24 March 2022 10:28

$$\text{Convexity} \approx \frac{PV_- + PV_0 - 2PV_0}{PV_0 \cdot \Delta y^2}$$ X

$$\frac{d}{di}\left(\frac{dB}{di|y}\right) \Rightarrow \text{convexity} \Rightarrow \text{curvature}$$

Immunization

↓ V_A > V_L i ↓ ↑ i ↑

Bonds Bonds i ↓

↑ PV_A < PV_L ↑

Iam

IL&FS → Scam

A.L.M

STL → cash X

30 60 90 i ↓

LTP

$PV_A > PV_L$ X

Match Assets ⇒ Liab. ALM

Portfolio Mgmt → higher return.

MTD ⟵ Asset.

Eg 10,000 p.y. (EOY) → 65   50, 53, 55

+ 100,000 at age 65

Assets ⇒ 3 Bonds

FV ⇒ 100,000

coupon rate → 10%

n = 10, 12, 15

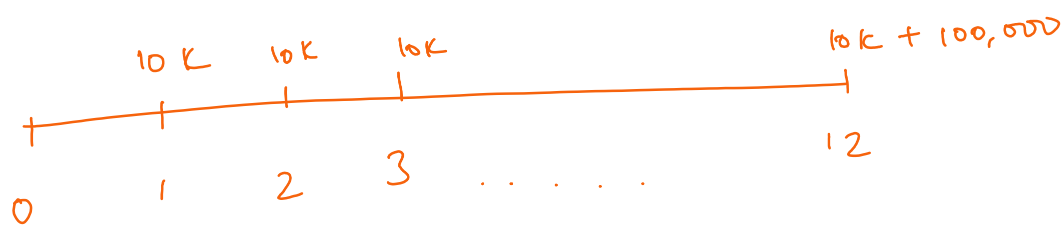
YTM 10%. 11%. 12%.

$PV_{10} = 100,000$

$PV_{12} = 93507.64$

$PV_{15} = 86,378.27$

27



$= 10000\, a_{\overline{12}|}^{@11\%} + 100,000\, v^{12}$

v^{12} ... $100,000\,(1+11\%)^{-12}$ $v =$ 1

$= 10{,}000\left(\frac{1-v}{11\%}\right) + 100{,}000\,(\ldots)$ $\quad \frac{1}{1+i}$

$y = 10\%$ fo

$PV_0 = 1v + 1v^2 = 1.7355$

(i) $0 \to 0$, $1, 2 \to 1$

$\Rightarrow$

$y \Rightarrow 10$

(ii) 1.7355

$AV_1 = 1.9090 \rightarrow 1 \Rightarrow 0.9090$

$AV_2 = 0.9090 \times 1.1 \Rightarrow 1$

(iii)

$PV_0 = 1.7355$

ALM X

Frankbhai → Actuary 1950

(1) $PV_A = PV_L$

(2) $D_A = D_L$

(3) $C_A \geq C_L$

5% →

Volatility :

$$A = \sum_{t=0}^{n} C_t v^t = \sum C_t (1+i)^{-t}$$

$$\frac{dA}{di} = \sum C_t \cdot -t \cdot (1+i)^{-(t+1)} = -\sum t \cdot C_t v^{t}$$

$$\nu = \frac{\frac{dA}{di}}{A} \Rightarrow \frac{-A'}{A}$$

$$\nu = \frac{-A'}{A}$$

DMT / Mac. duration

$$A = \sum c_t v^t = \sum c_t e^{-\delta t}$$

$$A' = \sum c_t e^{-\delta t}(-t) = -\sum t \cdot c_t \cdot e^{-\delta t}$$

$$= -\sum t \cdot c_t v^t$$

$10 - ZCB \Rightarrow$ < 10

$Z \Rightarrow 10$

$Z = (1+i) v(i)$

E.

=

Task volatility

Homework

3yr FIB

coupons → 5% p.a.

Redeemable at par

$v(5\%)$

↓

4%

$v(5\%) = 2.72325$

$P(4\%) = 102.78$

cou

ytm

5%

$$100 \longrightarrow 102.72$$

$$100(1 - 1\% \times 2.7)$$

$$DMT(i) = \frac{\sum t \cdot C_t \cdot v^t + n \times C_n \times v^n}{\sum C_t \cdot v^t}$$

8

0 1

$= 8v + 8v^2$

=

$= 8a_{\overline{10}|} +$

$= 1 \times 8v + 2 \times 8v^2 + 3 \times 8v^3 + \dots \ 108 \times 10$

$= 8 Ia_{\overline{10}|} + 1000v^{10}$

$$z = \frac{8\,Ia_{\overline{10|}} + 1000\,v^{10}}{8\,a_{\overline{10|}} + 100\,v^{10}}$$

0

$= 7.54$

$1 \times 2 \times$

Coupon → 3% RAP

$P = 3a$

7% ⟶ 8%

i) 5yrs

ii) 25 yrs.

