Lecture

INSTITUTE OF ACTUARIAL & QUANTITATIVE STUDIES

Class: FY BSc

Subject : Numerical methods

Subject Code: PUSAS201

Chapter: Unit 1 Chapter 1

Chapter Name: Errors, Dimension & Interpolation.

Today's Agenda

1. Define percentage
    1. Percentage increase or decrease

2. Change

3. Errors
    1. Types of errors

4. Dimensions

5. Interpolation
    1. Linear interpolation

1 Define percentage

A **percentage** is a number or ratio that can be expressed as a fraction of 100. The **percentage means,** a part per hundred.

To determine the percentage, we have to divide the value by the total value and then multiply the resultant by 100.

Percentage formula = (Value/Total value) × 100

1 Example

Calculate 10% of 80.

Solution:
Let 10% of 80 = X
10/100 * 80 = X
X = 8

To understand percentages look at the link below:

https://www.khanacademy.org/math/cc-sixth-grade-math/x0267d782:cc-6th-rates-and-percentages/cc-6th-percent-problems/v/finding-percentages-example

1.1 Percentage increase or decrease

The percentage increase is equal to the subtraction of the original number from a new number, divided by the original number and multiplied by 100.

% increase = [(New number – Original number)/Original number] x 100
Where,
increase in number = New number – original number

Similarly, a percentage decrease is equal to the subtraction of a new number from the original number, divided by the original number and multiplied by 100.

% decrease = [(Original number – New number)/Original number] x 100
Where decrease in number = Original number – New number
So basically if the answer is negative then there is a percentage decrease.

1.1 Example

In January Dylan worked a total of 35 hours, in February he worked 45.5 hours – by what percentage did Dylan's working hours increase in February?

Solution:
First we calculate the difference in hours between the new and old numbers.
45.5 – 35 hours = 10.5 hours.
We can see that Dylan worked 10.5 hours more in February than he did in January – this is his **increase**.

To work out the increase as a percentage it is now necessary to divide the increase by the original (January) number:
10.5 ÷ 35 = 0.3

Finally, to get the percentage we multiply the answer by 100. This simply means moving the decimal place two columns to the right.
0.3 × 100 = 30

Dylan therefore worked 30% more hours in February than he did in January.

2 Change

1. Absolute change:

Absolute change means the measures taken for estimating the absolute value on two numbers over a time period. The difference in the value of the variable could be positive or negative. Actual increase or decrease from a reference value to a new value

Absolute change = New value – original value

2. Proportionate change:

Proportionate change calculates the change relative to the original number in the population, dividing by the original amount. It is sometimes also called relative change.

Proportionate change = $$\frac{newvalue - originalvalue}{originalvalue}$$

2 Change

3. Percentage change:

It expresses the proportionate change as a percentage.

Percentage change = $\dfrac{absolutechange}{originalvalue} * 100$

2

Exampl
e

Example 1:

On September 20, a gallon of gas at my usual gas station cost 1.83.
On September 30, I noticed that the price had gone down to1.65 per gallon.

Absolute change in the price of gas is:
1.65 – 1.83 = – 0.18 dollars per gallon.

Relative change in the price of gas is:
(1.65 – 1.83)/1.83 = 0.0983

Percentage change in the price of gas is:
0.0983×100 = 9.83%

3 Errors

- **Error**, it is the difference between a true value and an estimate, or approximation, of that value. They are similar to change, but errors specifically refer to difference in actual (true) value and the expressed (experimented) value.

- Errors are of different types like absolute, proportional, percentage, round-off, truncated etc.

- Errors can occur due to inaccurate fitting, randomness, technical or computing defaults, human errors etc

- Calculating and analyzing errors help to study the goodness of any model which is built to estimate values. It also helps to know the accuracy and precision.

3.1 Types of Errors

1. Absolute error

It refers to the absolute value of the difference between he observed value of a quantity and the true value.

Absolute error = approx value – true value

2. Proportionate error:

It refers to the error relative to the true value. It is obtained by dividing the absolute error by the true value. Sometimes also called the relative error .

Proportionate error = $\dfrac{Absolute error}{true value}$

3.1 Types of Errors

3. Percentage error:
It is the proportionate error expressed as a percentage.

Percentage error = $$\frac{Asoluteerror}{truevalue} * 100$$

3.1 Example

Example 1:

If we are measuring the length of an eraser. The actual length is 35 mm and the measured length is 34.13 mm.

Absolute Error = (35 – 34.13) mm = 0.87 mm

Proportionate error = = 0.87/35 = 0.02485

Percentage error = 0.02485 13 × 100 = 2.485%

3.1 Example

The life insurance company Happy life Assurance ltd is trying to forecast the number of claims based on the experience in the past years. The company has developed a certain model and calculated the estimates as follows;

Motor Policy/ Attribute	Female	Male	Countryside	Inter city
Estimates	786	995	1589	1672

The true values are found from the books of the company and given below:

Motor Policy/ Attribute	Female	Male	Countryside	Inter city
True values	780	991	1582	1683

Calculate the absolute error, proportionate error and percentage error for the data. The company considers thee model as a good fit if the errors made at any point are not more than +/- 0.85%. Comment whether the particular model used is a good fit or not.

