INSTITUTE OF ACTUARIAL & QUANTITATIVE STUDIES

Subject: Numerical methods & Algebra

Chapter: Unit 3 & 4

Category: Assignment 2 solutions

1. The smallest 3 digit number that will leave a remainder of 2 when divided by 3 is 101.
The next number that will leave a remainder of 2 when divided by 3 is 104, 107, ...
The largest 3 digit number that will leave a remainder of 2 when divided by 3 is 998.

So, it is an AP with the first term being 101 and the last term being 998 and common difference being 3.

Sum of an AP = $\left(\frac{\text{First Term} + \text{Last Term}}{2}\right) * \text{Number of Terms}$

We know that in an A.P., the nth term $a_n = a_1 + (n - 1)*d$

In this case, therefore, 998 = 101 + (n - 1)* 3
i.e., 897 = (n - 1) * 3
Therefore, n - 1 = 299
Or n = 300.

Sum of the AP will therefore, be $\frac{101 + 998}{2} * 300$ = 164,850

2. $T_6 = 32$ and $T_8 = 128$

$\Rightarrow ar^5 = 32$ ..... (i) and

$ar^7 = 128$ .....(ii)

Dividing (ii) by (i), we have

$r^2 = 4$

r = 2

3. $(4 + 3xx)^5 = \sum_{ki=0}^{5} \binom{5}{i0} 3x^i x.4^{5-i}$

$$= \binom{5}{0}(4^5) + \binom{5}{1}(4^4)(3x)^1 + \binom{5}{2}(4^3)(3x)^2 + \binom{5}{3}(4^2)(3x)^3 + \binom{5}{4}(4^1)(3x)^4 + \binom{5}{5}(3x)^5$$

$$= 4^5 + (5)(4^4)(3x) + \frac{5(4)}{2!}(4^3)(3x)^2 + \frac{5(4)(3)}{3!}(4^2)(3x)^3 + (5)(4)(3x)^4 + (3x)^5$$

$$= \boxed{1024 + 3840x + 5760x^2 + 4320x^3 + 1620x^4 + 243x^5}$$

4. Put 1 – x = y.

$(1 - x + x^2)^4 = (y + x^2)^4$

$$= {}^4C_0\ y^4 (x^2)^0 + {}^4C_1\ y^3 (x^2)^1 + {}^4C_2\ y^2 (x^2)^2 + {}^4C_3\ y (x^2)^3 + {}^4C_4\ (x^2)^4$$
$$= y^4 + 4y^3 x^2 + 6y^2 x^4 + 4y x^6 + x^8$$
$$= (1-x)^4 + 4x^2 (1-x)^3 + 6x^4 (1-x)^2 + 4x^6 (1-x) + x^8$$
$$= 1 - 4x + 10x^2 - 16x^3 + 19x^4 - 16x^5 + 10x^6 - 4x^7 + x^8$$

5. 1)

$$(w+2)\left(\frac{w^2-10}{w+2} + w - 4\right) = (w-3)(w+2)$$
$$w^2 - 10 + (w-4)(w+2) = (w-3)(w+2)$$
$$w^2 - 10 + w^2 - 2w - 8 = w^2 - w - 6$$
$$w^2 - w - 12 = 0$$
$$(w-4)(w+3) = 0$$

$w - 4 = 0$, $w = 4$ OR $w + 3 = 0$, $w = -3$

6. 1)

$ax^2 + bx + c = 0$

$x^2 - 6x + 5 = 0$

$a = 1,\ b = -6,\ c = 5$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4 \cdot 1 \cdot (5)}}{2 \cdot 1}$$
$$x = \frac{6 \pm \sqrt{36 - 20}}{2}$$
$$x = \frac{6 \pm \sqrt{16}}{2}$$
$$x = \frac{6 \pm 4}{2}$$
$$x = \frac{6+4}{2},\quad x = \frac{6-4}{2}$$
$$x = \frac{10}{2},\quad x = \frac{2}{2}$$
$$x = 5,\quad x = 1$$

2)

$$ax^2 + bx + c = 0$$
$$x^2 + 6x + 4 = 0$$

$a = 1,\ b = 6,\ c = 4$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(6) \pm \sqrt{(6)^2 - 4 \cdot 1 \cdot (4)}}{2 \cdot 1}$$

$$x = \frac{-6 \pm \sqrt{36 - 16}}{2}$$

$$x = \frac{-6 \pm \sqrt{20}}{2}$$

$$x = \frac{-6 \pm 2\sqrt{5}}{2}$$

$$x = \frac{2(-3 \pm 2\sqrt{5})}{2}$$

$$x = -3 \pm 2\sqrt{5}$$

$$x = -3 + 2\sqrt{5},\quad x = -3 - 2\sqrt{5}$$

7. 1)

We have, $a = 1$, $b = 4$ and $c = -21$.

