INSTITUTE OF ACTUARIAL & QUANTITATIVE STUDIES

Class: SY BSc

Subject : Introduction to Derivatives and Financial Markets

Chapter: Unit 3 Chapter 3

Chapter Name: Determination of forward and futures prices

Precap of Previous Chapter

- Hedging is undertaken with basic aim to offsetting the risk faced in various positions taken in the market. Hedge could be of different types – perfect, dynamic or static hedge and a short or long hedge.

- The payoff from a long position in a forward contract on one unit of an asset is S_T - K. The payoff from a short position in a forward contract on one unit of an asset is K - S_T.

- Basis is the difference between spot and future prices at any point in time. Basis risk arises because of the uncertainty about the basis when the hedge is closed out.

- The effective price that is obtained for the asset with hedging is

$$S_2 + F_1 - F_2 = F_1 + b_2$$

Precap of Previous Chapter

- The hedge ratio is the ratio of the size of the position taken in futures contracts to the size of the exposure.

- The minimum variance hedge ratio is $h^* = \rho \frac{\sigma_S}{\sigma_F}$

- The optimal number of futures contracts required is $N^* = \frac{h^* Q_A}{Q_F}$

- A stock index tracks changes in the value of a hypothetical portfolio of stocks.

- Stock index futures can be used to hedge a well-diversified equity portfolio.

- Reasons to Hedge an Equity Portfolio include:

- removes the risk arising from market moves
- reduces the systematic risk
- provides short-term protection

Today's Agenda

1. Introduction
    1. Types of Assets
    2. Investment Assets
    3. Consumption Assets
2. Short Selling
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    3. Notations
4. Forward Price of an Investment Asset
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5. Valuing Forward Contracts
    1. Value of Long & Short Forward Contract
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6. Futures price of Stock Index
7. Futures and Forwards on Currencies
    1. Use of Currency Futures
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8. Futures on Commodities
    1. Pricing commodity futures What is Financial Market?
        - A. Commodities that are Investment Assets
        - B. Commodities with Income and Storage costs
        - C. Consumption Commodities
9. Are forward and futures price equal?

1 Introduction

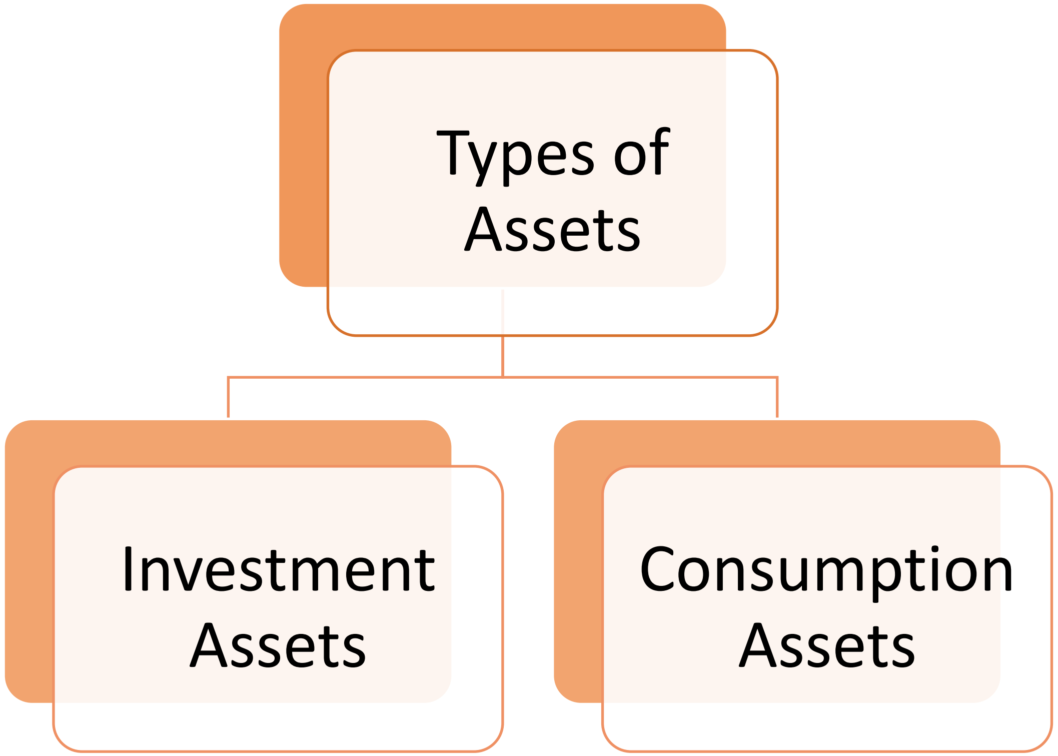
We will examine how forward prices and futures prices are related to the spot price of the underlying asset.

