INSTITUTE OF ACTUARIAL & QUANTITATIVE STUDIES

Subject: Numerical methods

Chapter: Matrices

Category:

Solutions:

1.

i)

$$\begin{pmatrix} 5 & 2 \\ 4 & 9 \\ 10 & -3 \end{pmatrix} + \begin{pmatrix} -11 & 0 \\ 7 & 1 \\ -6 & -8 \end{pmatrix} = \begin{pmatrix} 5+(-11) & 2+0 \\ 4+7 & 9+1 \\ 10+(-6) & -3+(-8) \end{pmatrix}$$

$$= \begin{pmatrix} -6 & 2 \\ 11 & 10 \\ 4 & -11 \end{pmatrix}$$

ii)

$$\begin{pmatrix} 7 & 3 \\ 5 & 9 \\ 11 & -2 \end{pmatrix} - \begin{pmatrix} -3 & 0 \\ 8 & 1 \\ -3 & -4 \end{pmatrix} = \begin{pmatrix} 7-(-3) & 3-0 \\ 5-8 & 9-1 \\ 11-(-3) & -2-(-4) \end{pmatrix}$$

$$= \begin{pmatrix} 10 & 3 \\ -3 & 8 \\ 14 & 2 \end{pmatrix}$$

2. x = 4

3.

Since corresponding entries of equal matrices are equal, So

$x = 3$ ---(i)

$3x - y = 2$ ---(ii)

$2x + z = 4$ ---(iii)

$3y - w = 7$ ---(iv)

Put the value of $x = 3$ from equation on (i) in eqation(ii),

$$3x - y = 2$$
$$3(3) - y = 2$$
$$9 - y = 2$$
$$y = 9 - 2$$
$$y = 7$$

Put the value of $y = 7$ in equation (iv),

$$3y - w = 7$$
$$3(7) - w = 7$$
$$w = 21 - 7$$
$$w = 14$$

Put the value of $x = 3$ in equation(iii),

$$2x + z = 4$$
$$2(3) + z = 4$$
$$6 + z = 4$$
$$z = 4 - 6$$
$$z = -2$$

Hence,

$$x = 3. y = 7. z = -2. w = 14$$

4. Given matrix equation can be written as,

$$A=\begin{bmatrix}9 & -1 & 4\\ -2 & 1 & 3\end{bmatrix}-\begin{bmatrix}1 & 2 & -1\\ 0 & 4 & 9\end{bmatrix}$$

$$A=\begin{bmatrix}9-1 & -1-2 & 4+1\\ -2-0 & 1-4 & 3-9\end{bmatrix}$$

5.

$$2x-3y=1$$
$$x+y=3$$
$$x=3-y$$
$$2(3-y)-3y=1$$
$$-5y=-5$$
$$y=1$$
$$x=3-1$$
$$x=2$$

6. Since the number of columns of matrix D equals the number of rows of matrix F, the product of DF is defined.

$$DF=\begin{bmatrix}6 & -2\\ 3 & 7\end{bmatrix}\begin{bmatrix}1 & -2\\ -3 & 4\end{bmatrix}$$

$$=\begin{bmatrix}(6)(1)+(-2)(-3) & (6)(-2)+(-2)(4)\\ (3)(1)+(7)(-3) & (3)(-2)+(7)(4)\end{bmatrix}$$

$$=\begin{bmatrix}12 & -20\\ -18 & 22\end{bmatrix}$$

7.

i) |A| = 0[0-5(-5)] – (a-b)[0(b-a)-5(-k)]+k[-5(b-a)-0(-k)]

=0[0+25]-(a-b)[0+5k]+k[-5b+5a-0]

=0-(a-b)(5k)-5bk+5ak

=-5ak+5bk-5bk+5ak

=0

|A|=0 therefore it is a singular matrix

ii) |A| =3(1-1)-8(-4+4)+1(-4+4)

= 3(0)-8(0)+1(0)

= 0

|A|= 0, therefore it is a singular matrix

8.

$$\text{Let } A = \begin{bmatrix} 2 & 5 & 1 \\ -5 & 4 & 6 \\ -1 & -6 & 3 \end{bmatrix}$$

$$\therefore A^T = \begin{bmatrix} 2 & -5 & -1 \\ -5 & 4 & -6 \\ 1 & 6 & 3 \end{bmatrix}$$

$$\therefore A^T = \begin{bmatrix} -2 & 5 & 1 \\ -5 & -4 & 6 \\ -1 & -6 & -3 \end{bmatrix}$$

$A \neq A^T$ and $A \neq -A^T$

Therefore, A is neither symmetric nor skew-symmetric.