Find the value of m and n.

$m = 4/2 = 2$

$n = -21 - (16/4) = -21 - 4 = -25$

So, the equation is solved as,

$(x + 2)^2 - 25 = 0$

$x + 2 = \pm 5$

$x = 3, -7$

2)

We have, a = 1, b = 10 and c = 21.

Find the value of m and n.

m = 10/2 = 5

n = 21 – (100/4) = 21 – 25 = –4

So, the equation is solved as,

$(x + 5)^2 – 4 = 0$

$x + 5 = \pm 2$

$x = -3, -7$

8. 1)

The given inequality is $\frac{7x+5}{3x-1} < 5$

$$\Rightarrow \frac{7x+5}{3x-1} - 5 < 0$$

$$\Rightarrow \frac{7x+5-15x+5}{3x-1} < 0$$

$$\Rightarrow \frac{-8x+10}{3x-1} < 0$$

$$\Rightarrow \frac{8x-10}{3x-1} > 0$$

$$\Rightarrow \frac{4x-5}{3x-1} > 0$$

The critical points are x = 1/3, 5/4

The given inequality is positive, the solution is x ε (-∞, 1/3)∪(5/4, ∞)

2)

The given inequality is $\frac{1}{x+3} \le 11$

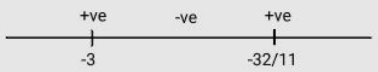
$$\Rightarrow \frac{1}{x+3} - 11 \le 0$$

$$\Rightarrow \frac{1-11x-33}{x+3} \le 0$$

$$\Rightarrow \frac{-11x-32}{x+3} \le 0$$

$$\Rightarrow \frac{11x+32}{x+3} \ge 0$$

The critical points are x = -3, -32/11



The solution is (-∞, -3) ∪ (-32/11, ∞)

9. 1) $|x^2 - 5x + 6| = |(x-2)(x-3)| = |f(x)|$

As per modulus definition,

$|f(x)| = f(x)$; if f(x) is positive

$|f(x)| = -f(x)$; if f(x) is negative

$f(x) = (x-2)(x-3)$ is positive or zero when $x = (-\infty, 2] \cup [3, \infty)$

$f(x) = (x-2)(x-3)$ is negative when $x = (2, 3)$

So, $|x^2 - 5x + 6| = (x^2 - 5x + 6)$ when $x = (-\infty, 2] \cup [3, \infty)$ and

$|x^2 - 5x + 6| = -(x^2 - 5x + 6)$ when $x = (2, 3)$

2)

Every modulus is a non-negative number and if two non-negative numbers add up to get zero then individual numbers itself equal to zero simultaneously.

$x^2 - 5x + 6 = 0$ for x = 2 or 3

$x^2 - 8x + 12 = 0$ for x = 2 or 6

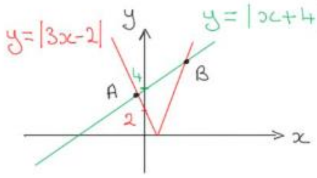
Both the equations are zero at x = 2

So, x = 2 is the only solution for this equation.

10.

Solve $|3x-2| \geq |x+4|$

Method 1 : Graphical



For $|3x-2| \geq |x+4|$

Look for the values of x for which the graph of $y=|3x-2|$ is above the graph of $y=|x+4|$

For A: $x+4=-(3x-2)$

$\therefore x+4=-3x+2$

$\therefore 4x=-2$

$\therefore x=-\frac{2}{4}$

$\therefore x=-\frac{1}{2}$

For B:

$x+4=3x-2$

$\therefore 2x=6$

$\therefore x=3$

$\therefore x \leq -\frac{1}{2}$ or $x \geq 3$