Forward contracts are easier to analyse than futures contracts because there is no daily settlement—only a single payment at maturity.

It can be shown that the forward price and futures price of an asset are usually very close when the maturities of the two contracts are the same. This is convenient because it means that results obtained for forwards are usually also true for futures.

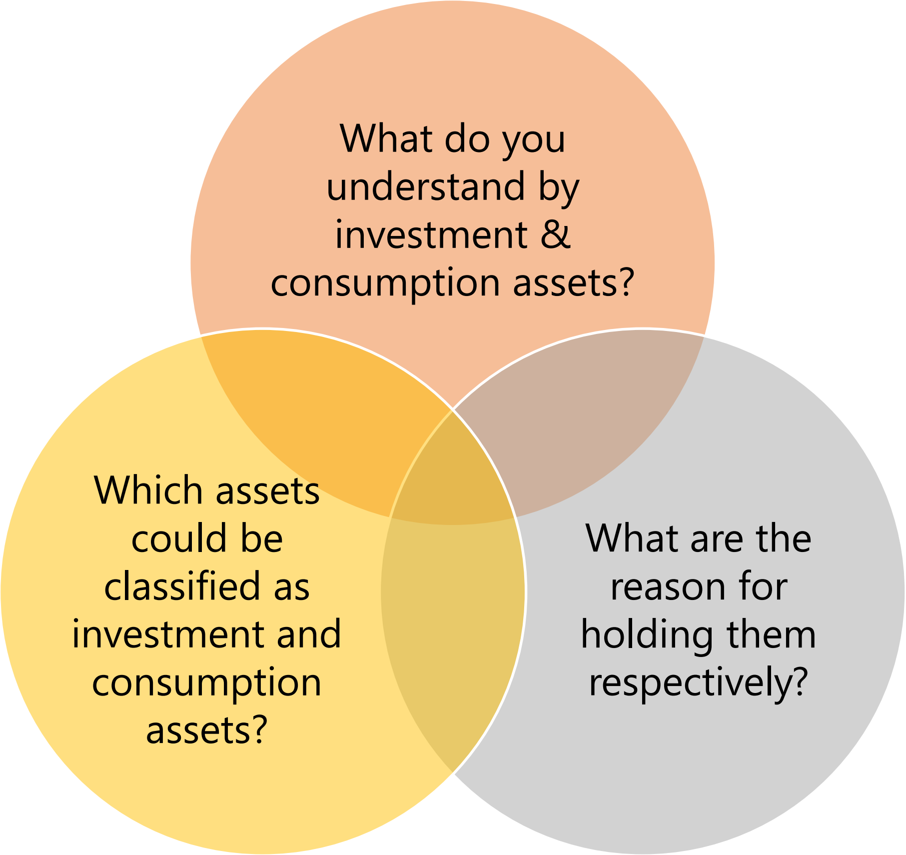
1.1 Types of Assets

We will classify the assets into 2 types



1.1 Types of Assets

When considering forward and futures contracts, it is important to distinguish between investment assets and consumption assets.



1.1 Types of Assets - Activity

Classify the following assets into investment or Consumption assets.

- Bonds
- Coffee
- Shares
- Iron
- Real Estate
- Gold
- Certificate of Deposit
- Silver
- Copper

1.2 Investment Assets

An investment asset is an asset that is held for investment purposes by significant numbers of investors.

Example - Stocks and bonds are clearly investment assets.

Gold and silver are also examples of investment assets. Note that investment assets do not have to be held exclusively for investment. (Silver, for example, has a number of industrial uses.) However, they do have to satisfy the requirement that they are held by significant numbers of investors solely for investment.

1.3 Consumption Assets

A consumption asset is an asset that is held primarily for consumption. It is not usually held for investment.

Example - Consumption assets are commodities such as copper, oil, and pork bellies.

In this chapter we will see that we can use arbitrage arguments to determine the forward and futures prices of an investment asset from its spot price and other observable market variables. We cannot do this for consumption assets.