9. The first step is to write down the augmented matrix for the system of equations

$$\left[\begin{array}{ccc|c} 2 & 5 & 2 & -38 \\ 3 & -2 & 4 & 17 \\ -6 & 1 & -7 & -12 \end{array}\right]$$

So now, let’s start with the elementary row operation.

$$\left[\begin{array}{ccc|c} 2 & 5 & 2 & -38 \\ 3 & -2 & 4 & 17 \\ -6 & 1 & -7 & -12 \end{array}\right] \quad \begin{array}{c} R_1 - R_2 \to R_1 \\ \to \end{array} \quad \left[\begin{array}{ccc|c} -1 & 7 & -2 & -55 \\ 3 & -2 & 4 & 17 \\ -6 & 1 & -7 & -12 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} -1 & 7 & -2 & -55 \\ 3 & -2 & 4 & 17 \\ -6 & 1 & -7 & -12 \end{array}\right] \quad \begin{array}{c} -R_1 \\ \to \end{array} \quad \left[\begin{array}{cccc} 1 & -7 & 2 & 55 \\ 3 & -2 & 4 & 17 \\ -6 & 1 & -7 & -12 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & -7 & 2 & 55 \\ 3 & -2 & 4 & 17 \\ -6 & 1 & -7 & -12 \end{array}\right] \quad \begin{array}{c} R_2 - 3R_1 \to R_2 \\ R_3 + 6R_1 \to R_3 \\ \to \end{array} \quad \left[\begin{array}{cccc} 1 & -7 & 2 & 55 \\ 0 & 19 & -2 & -148 \\ 0 & -41 & 5 & 318 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & -7 & 2 & 55 \\ 0 & 19 & -2 & -148 \\ 0 & -41 & 5 & 318 \end{array}\right] \quad \begin{array}{c} \frac{1}{19}R_2 \\ \to \end{array} \quad \left[\begin{array}{ccc|c} 1 & -7 & 2 & 55 \\ 0 & 1 & -\frac{2}{19} & -\frac{148}{19} \\ 0 & -41 & 5 & 318 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & -7 & 2 & 55 \\ 0 & 1 & -\frac{2}{19} & -\frac{148}{19} \\ 0 & -41 & 5 & 318 \end{array}\right] \quad \xrightarrow{R_3+41R_2\to R_3} \quad \left[\begin{array}{ccc|c} 1 & -7 & 2 & 55 \\ 0 & 1 & -\frac{2}{19} & -\frac{148}{19} \\ 0 & 0 & \frac{13}{19} & -\frac{26}{19} \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & -7 & 2 & 55 \\ 0 & 1 & -\frac{2}{19} & -\frac{148}{19} \\ 0 & 0 & \frac{13}{19} & -\frac{26}{19} \end{array}\right] \quad \xrightarrow{\frac{19}{13}R_3} \quad \left[\begin{array}{ccc|c} 1 & -7 & 2 & 55 \\ 0 & 1 & -\frac{2}{19} & -\frac{148}{19} \\ 0 & 0 & 1 & -2 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & -7 & 2 & 55 \\ 0 & 1 & -\frac{2}{19} & -\frac{148}{19} \\ 0 & 0 & 1 & -2 \end{array}\right] \quad \xrightarrow[R_2+\frac{2}{19}R_3\to R_2]{R_1-2R_3\to R_1} \quad \left[\begin{array}{ccc|c} 1 & -7 & 0 & 59 \\ 0 & 1 & 0 & -8 \\ 0 & 0 & 1 & -2 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & -7 & 0 & 59 \\ 0 & 1 & 0 & -8 \\ 0 & 0 & 1 & -2 \end{array}\right] \quad \xrightarrow{R_1+7R_2\to R_1} \quad \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -8 \\ 0 & 0 & 1 & -2 \end{array}\right]$$