Effective duration

(1.) A.K.A volatility

(2.) $v(i) = -\dfrac{A'}{A}$

$$= \frac{\sum_{t=1}^{n} t \cdot C_t \cdot v^{t+1}}{\sum_{t=1}^{n} C_t \cdot v^t}$$

AKA

$\tau =$

$\tau =$

(3.) i to $i+\varepsilon$, $\approx \varepsilon v(i)$

$A(1 - \varepsilon v(i))$

(4.)

Q3 Asset A $\rightarrow$ 11 - yr ZCB

B $\rightarrow$ 9663 $\rightarrow$ time

26910 ... time

Asset A

$$P_A = 100(1+i)^{-11}$$

$$\vartheta_A = \frac{-P_A'}{P_A} = \cancel{-}\frac{\left[1\cancel{0}0 \cdot (\cancel{-}11)(1+i\right.}{1\cancel{0}0\,\cancel{(1+i)^{-11}}}$$

$$C_A = \frac{P_A''}{P_A} = \frac{1\cancel{0}0(\cancel{-}11)(\cancel{-}12)(1+i)}{1\cancel{0}0\,\cancel{(1+i)^{-11}}}$$

Asset B

$$P_B = 9663(1+i)^{-5} + 26910\,($$

$$\Big[9663(-5)(1+i)^{-6} + 26910(-20)(1+$$

$$\nu_B = \frac{-P_B'}{P_B} = \frac{-[9663(-5)(1+i)^{-6} \cdots}{9663(1+i)^{-5} + 26910(1+i)^{-20}}$$

$$C_B = \frac{P_B''}{P_B} = \frac{9663(-5)(-6)(1+i)^{-7} + 26910(-20)(-21)\cdots}{9663(1+i)^{-5} + 26910(1+i)^{-20}}$$

$V_A(i) =$ P.V. of Assets @ eff. roi i

$V_L(i) =$

$\nu_A(i) =$

$\nu_L(i) =$

$C_A(i) =$

$C_L(i) =$

At rate of int. i_0, the fund is imm.

in the rate of int of ε, if and only if

and

$$S(i_0) = V_A(i_0) - V_L(i_0) = 0$$

$$S(i_0 + \varepsilon)$$

$0 \Rightarrow S'(i_0) = 0$

+ve

$$S(i_0+\varepsilon) = \cancel{S(i_0)}^{0} + \varepsilon \cancel{S'(i_0)} + \frac{\varepsilon^2}{2} \cancel{S''(i_0)}^{+ve} +$$

$$f(x+h) = f(x) + hf'(x) + \frac{h^2}{2!} \times f''(x$$

$\varepsilon \Rightarrow$ +ve / -ve

$$S'(i_0) = \frac{V_A'(i_0)}{V_A(i_0)} = \frac{V_L'(i_0)}{V_L(i_0)} \Rightarrow V_A'(i_0)$$

0

$$\tau_A(i_0) =$$

$$S''(i_0) > 0 \Rightarrow V_A''(i_0) > V_L''(i_0) \Rightarrow C_A$$

$V_A(i_0)$ $V_L(i_0)$

Redington's immunisation:

(i) $V_A(i_0) = V_L(i_0)$

(ii) $V'_A(i_0) = V'_L(i_0)$ ⟹

(iii) $C_A(i_0) > C_L(i_0)$

5yr → x 10yr → y

$V_A(7\%) = xv^5 + yv^{10}$

$V_L(7\%) = 50{,}000\,v^6 + 50000\,v^8 = 62{,}417.$

∴ eqn (1) → $xv^5 + yv^{10} = 62{,}417.57$

$V'_A(7\%) = x(-5)v^6 + y(-10)v^{11}$

$V'_L(7\%) = 50{,}000\left[(-6)v^7 + (-8)v^9\right]$

As per R.I.T

$$\frac{-V_A'(7\%)}{V_A(7\%)} = \frac{-V_L'(7\%)}{V_L(7\%)}$$

$$\frac{x(5)v^6 + y(10)v^{11}}{\cancel{V_A(7\%)}\,V_L(7\%)} = \frac{50{,}000\left[6v^7 + 8v^9\right]}{62417.57}$$

$$5xv^6 + 10yv^{11} = 404{,}398$$ — (2

$$V_A''(7\%) = (-5)(-6)v^7 \times 53709.12$$

$>$

$$V_L''(7\%) = 50{,}000(-6)(-7)v^8 + 50{,}000$$

$\underset{\sim}{7\%} \longrightarrow 6.5\%$

$V_A(6.5\%) \quad 47454\,\underset{\sim}{v}^{10} + 53709.12\,\underset{\sim}{v}^{5}$

$V_L(6.5\%) = 50{,}000\left[v^6 + v^8\right] \quad 64{,}478$

@ $7\% \longrightarrow 7.5\%$

$V_A(7.5\%) \qquad\qquad V_L(7.5\%)$

$= 60{,}437 \quad > \quad 60{,}433$