2 Short Selling

We understand what buying means. We understand what selling means. But.....
What do you mean or understand by short selling?

2 Short Selling

Short selling involves selling an asset that is not owned. It is something that is possible for some—but not all—investment assets.

Some of the arbitrage strategies presented in this chapter involve short selling. This trade, usually simply referred to as "shorting.

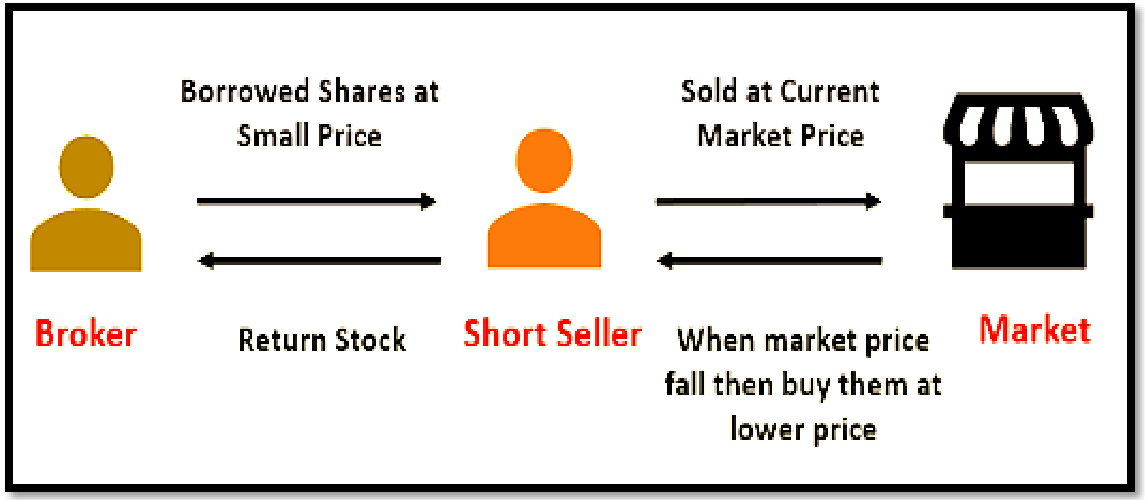
An investor with a short position must pay to the broker any income, such as dividends or interest, that would normally be received on the securities that have been shorted. The broker will transfer this income to the account of the client from whom the securities have been borrowed.

2 Short Selling - Example

An investor instructs a broker to short 500 IBM shares.

The broker will carry out the instructions by borrowing the shares from another client and selling them in the market in the usual way. The investor can maintain the short position for as long as desired, provided there are always shares for the broker to borrow.

At some stage, however, the investor will close out the position by purchasing 500 IBM shares.



2 Short Selling - Example

Let's suppose the investor closes out the position.

Consider the position of an investor who shorts 500 shares in April when the price per share is $120 and closes out the position by buying them back in July when the price per share is $100. Suppose that a dividend of $1 per share is paid in May. The investor receives 500 x $120 = $60,000 in April when the short position is initiated. The dividend leads to a payment by the investor of 500 x $1 = $500 in May. The investor also pays 500 x $100 = $50,000 for shares when the position is closed out in July.

The net gain, therefore, is = $60,000 - $500 - $50,000 = $9,500
assuming there is no fee for borrowing the shares.

? Suppose instead of shorting the investor purchases the 500 shares in April and sells them in July, receiving the dividend in May. Calculate the investors net gain. Also suggest the ideal strategy from the two.

Question

1. The price of a futures contract at the expiration date of the contract
(a) equals the price of the underlying asset.
(b) equals the price of the counterparty.
(c) equals the hedge position.
(d) equals the value of the hedged asset.
(e) none of the above.

Answer: A

2. If a firm must pay for goods it has ordered with foreign currency, it can hedge its foreign exchange rate risk by ______ foreign exchange futures ______.
(a) selling; short
(b) buying; long
(c) buying; short
(d) selling; long

Answer: B

Question

3. Futures markets have grown rapidly because futures
(a) are standardized.
(b) have lower default risk.
(c) are liquid.
(d) all of the above.

Answer: D

4. Futures contracts trade with every month as a delivery month. A company is hedging the purchase of the underlying asset on June 15. Which futures contract should it use?
A. The June contract
B. The July contract
C. The May contract
D. The August contract

Answer: B

3.1 Certain Definitions

An **interest rate** refers to the amount charged by a lender to a borrower for any form of debt. It is listed as a current liability and part of given, generally expressed as a percentage of the principal.