From the final augmented matrix we get the solution to the system is $x=3, y=-8, z=-2$

10. Given, AX=B

$$\therefore \begin{bmatrix} 1 & 2 & 3 \\ -1 & 1 & 2 \\ 1 & 2 & 4 \end{bmatrix} X = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

Applying $R_2 \rightarrow R_2 + R_1$ and $R_3 \rightarrow R_3 - R_1$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 3 & 5 \\ 0 & 0 & 1 \end{bmatrix} X = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

Applying $R_2 \rightarrow \left(\frac{1}{3}\right) R_2$, we get

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & \frac{5}{3} \\ 0 & 0 & 1 \end{bmatrix} X = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

Applying $R_1 \to R_1 - 2R_2$, we get

$$\begin{bmatrix} 1 & 0 & -\frac{1}{3} \\ 0 & 1 & \frac{5}{3} \\ 0 & 0 & 1 \end{bmatrix} X = \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix}$$

Applying $R_1 \to R_1 + \left(\frac{1}{3}\right) R_3$ and $R_2 \to R_2 - \left(\frac{5}{3}\right) R_3$, we get

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} X = \begin{bmatrix} -\frac{1}{3} \\ -\frac{7}{3} \\ 2 \end{bmatrix}$$

$$\therefore X = \begin{bmatrix} -\frac{1}{3} \\ -\frac{7}{3} \\ 2 \end{bmatrix}$$

11.

Write the augmented matrix $[A|I]$

$$\left[\begin{array}{ccc|ccc} -2 & 2 & 0 & 1 & 0 & 0 \\ 2 & 1 & 3 & 0 & 1 & 0 \\ -2 & 4 & -2 & 0 & 0 & 1 \end{array}\right]$$

step 1

$$\begin{array}{c} \\ R_2 + R_1 \\ R_3 - R_1 \end{array} \left[\begin{array}{ccc|ccc} -2 & 2 & 0 & 1 & 0 & 0 \\ 0 & 3 & 3 & 1 & 1 & 0 \\ 0 & 2 & -2 & -1 & 0 & 1 \end{array}\right]$$

step 2

$$\begin{array}{c} \\ \\ R_3 - (2/3)R_2 \end{array} \left[\begin{array}{ccc|ccc} -2 & 2 & 0 & 1 & 0 & 0 \\ 0 & 3 & 3 & 1 & 1 & 0 \\ 0 & 0 & -4 & -5/3 & -2/3 & 1 \end{array}\right]$$

step 3

$$(-1/4)R_3 \begin{bmatrix} -2 & 2 & 0 & | & 1 & 0 & 0 \\ 0 & 3 & 3 & | & 1 & 1 & 0 \\ 0 & 0 & 1 & | & 5/12 & 1/6 & -1/4 \end{bmatrix}$$

step 4

$$R_2 - 3 \times R_3 \begin{bmatrix} -2 & 2 & 0 & | & 1 & 0 & 0 \\ 0 & 3 & 0 & | & -1/4 & 1/2 & 3/4 \\ 0 & 0 & 1 & | & 5/12 & 1/6 & -1/4 \end{bmatrix}$$

step 5

$$(1/3)R_2 \begin{bmatrix} -2 & 2 & 0 & | & 1 & 0 & 0 \\ 0 & 1 & 0 & | & -1/12 & 1/6 & 1/4 \\ 0 & 0 & 1 & | & 5/12 & 1/6 & -1/4 \end{bmatrix}$$

step 6

$$R_1 - 2 \times R_2 \begin{bmatrix} -2 & 0 & 0 & | & 7/6 & -1/3 & -1/2 \\ 0 & 1 & 0 & | & -1/12 & 1/6 & 1/4 \\ 0 & 0 & 1 & | & 5/12 & 1/6 & -1/4 \end{bmatrix}$$

step 7

$$(-1/2)R_1 \begin{bmatrix} 1 & 0 & 0 & | & -7/12 & 1/6 & 1/4 \\ 0 & 1 & 0 & | & -1/12 & 1/6 & 1/4 \\ 0 & 0 & 1 & | & 5/12 & 1/6 & -1/4 \end{bmatrix}$$

Hence

$$A^{-1} = \begin{bmatrix} -7/12 & 1/6 & 1/4 \\ -1/12 & 1/6 & 1/4 \\ 5/12 & 1/6 & -1/4 \end{bmatrix}$$

12.