3.1 Example

Motor Policy/ Attribute	Female	Male	Countryside	Inter city
True value	780	991	1582	1683
Estimated value	786	995	1589	1672
Absolute Error = Estimated – True	+6	+4	+7	-11
	0.0076923	0.0040363	0.0044248	-0.006536
	0.76923%	0.40363%	0.44248%	-0.6536%
We see that none of the attribute has an error above +/- 0.85%. Hence we can conclude that the model is good fit for estimation.				

4 Dimension

Dimensions in mathematics are the measure of the size or distance of an object or region or space in one direction. In simpler terms, it is the measurement of the length, width, and height of anything.

- Dimensions are used to show what a value represents
- Numbers or coefficients (including constants such as p) have a dimension of zero and they are referred to as dimensionless.
- If two values that have the same dimension are divided, then the resulting value is dimensionless.
- Dimensions can give us a convenient way of telling if a formula is correct.

5 Interpolation

- **Interpolation**, in mathematics, means the determination or estimation of the value of $f(x)$, or a function of x, from certain known values of the function.

- During certain times exact values of functions can not be found by algebraic methods. At such times a close approximate estimate can be given if we know few values of the function close to the true value that we are evaluating.

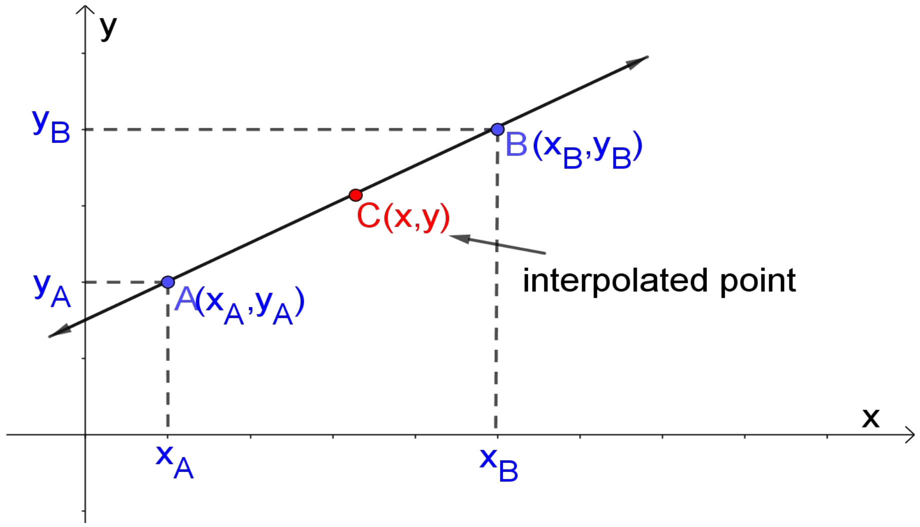
- Suppose the true value x lies between the two known values, say x1 and x2 then it is called interpolation; otherwise extrapolation. *Extrapolation* is used to find data points outside the range of known data points.

- In practice, different methods of interpolation are used to predict unknown values for any geographic point data, such as elevation, rainfall, chemical concentrations, noise levels, and so on.

5.1 Linear Interpolation

The most used method for interpolation is the linear interpolation which we will study in detail. Consider the following steps to understand what linear interpolating exactly is.

- Let us say that we have two known points A(Xa,Ya) and B(Xb,Yb).
- Now we want to estimate what Y value we would get for some X value that is between Xa and Xb. Call this Y value estimate — an *interpolated* value.
- By using linear interpolation it means to draw a straight line between X_A,Y_A and X_B,Y_B.
- We look to see the Y value on the line for our chosen X. This is *linear interpolation*. The



Formula used for linear interpolation is;

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

5.1 Example

Example 1:

Find the value of y at x = 4 given some set of values (2, 4), (6, 7)?

Solution:

Given the known values are,

$x = 4$; $x_1 = 2$; $x_2 = 6$; $y_1 = 4$; $y_2 = 7$

The interpolation formula is,

$$y = y_1 + \frac{(x - x_1)(y_2 - y_1)}{x_2 - x_1}$$

$$y = 4 + \frac{(4 - 2)(7 - 4)}{6 - 2}$$

$$y = 4 + \frac{6}{4}$$

$$y = \frac{11}{2}$$

5.1 **Exampl
e**

Example 2:

A gardener planted a tomato plant and she measured and kept track of its growth every other day. This gardener is a curious person, and she would like to estimate how tall her plant was on the fourth day. Her table of observations looked like this:

Day	Height (mm)
1	0
3	4
5	8
7	12
9	16

Calculate the Height of the tomato plant on day four for the gardener, assuming linear growth.

5.1 Example

It is given that there is linear growth, which means there was a linear relationship between the number of days measured and the plant's height growth. Using the data given the first set of values for day three are (3,4), the second set of values for day five are (5,8), and the value for *x* is 4 since we want to find the height, y, on the fourth day.

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - 4 = \frac{8 - 4}{5 - 3}(x - 3)$$

$y - 4 = 2(4 - 3)$ [Putting x = 4]

$y = 2 * 1 + 4 = 6$

The height of the tomato plant on fourth day was 6mm.