The **discount factor**, is the factor by which a future cash flow must be multiplied in order to obtain the present value. So if suppose i is the interest rate and v is the discount rate then, $v = 1/(1+i)$.

The **price of a futures contract** is what is agreed upon by both the parties and mentioned in the contract. We will look in this chapter at how to price forwards/futures contracts. Price is fixed throughout the contract tenure.

The **value of a futures contract** is what comes into picture due to the daily marking-to-market of prices. The value keeps fluctuating based on changes in spot prices and futures prices.

3.2 Assumptions

Before pricing we make certain assumptions about the market. In this chapter we will assume that the following are all true for the market participants:

1. The market participants are subject to no transaction costs when they trade.

2. The market participants are subject to the same tax rate on all net trading profits.

3. The market participants can borrow money at the same risk-free rate of interest as they can lend money.

4. The market participants take advantage of arbitrage opportunities as they occur.

3.3 Notations

The following notations will be used throughout this chapter:

T : Time until delivery date in a forward or futures contract (in years).

S_0 : Price of the asset underlying the forward or futures contract today, i.e. at time 0.

F_0 : Forward or futures price today, i.e. at time 0.

r : Zero-coupon risk-free rate of interest per annum, expressed with continuous compounding, for an investment maturing at the delivery date (i.e., in T years).

4 Forward Price of an Investment Asset

4.1 No Income

The easiest forward contract to value is one written on an investment asset that provides the holder with no income. Non-dividend-paying stocks and zero-coupon bonds are examples of such investment assets.

We consider a forward contract on an investment asset with price S_0 that provides no income. Using our notation, T is the time to maturity, r is the risk-free rate, and F_0 is the forward price. Assuming no arbitrage.

The relationship between F_0 and S_0 is:
$F_0 = S_0 \text{ x } e^{rT}$

Question

Consider a long forward contract to purchase a non-dividend-paying stock in 3 months. Assume the current stock price is $40 and the 3-month risk-free interest rate is 5% per annum.

Calculate the forward price.

Solution

We have T = 3/12, r = 0:05, and S_0 = 40.

The forward price, F_0, is given by

F_0 = 40 x $e^{0.05x(\frac{3}{12})}$ = \$40.50

This would be the delivery price in a contract negotiated today. This is the correct price for the forward. Any price other than this will open up arbitrage opportunities in the market.

Let’s see how!

4 Forward Price of an Investment Asset

Arbitrage Opportunity

Suppose that the forward price is relatively high at $43. Since this price is higher than the actual price arbitrage opens up. How do you think can an arbitrageur lock in profits in this scenario?

4 Forward Price of an Investment Asset

Arbitrage Opportunity

Strategy to Lock Profits
The forward price is relatively high at $43. An arbitrageur can borrow $40 at the risk-free interest rate of 5% per annum, buy one share, and short a forward contract to sell one share in 3 months. At the end of the 3 months, the arbitrageur delivers the share and receives $43. The sum of money required to pay off the loan is 40 x $e^{0.05x(\frac{3}{12})}$ = $40:50

By following this strategy, the arbitrageur locks in a profit of $43:00 - $40:50 = $2:50 at the end of the 3-month period.

Suppose now that the forward price is relatively low at $39. This also leads to arbitrage. Think of an arbitrage strategy to lock profits.

4 Forward Price of an Investment Asset

4.2 Known Income

We consider a forward contract on an investment asset that will provide a perfectly predictable cash income to the holder. Examples are stocks paying known dividends and coupon-bearing bonds.

Let, I = present value of the income during the life of the contract.

Forward price when an investment asset will provide income with a present value of I during the life of a forward contract, we have

$F_0 = (S_0 - I) \times e^{rT}$

Question

Consider a 10-month forward contract on a stock when the stock price is \$50. We assume that the risk-free rate of interest (continuously compounded) is 8% per annum for all maturities. We also assume that dividends of \$0.75 per share are expected after 3 months, 6 months, and 9 months.

Calculate the forward price for this contract.

