option is always worth at least as much as a European option on the same asset with the same strike price and exercise date.

The holder of an American option has all the same rights as the holder of a European option and more. It must therefore be worth at least as much. If it were not, an arbitrageur could short the European option and take a long position in the American option.

Q 6)

List the six factors that affect stock option prices.

The six factors affecting stock option prices are the stock price, strike price, risk-free interest rate, volatility, time to maturity, and dividends.

Q 7)

The price of a non-dividend-paying stock is \$19 and the price of a 3-month European call option on the stock with a strike price of \$20 is \$1. The risk-free rate is 4% per annum. What is the price of a 3-month European put option with a strike price of \$20?

In this case, $c = 1$, $T = 0.25$, $S_0 = 19$, $K = 20$, and $r = 0.04$. From put-call parity

$$p = c + Ke^{-rt} - S_0$$

or

$$p = 1 + 20e^{-0.04 \times 0.25} - 19 = 1.80$$

so that the European put price is \$1.80.

Q 8)

A 4-month European call option on a dividend-paying stock is currently selling for \$5. The stock price is \$64, the strike price is \$60, and a dividend of \$0.80 is expected in 1 month. The risk-free interest rate is 12% per annum for all maturities. What opportunities are there for an arbitrageur?

The present value of the strike price is $60e^{-0.12 \times 4/12} = \57.65 . The present value of the dividend is $0.80e^{-0.12 \times 1/12} = 0.79$. Because

$$5 < 64 - 57.65 - 0.79$$

the condition in equation (10.8) is violated. An arbitrageur should buy the option and short the stock. This generates $64 - 5 = \$59$. The arbitrageur invests \$0.79 of this at 12% for one month to pay the dividend of \$0.80 in one month. The remaining \$58.21 is invested for four months at 12%. Regardless of what happens a profit will materialize.

If the stock price declines below \$60 in four months, the arbitrageur loses the \$5 spent on the option but gains on the short position. The arbitrageur shorts when the stock price is \$64, has to pay dividends with a present value of \$0.79, and closes out the short position when the stock price is \$60 or less. Because \$57.65 is the present value of \$60, the short position generates at least $64 - 57.65 - 0.79 = \$5.56$ in present value terms. The present value of the arbitrageur's gain is therefore at least $5.56 - 5.00 = \$0.56$.

If the stock price is above \$60 at the expiration of the option, the option is exercised. The arbitrageur buys the stock for \$60 in four months and closes out the short position. The present value of the \$60 paid for the stock is \$57.65 and as before the dividend has a present value of \$0.79. The gain from the short position and the exercise of the option is therefore exactly $64 - 57.65 - 0.79 = \$5.56$. The arbitrageur's gain in present value terms is exactly $5.56 - 5.00 = \$0.56$.

Q 9)

The price of a European call that expires in 6 months and has a strike price of \$30 is \$2. The underlying stock price is \$29, and a dividend of \$0.50 is expected in 2 months and again in 5 months. The term structure is flat, with all risk-free interest rates being 10%. What is the price of a European put option that expires in 6 months and has a strike price of \$30?

Using the notation in the chapter, put-call parity [equation (10.10)] gives

$$c + Ke^{-rt} + D = p + S_0$$

or

$$p = c + Ke^{-rt} + D - S_0$$

In this case

$$p = 2 + 30e^{-0.1 \times 6/12} + (0.5e^{-0.1 \times 2/12} + 0.5e^{-0.1 \times 5/12}) - 29 = 2.51$$

In other words the put price is \$2.51.

Q 10)

The price of an American call on a non-dividend-paying stock is \$4. The stock price is \$31, the strike price is \$30, and the expiration date is in 3 months. The risk-free interest rate is 8%. Derive upper and lower bounds for the price of an American put on the same stock with the same strike price and expiration date

$$S_0 - K \leq C - P \leq S_0 - Ke^{-rt}$$

In this case

$$31 - 30 \leq 4 - P \leq 31 - 30e^{-0.08 \times 0.25}$$

or

$$1.00 \leq 4.00 - P \leq 1.59$$

or

$$2.41 \leq P \leq 3.00$$

Upper and lower bounds for the price of an American put are therefore \$2.41 and \$3.00.

Q 11)

What is meant by a protective put? What position in call options is equivalent to a protective put?

A protective put consists of a long position in a put option combined with a long position in the underlying shares. It is equivalent to a long position in a call option plus a certain amount of cash. This follows from put-call parity:

$$p + S_0 = c + Ke^{-rt} + D$$

Q 12)

Explain two ways in which a bear spread can be created.

A bear spread can be created using two call options with the same maturity and different strike prices. The investor shorts the call option with the lower strike price and buys the call option with the higher strike price. A bear spread can also be created using two put options with the same maturity and different strike prices. In this case, the investor shorts the put option with the lower strike price and buys the put option with the higher strike price.

Q 13)

Call options on a stock are available with strike prices of \$15, \$17½, and \$20, and expiration dates in 3 months. Their prices are \$4, \$2, and \$12, respectively. Explain how the options can be used to create a butterfly spread. Construct a table showing how profit varies with stock price for the butterfly spread.

An investor can create a butterfly spread by buying call options with strike prices of \$15 and \$20 and selling two call options with strike prices of \$17½. The initial investment is $4 + \frac{1}{2} - 2 \times 2 = \$\frac{1}{2}$. The following table shows the variation of profit with the final stock price:

Stock Price, S_T	Profit
$S_T < 15$	$-\frac{1}{2}$
$15 < S_T < 17\frac{1}{2}$	$(S_T - 15) - \frac{1}{2}$
$17\frac{1}{2} < S_T < 20$	$(20 - S_T) - \frac{1}{2}$
$S_T > 20$	$-\frac{1}{2}$

Q 14)

Suppose that put options on a stock with strike prices \$30 and \$35 cost \$4 and \$7, respectively. How can the options be used to create (a) a bull spread and (b) a bear spread? Construct a table that shows the profit and payoff for both spreads.

A bull spread is created by buying the \$30 put and selling the \$35 put. This strategy gives rise to an initial cash inflow of \$3. The outcome is as follows:

<i>Stock Price</i>	<i>Payoff</i>	<i>Profit</i>
$S_T \geq 35$	0	3
$30 \leq S_T < 35$	$S_T - 35$	$S_T - 32$
$S_T < 30$	-5	-2

A bear spread is created by selling the \$30 put and buying the \$35 put. This strategy costs \$3 initially. The outcome is as follows:

<i>Stock Price</i>	<i>Payoff</i>	<i>Profit</i>
$S_T \geq 35$	0	-3
$30 \leq S_T < 35$	$35 - S_T$	$32 - S_T$
$S_T < 30$	5	2

Q 15)

What is the difference between a strangle and a straddle?

The difference is that the strangle has two different strike prices, while the straddle has a common strike price.