INSTITUTE OF ACTUARIAL & QUANTITATIVE STUDIES

Subject: Numerical methods

Chapter: Unit 1

Category: Practice questions

Answer 1.

True value X= 3.1415926, And Approx. value X_1= 3.1428571.

Absolute error, $E_A = X - X_1 = \delta X$

E_A= 3.1415926- 3.1428571 = -0.0012645

Relative error = (absolute error)/(true value of quantity)

E_R = E_A/X = (Absolute Error)/X, E_A=(-0.0012645)/3.1415926 = -0.000402ans.

Percentage Error,

E_P= 100 × E_A/X = 100 × (-0.000402) = - 0.0402ans.

Answer 2.

$$– \text{Absolute error} = \text{True value} – \text{measured value}$$
$$= 1.605 – 1.603 = 0.002 \text{ inch } \textit{Ans}$$
$$\text{Relative Error} = \frac{\text{Absolute Error}}{\text{True Value}} \times 100 = \frac{0.002}{1.605} \times 100$$
$$= 0.1246\% \ \textit{Ans}$$

Answer 3.

165, 127.5

Answer 4.

12 %

Answer 5.

Here, x0 (first element) is 0, h(length of interval). is 0.5, x is 1.8.

puing the values in the formula

x = x0 + nh,

1.8 = 0 + n(0.5)

n = 3.6.

Answer 6

Here it is obvious that given values are

x = 2

x_1 = -1

x_2 = 3

y_1 = 4

y_2 = 6

Thus, applying the formula,

$y = y_1 + y_2 - y_1 \text{ x } 2 - x_1 \times (x - x1)$

Put all known values, we get

y = 4 + 6 − 43 − (−1) × (2−(−1))

i.e. y = 4 + 24(3)

y = 112
y = 5.5
Thus the value of f(2) is 5.5.

Answer 7.

Let $f(x) = x^3 - 4x - 9$

Since f(2) is negative and f(3) is positive, a root lies between 2 and 3

First approximation to the root is

$$x_1 = \frac{1}{2}(2+3) = 2.5$$

Then $f(x_1) = 2.5^3 - 4(2.5) - 9 = -3.375$

i.e. negative. The root lies between x_1 and 3. Thus the second approximation to the root is

$$x_2 = \frac{1}{2}(x_1+3) = 2.75$$

Then $f(x_2) = (2.75)^3 - 4(2.75) - 9 = 0.7969$ i.e. positive.

The root lies between x_1 and x_2. Thus the third approximation to the root is

$$x_3 = \frac{1}{2}(x_1+x_2) = 2.625$$

Then $f(x_3) = (2.625)^3 - 4(2.625) - 9 = -1.4121$ i.e negative

The root lies between x_2 and x_3. Thus the fourth approximation to the root is

$$x_4 = \frac{1}{2}(x_2+x_3) = 2.6875$$

Answer 8.

Clarification: Function $f(x) = 3x^2 - 5x - 2 = 0$. The Iteration table is given as follows

No.	a	b	c	f(a)*f(c)
1	0	3	2.25	Positive
2	1.5	3	2.25	Negative
3	1.5	2.25	1.875	Positive
4	1.875	2.25	2.0625	Negative
5	1.875	2.0625	1.96875	Positive
6	1.96875	2.0625	2.015625	Negative
7	1.96875	2.015625	1.9921875	Positive
8	1.9921875	2.015625	2.00390625	Negative
9	1.9921875	2.00390625	1.99806875	Positive
10	1.998046875	2.00390625	2.000976563	Negative

The difference between final values is less than 0.01 hence we stop the iterations. So one of the roots of $f(x) = 3x^2 - 5x-2 = 0$ is approximately 2.000976563.

Answer 9.