Solution

The present value of the dividends, I, is

$$I = 0.75e^{-0.08\times 3/12} + 0.75e^{-0.08\times 6/12} + 0.75e^{-0.08\times 9/12} = 2.162$$

The variable T is 10 months, so that the forward price, F_0, is given by

$$F_0 = (50 - 2.162)e^{0.08\times 10/12} = \$51.14$$

4 Forward Price of an Investment Asset

4.3 Known Yield

We now consider the situation where the asset underlying a forward contract provides a known yield rather than a known cash income. This means that the income is known when expressed as a percentage of the asset's price at the time the income is paid. We will normally measure yields with continuous compounding.

Define q as the average yield per annum on an asset during the life of a forward contract with continuous compounding.

The forward price is then given as:

$F_0 = S_0 \text{ x } e^{(r-q)T}$

Question

Consider a 6-month forward contract on an asset that is expected to provide income equal to 2% of the asset price once during a 6-month period. The risk- free rate of interest (with continuous compounding) is 10% per annum. The asset price is $25.

Calculate the forward price.

Solution

In this case, S_0 = 25, r = 0:10, and T = 0:5. The yield is 4% per annum with semiannual compounding. This is 3.96% per annum with continuous compounding. It follows that q = 0:0396, so that the forward price, F_0 , is given by

$$F_0 = 25e^{(0.10-0.0396)\times 0.5} = \$25.77$$

Note:
Suppose that Rc is a rate of interest with continuous compounding and Rm is the equivalent rate with compounding m times per annum, then

$$R_c = m \ln\left(1 + \frac{R_m}{m}\right)$$

5 Valuing Forward Contracts

Example

To hedge against falling commodity prices, a wheat farmer takes a short position in 10 wheat futures contracts on November 21, 2019. Since each contract represents 5,000 bushels, the farmer is hedging against a price decline on 50,000 bushels of wheat. If the price of one contract is $4.50 on November 21, 2019, the wheat farmer's account will be recorded as $4.50 x 50,000 bushels = $225,000.

Day	Futures Price	Change in Value	Gain/Loss	Cumulative Gain/Loss	Account Balance
1	$4.50				225,000
2	$4.55	+0.05	-2,500	-2,500	222,500
3	$4.53	-0.02	+1,000	-1,500	223,500
4	$4.46	-0.07	+3,500	+2,000	227,000
5	$4.39	-0.07	+3,500	+5,500	230,500

The daily mark to market settlements will continue until the expiration date of the futures contract or until the farmer closes out his position by going long on a contract with the same maturity.

5 Valuing Forward Contracts

The value of a forward contract at the time it is first entered into is zero.

Later it may prove to have a positive or negative value. It is important for banks and institutions to value the contract each day. (This is referred to as marking to market the contract – as in the example before).

Let, K = the delivery price for a contract that was negotiated some time ago.
The variable F_0 is the forward price that would be applicable if we negotiated the contract today.
In addition, we have f to be the value of forward contract today.

5.1 Value of Long & Short Forward

The value of a long forward, (both those on investment assets and those on consumption assets), is,

$$f = (F_0 - K)\, e^{-rT}$$

i.e. $$f = S_0 - Ke^{-rT}$$

The value of a short forward contract (both those on investment assets and those on consumption assets), is,

$$f = (K - F_0)\, e^{-rT}$$

Note: $F_0 = S_0 e^{rT}$
Therefore, $S_0 = F_0 e^{-rT}$

Question

A long forward contract on a non-dividend-paying stock was entered into some time ago. It currently has 6 months to maturity. The risk-free rate of interest (with continuous compounding) is 10% per annum, the stock price is $25, and the delivery price is $24.

Calculate the value of the forward contract.

Solution

In this case, S_0 = 25, r = 0:10, T = 0:5, and K = 24.

The 6-month forward price, F_0, is given by

$$F_0 = 25e^{0.1\times0.5} = \$26.28$$

The value of the forward contract is

$$f = (26.28 - 24)e^{-0.1\times0.5} = \$2.17$$

5.2 Value of Forward contract with Income

Expression for the value of a long forward contract on an investment asset that provides a known income with present value I :

$$f = S_0 - I - Ke^{-rT}$$

Expression for the value of a long forward contract on an investment asset that provides a known yield at rate q

$$f = S_0e^{-qT} - Ke^{-rT}$$

Note: $F_0 = S_0e^{rT}$
Therefore, $S_0 = F_0e^{-rT}$

Have a Read! – A systems Error?