We first find the minors.

$$M_{1,1} = Det\begin{bmatrix} . & . & . \\ . & 2 & 2 \\ . & 0 & 1 \end{bmatrix} = 2\ ,\ M_{1,2} = Det\begin{bmatrix} . & . & . \\ 3 & . & 2 \\ 0 & . & 1 \end{bmatrix} = 3\ ,$$

$$M_{1,3} = Det\begin{bmatrix} . & . & . \\ 3 & 2 & . \\ 0 & 0 & . \end{bmatrix} = 0$$

$$M_{2,1} = Det\begin{bmatrix} . & 0 & 3 \\ . & . & . \\ . & 0 & 1 \end{bmatrix} = 0\ ,\ M_{2,2} = Det\begin{bmatrix} -1 & . & 3 \\ . & . & . \\ 0 & . & 1 \end{bmatrix} = -1$$

$$M_{2,3} = Det\begin{bmatrix} -1 & 0 & . \\ . & . & . \\ 0 & 0 & . \end{bmatrix} = 0$$

$$M_{3,1} = Det\begin{bmatrix} . & 0 & 3 \\ . & 2 & 2 \\ . & . & . \end{bmatrix} = -6\ ,\ M_{3,2} = Det\begin{bmatrix} -1 & . & 3 \\ 3 & . & 2 \\ . & . & . \end{bmatrix} = -11$$

$$M_{3,3} = Det\begin{bmatrix} -1 & 0 & . \\ 3 & 2 & . \\ . & . & . \end{bmatrix} = -2$$

Matrix C of cofactors whose entries defined are defined as

$$C_{i,j} = (-1)^{i+j} M_{i,j}$$

$$C = \begin{bmatrix} 2 & -3 & 0 \\ 0 & -1 & 0 \\ -6 & 11 & -2 \end{bmatrix}$$

We need to find D the determinant of A using the third row (it has 2 zeros!)

$$D = A_{3,3} M_{3,3} = -2$$

The inverse of A is given by

$$A^{-1} = \frac{1}{D} C^T = -\frac{1}{2}\begin{bmatrix} 2 & 0 & -6 \\ -3 & -1 & 11 \\ 0 & 0 & -2 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 3 \\ \frac{3}{2} & \frac{1}{2} & -\frac{11}{2} \\ 0 & 0 & 1 \end{bmatrix}$$

13. The total amount of money the bookshop will receive from sale of all the books can be represented in the form of a matrix as:

$$12\begin{bmatrix}10 & 8 & 10\end{bmatrix}\begin{bmatrix}80\\60\\40\end{bmatrix}$$

$= 12\,[10 \times 80 + 8 \times 60 + 10 \times 40]$

$= 12\,(800 + 480 + 400)$

$= 12\,(1680)$

$= 20160$

14. If λλ is an eigenvalue of AA, then λλ satisfies

$$0 = \det(A - \lambda I) = \det\left(\begin{bmatrix}2 & -3 & 0\\2 & -5 & 0\\0 & 0 & 3\end{bmatrix} - \begin{bmatrix}\lambda & 0 & 0\\0 & \lambda & 0\\0 & 0 & \lambda\end{bmatrix}\right)$$
$$= \begin{vmatrix}2-\lambda & -3 & 0\\2 & -5-\lambda & 0\\0 & 0 & 3-\lambda\end{vmatrix} = (3-\lambda)\begin{vmatrix}2-\lambda & -3\\2 & -5-\lambda\end{vmatrix}$$
$$= (3-\lambda)\,[(2-\lambda)(-5-\lambda)+6] = (3-\lambda)(-10+5\lambda-2\lambda+\lambda^2+6)$$
$$= (3-\lambda)(\lambda^2+3\lambda-4) = -(\lambda-3)(\lambda+4)(\lambda-1).$$