$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$

Let initial guess x_0 = 2.25

f(2.25) = $(2.25)^3$ − 3×2.25 − 5 = −0.3594

f '(2.25) = 3 × $(2.25)^2$ − 3 = 12.1875

Therefore, 1st iteration:

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

$x_1 = 2.25 - \frac{(-0.3594)}{12.1875}$

$x_1 = 2.2794$

2nd iteration:

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$x_2 = 2.2794 - \frac{(2.2794)^3-(3\times 2.2794)-5}{(3\times(2.2794)^2)-3}$

$x_2 = 2.279$

Answer 10.

Here is the derivative of the function since we'll need that.

$$f'(x) = 3x^2 - 14x + 8$$

We just now need to run through the formula above twice.

Hide Step 2 ▾

The first iteration through the formula for x_1 is,

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 5 - \frac{f(5)}{f'(5)} = 5 - \frac{-13}{13} = 6$$

Hide Step 3 ▾

The second iteration through the formula for x_2 is,

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 6 - \frac{f(6)}{f'(6)} = 6 - \frac{9}{32} = 5.71875$$

So, the answer for this problem is $\boxed{x_2 = 5.71875}$.

Answer 11:

This is an example of linear growth and hence the linear interpolation formula is very much suitable here. We may take (3,4) as the first data point and (5,8) as the second data point.

We have values as:

- $Y_1 = 4$
- $X_1 = 3$
- $Y_2 = 8$
- $X_2 = 5$

$$y = y_1 + \frac{(x-x_1)(y_2-y_1)}{x_2-x_1}$$

Substituting the values we have:

$$y = 4 + \frac{(x-3)(8-4)}{5-3}$$

$$y = 4 + 2(x-3)$$

$$y = 2x - 2$$

Thus for forth day x = 4.

Hencey=2×4–2

Y = 6

Therefore, at forth day height will be 6 units.

Answer 12:

Given: $x^2-3 = 0$

Let $f(x) = x^2-3$

Now, find the value of f(x) at a= 1 and b=2.

$f(x=1) = 1^2-3 = 1 - 3 = -2 < 0$

$f(x=2) = 2^2-3 = 4 - 3 = 1 > 0$

The given function is continuous, and the root lies in the interval [1, 2].

Let "t" be the midpoint of the interval.

I.e., t = (1+2)/2

t =3 / 2

t = 1.5

Therefore, the value of the function at "t" is

$f(t) = f(1.5) = (1.5)^2-3 = 2.25 - 3 = -0.75 < 0$

If f(t)<0, assume a = t.

and

If f(t)>0, assume b = t.

f(t) is negative, so a is replaced with t = 1.5 for the next iterations.

The iterations for the given functions are:

Iterations	a	b	t	f(a)	f(b)	f(t)
1	1	2	1.5	-2	1	-0.75
2	1.5	2	1.75	-0.75	1	0.062
3	1.5	1.75	1.625	-0.75	0.0625	-0.359
4	1.625	1.75	1.6875	-0.3594	0.0625	-0.1523
5	1.6875	1.75	1.7188	-01523	0.0625	-0.0457
6	1.7188	1.75	1.7344	-0.0457	0.0625	0.0081
7	1.7188	1.7344	1.7266	-0.0457	0.0081	-0.0189

So, at the seventh iteration, we get the final interval [1.7266, 1.7344]

Hence, 1.7344 is the approximated solution.

Answer 13:

Given measures are,
$f(x) = x^2 - 2 = 0$, $x_0 = 2$

Newton's method formula is: $x_1 = x_0 -$

$\frac{f(x_0)}{f'(x_0)}$

To calculate this we have to find out the first derivative f'(x)
$f'(x) = 2x$
So, at $x_0 = 2$,
$f(x_0) = 2^2 - 2 = 4 - 2 = 2$
$f'(x_0) = 2$

$\times$
$2 = 4$

Substituting these values in the formula,

$x_1 = 2 -$

$\frac{2}{4}$
$=$
$\frac{6}{4}$
$=$
$\frac{3}{2}$