A foreign exchange trader working for a bank enters into a long forward contract to buy 1 million pounds sterling at an exchange rate of 1.5000 in 3 months. At the same time, another trader on the next desk takes a long position in 16 contracts for 3-month futures on sterling. The futures price is 1.5000 and each contract is on 62,500 pounds. The positions taken by the forward and futures traders are therefore the same. Within minutes of the positions being taken the forward and the futures prices both increase to 1.5040. The bank's systems show that the futures trader has made a profit of $4,000, while the forward trader has made a profit of only $3,900. The forward trader immediately calls the bank's systems department to complain. Does the forward trader have a valid complaint?

The answer is no! The daily settlement of futures contracts ensures that the futures trader realizes an almost immediate profit corresponding to the increase in the futures price. If the forward trader closed out the position by entering into a short contract at 1.5040, the forward trader would have contracted to buy 1 million pounds at 1.5000 in 3 months and sell 1 million pounds at 1.5040 in 3 months. This would lead to a $4,000 profit—but in 3 months, not today. The forward trader's profit is the present value of $4,000. ~~This is consistent with equation (5.4)~~.

The forward trader can gain some consolation from the fact that gains and losses are treated symmetrically. If the forward/futures prices dropped to 1.4960 instead of rising to 1.5040, then the futures trader would take a loss of $4,000 while the forward trader would take a loss of only $3,900.

Question

A 1-year long forward contract on a non-dividend-paying stock is entered into when the stock price is $40 and the risk-free rate of interest is 10% per annum with continuous compounding.

(a) What are the forward price and the initial value of the forward contract?

(b) Six months later, the price of the stock is $45 and the risk-free interest rate is still 10%. What are the forward price and the value of the forward contract?

Solution

(a) The forward price, F_0, is given by equation (5.1) as:

$$F_0 = 40e^{0.1\times 1} = 44.21$$

or \$44.21. The initial value of the forward contract is zero.

(b) The delivery price K in the contract is \$44.21. The value of the contract, f, after six months is given by equation (5.5) as:

$$f = 45 - 44.21e^{-0.1\times 0.5}$$

$$= 2.95$$

i.e., it is \$2.95. The forward price is:

$$45e^{0.1\times 0.5} = 47.31$$

6 Futures price of Stock Index

A stock index can usually be regarded as the price of an investment asset that pays dividends. The investment asset is the portfolio of stocks underlying the index, and the dividends paid by the investment asset are the dividends that would be received by the holder of this portfolio.

It is usually assumed that the dividends provide a known yield rather than a known cash income.

If q is the dividend yield rate, the futures price F_0 is as:

$F_0 = S_0\, e^{(r-q)T}$

Questions

1. The risk-free rate of interest is 7% per annum with continuous compounding, and the dividend yield on a stock index is 3.2% per annum. The current value of the index is 150. What is the 6-month futures price?

2. Suppose that the risk-free interest rate is 10% per annum with continuous compounding and that the dividend yield on a stock index is 4% per annum. The index is standing at 400, and the futures price for a contract deliverable in four months is 405. What arbitrage opportunities does this create?

Solution

1.

the six month futures price is

$$150e^{(0.07-0.032)\times 0.5} = 152.88$$

Solution

2.

The theoretical futures price is

$$400e^{(0.10-0.04)\times 4/12} = 408.08$$

The actual futures price is only 405. This shows that the index futures price is too low relative to the index. The correct arbitrage strategy is

1. Buy futures contracts
2. Short the shares underlying the index.

7 Futures and Forwards on Currencies

Currency futures are contracts for currencies that specify the price of exchanging one currency for another at a future date.

The rate for currency futures contracts is derived from spot rates of the currency pair.

INSTRUMENT TYPE	SYMBOL	EXPIRY DATE	OPTION TYPE	STRIKE PRICE	SPREAD	LAST PRICE	CHNG	%CHNG	VOLUME (Contracts)	VALUE *(₹ Lakhs)
Currency Futures	USDINR	28-Jun-2021	-	-	0.0100	**73.4100**	-0.0250	-0.0340	12,66,149	9,29,692.74
Currency Futures	GBPINR	28-Jun-2021	-	-	0.0025	**103.6100**	0.3450	0.3341	2,86,124	2,96,244.43
Currency Futures	USDINR	28-Jul-2021	-	-	0.0050	**73.6725**	-0.0200	-0.0271	1,46,372	1,07,857.84
Currency Futures	EURINR	28-Jun-2021	-	-	0.0200	**89.0000**	0.0175	0.0197	1,26,584	1,12,711.84
Currency Futures	GBPINR	28-Jul-2021	-	-	0.0075	**103.9300**	0.3475	0.3355	43,678	45,374.48
Currency Futures	USDINR	27-Aug-2021	-	-	0.0125	**73.9325**	-0.0325	-0.0439	26,280	19,435.35
Currency Futures	JPYINR	28-Jun-2021	-	-	0.0375	**66.8400**	0.1175	0.1761	24,904	16,628.12

7.1 Use of Currency Futures

- Currency futures are used to hedge the risk of receiving payments in a foreign currency. Currency futures may be contrasted with non-standardized currency forwards, which trade OTC.

- Also many participants are speculators who close out their positions before futures expire. They do not end up delivering the physical currency. Rather, they make or lose money based on the price change in the futures contracts themselves.

7.2 Pricing Currency Futures

The underlying asset is one unit of the foreign currency.

We define the variable S_0 as the current spot price of one unit of the foreign currency and F_0 as the forward or futures price of one unit of the foreign currency.

A foreign currency has the property that the holder of the currency can earn interest at the risk-free interest rate prevailing in the foreign country. We define r_f as the value of the foreign risk-free interest rate when money is invested for time T. The variable r is the risk-free rate when money is invested for this period of time.

The relationship between F_0 and S_0 is;

$$F_0 = S_0 \, e^{(r-r_f)T}$$

This is the well-known interest rate parity relationship.

Questions

The 2-month interest rates in Switzerland and the United States are, respectively, 2% and 5% per annum with continuous compounding. The spot price of the Swiss franc is \$0.8000. The futures price for a contract deliverable in 2 months is \$0.8100. What arbitrage opportunities does this create?

Solution

The theoretical futures price is

$$0.8000e^{(0.05-0.02)\times 2/12} = 0.8040$$

The actual futures price is too high. This suggests that an arbitrageur should buy Swiss francs and short Swiss francs futures.

8 Futures on Commodities

Commodity futures contract is an agreement to buy or sell a predetermined amount of a commodity at a specific price on a specific date in the future.

Commodity Futures Trading

Name	Month	Last	High	Low	Chg	Chg%	Time
NCDEX Jeera	Jun 21	13,450.00	13,610.00	13,400.00	0.00	0.00%	14/06
NCDEX Coriander	Jun 21	6,690.00	6,692.00	6,690.00	+0.00	+0.00%	15/06
NCDEX Guar Gum	Jun 21	6,230.00	6,230.00	6,230.00	-60.00	-0.95%	15:43:09
NCDEX Soybean	Jun 21	7,300.00	7,360.00	7,280.00	+25.00	+0.34%	16:59:42
MCX Gold 1 Kg	Aug 21	48,439	48,564	48,367	+15	+0.03%	18:52:42
MCX Silver	Jul 21	71,581	71,750	71,444	+333	+0.47%	18:53:02
MCX Crude Oil WTI	Jun 21	5,282	5,340	5,269	+2	+0.04%	18:54:02
MCX Natural Gas	Jun 21	235.50	236.90	233.20	-2.20	-0.93%	18:53:59

8.1 Pricing Commodity Futures

8.1.A Commodities that are Investment assets

First, we look at the futures prices of commodities that are investment assets such as gold and silver.

The forward price (no income or storage costs) is given by:

$F_0 = S_0\ e^{rT}$

How will you incorporate storage costs of commodities into pricing?

8.1 Pricing Commodity Futures

8.1.B Income and Storage Costs

Gold and silver can therefore provide income to the holder. Like other commodities they also have storage costs.

Storage costs can be treated as negative income. If U is the present value of all the storage costs, net of income, during the life of a forward contract, then forward price is given by:
$F_0 = (S_0 + U)e^{rT}$

If the storage costs (net of income) incurred at any time are proportional to the price of the commodity, they can be treated as negative yield, denoted by u. Thus:
$F_0 = S_0\ e^{(r+u)T}$

Question

Consider a 1-year futures contract on an investment asset that provides no income. It costs $2 per unit to store the asset, with the payment being made at the end of the year. Assume that the spot price is $450 per unit and the risk-free rate is 7% per annum for all maturities.

Calculate the futures price.

Solution

This corresponds to r = 0:07, S0 = 450, T = 1, and

$$U = 2e^{-0.07\times 1} = 1.865$$

The futures price is given by:

$$F_0 = (450 + 1.865)e^{0.07\times 1} = \$484.63$$

8.1 Pricing Commodity Futures

8.1.C Consumption Commodities

Commodities that are consumption assets rather than investment assets usually provide no income, but can be subject to significant storage costs. However they do have some convenience yields.

Users of a consumption commodity may feel that ownership of the physical commodity provides benefits that are not obtained by holders of futures contracts. The benefits from holding the physical asset are sometimes referred to as the convenience yield provided by the commodity.

If the amount of storage costs is known and has a present value U, the convenience yield is y, then;
$F_0 e^{yT} = (S_0 + \mathrm{U})\, e^{rT}$

If the storage costs per unit are a constant proportion, u, of the spot price, then y is defined so that
$F_0 e^{yT} = S_0\, e^{(r+u)T}$

Question

The spot price of silver is $15 per ounce. The storage costs are $0.24 per ounce per year payable quarterly in advance. Assuming that interest rates are 10% per annum for all maturities, calculate the futures price of silver for delivery in 9 months.

Solution

The present value of the storage costs for nine months are

$$0.06 + 0.06e^{-0.10\times0.25} + 0.06e^{-0.10\times0.5} = 0.176$$

or \$0.176. The futures price is from equation (5.11) given by F_0 where

$$F_0 = (9.000 + 0.176)e^{0.1\times0.75} = 9.89$$

i.e., it is \$9.89 per ounce.

9 Are Forward and Futures prices equal?

What do you think about forward and futures prices being equal or not?

9 Are Forward and Futures prices equal?

- When the short-term risk-free interest rate is constant, the forward price for a contract with a certain delivery date is in theory the same as the futures price for a contract with that delivery date.

- The theoretical differences between forward and futures prices for contracts that last only a few months are in most circumstances sufficiently small to be ignored. In practice, there are a number of factors not reflected in theoretical models that may cause forward and futures prices to be different. These include taxes, transactions costs, and the treatment of margins.

- Despite all these, for most purposes it is reasonable to assume that forward and futures prices are the same.

Quick Recap

- An investment asset is an asset that is held for investment purposes by significant numbers of investors.
- A consumption asset is an asset that is held primarily for consumption. It is not usually held for investment.
- Short selling involves selling an asset that is not owned.

Quick Recap

- We use the following notations - T: Time until delivery, S_0: Current price of the asset, F_0: Current price of the forward, r: risk-free rate.

- Forward price of an investment asset with no income is given as F_0 = S_0 x e^{rT}.

- Forward price of an investment asset with known income is given as F_0 = (S_0 - I) x e^{rT}.

- Forward price of an investment asset with known yield is given as F_0 = S_0 x $e^{(r-q)T}$.

Quick Recap

Summary of results for a contract with time to maturity T on an investment asset with price S_0 when the risk-free interest rate for a T-year period is r.

Asset	*Forward/futures price*	*Value of long forward contract with delivery price K*
Provides no income:	S_0e^{rT}	$S_0 - Ke^{-rT}$
Provides known income with present value I:	$(S_0 - I)e^{rT}$	$S_0 - I - Ke^{-rT}$
Provides known yield q:	$S_0e^{(r-q)T}$	$S_0e^{-qT} - Ke^{-rT}$

Quick Recap

- A stock index can usually be regarded as the price of an investment asset that pays dividends and it can be priced as $F_0 = S_0 \, e^{(r-q)T}$.

- Currency futures are contracts for currencies that specify the price of exchanging one currency for another at a future date.

- When the underlying asset is one unit of the foreign currency, the futures is priced as $F_0 = S_0 \, e^{(r-r_f)T}$

Quick Recap

- Commodity futures contract is an agreement to buy or sell a predetermined amount of a commodity at a specific price on a specific date in the future.

Asset	Price
Commodities that are investment asset	$F_0 = S_0\, e^{rT}$
Commodity with net storage and income	$F_0 = (S_0 + \mathrm{U})e^{rT}$ $F_0 = S_0\, e^{(r+u)T}$
Consumption commodity with storage costs and convenience yield	$F_0 e^{yT} = (S_0+\mathrm{U})\, e^{rT}$ $F_0 e^{yT} = S_0\, e^{(r+u)T}$

- For most purposes it is reasonable to assume that forward and futures prices are the same.