In the above calculation, we calculated the determinant of the 3×33×3 matrix by expanding along the third row. From the above equation, we can conclude that the eigenvalues of AA are λ=3,1,−4λ=3,1,−4.
We will first find the eigenvectors corresponding to the eigenvalue λ = - 4

$$\begin{bmatrix}0\\0\\0\end{bmatrix} = (A+4I)\mathbf{x} = \left(\begin{bmatrix}2 & -3 & 0\\2 & -5 & 0\\0 & 0 & 3\end{bmatrix} + 4\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}\right)\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}$$
$$= \begin{bmatrix}6 & -3 & 0\\2 & -1 & 0\\0 & 0 & 7\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}.$$

The augmented matrix for this system is given by:

$$\left[\begin{array}{ccc|c} 6 & -3 & 0 & 0 \\ 2 & -1 & 0 & 0 \\ 0 & 0 & 7 & 0 \end{array}\right] \xrightarrow[\frac{1}{7}R_3]{R_2-\frac{1}{6}R_1} \left[\begin{array}{ccc|c} 6 & -3 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$$\xrightarrow[R_2\leftrightarrow R_3]{\frac{1}{6}R_1} \left[\begin{array}{ccc|c} 1 & -1/2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Therefore, the solution to this system satifies x3=0x3=0 and x1−(1/2)x2=0x1−(1/2)x2=0, the latter of which reduces to x1=(1/2)x2x1=(1/2)x2. Thus any eigenvalue xx of AA corresponding to λ=−4λ=−4 must be of the form

$$\mathbf{x} = x_2 \begin{bmatrix} 1/2 \\ 1 \\ 0 \end{bmatrix}$$

15.Let the cost of a T.V be Rs x and the cost of a V.C.R be Rs y.
According to the first condition,

3x+2y= 35000 (i)

The required profit per T.V is Rs 1000 and per V.C.R is Rs 500
Therefore, selling price of one T.V is Rs(x+1000) and selling price of a V.C.R is Rs(y+500)

According to the second condition,

2(x+1000)+1(y+500)=21500
2x+2000+y+500=21500
2x+y=19000 (ii)

Writing the above equations in matrix form

$$\begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 35000 \\ 19000 \end{bmatrix}$$

Applying $R_2 \rightarrow 2R_2 - R_1$, we get

$$\begin{bmatrix} 3 & 2 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 35000 \\ 3000 \end{bmatrix}$$

Applying $R_1 \leftrightarrow R_2 - R_1$, we get

$$\begin{bmatrix} 1 & 0 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 300 \\ 35000 \end{bmatrix}$$

Hence, the original matrix is reduced to a lower triangular matrix.

$$\therefore \begin{bmatrix} x+0 \\ 3x+2y \end{bmatrix} = \begin{bmatrix} 3000 \\ 35000 \end{bmatrix}$$

By equality of matrices, we get
X= 3000
3x+2y=35000 (iii)
Substituting x=3000 in equation (iii), we get
3(3000) +2y= 35000
9000+2Y=35000
2y=26000
Y= 13000
The cost price of a T.V is Rs 3000 and the cost price of a V.C.R is Rs13000
Hence,
Selling price of T.V = 3000+1000 =4000
Selling price of V.C.R = 13000+500= 13500

16. First find the determinant of the matrix and check if it is singular ,

$$\therefore |A| = \begin{vmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{vmatrix}$$

$= 2(3 - 0) - 0(15 - 0) - 1(5 - 0)$

$= 6 - 0 - 5$

$= 1 \neq 0$

$\therefore A^{-1}$ exists.

$A.A^{-1} = I$

$$\therefore \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Applying $R_1 \leftrightarrow R_2$, we get

$$\begin{bmatrix} 5 & 1 & 0 \\ 2 & 0 & -1 \\ 0 & 1 & 3 \end{bmatrix} A^{-1} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Applying $R_1 \rightarrow R_1 - 2R_2$, we get

$$\begin{bmatrix} 1 & 1 & 2 \\ 2 & 0 & -1 \\ 0 & 1 & 3 \end{bmatrix} A^{-1} = \begin{bmatrix} -2 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Applying $R_2 \rightarrow R_2 - 2R_1$, we get

$$\begin{bmatrix} 1 & 1 & 2 \\ 0 & -2 & -5 \\ 0 & 1 & 3 \end{bmatrix} A^{-1} = \begin{bmatrix} -2 & 1 & 0 \\ 5 & -2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Applying $R_2 \rightarrow R_2 - 3R_3$, we get

$$\begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 4 \\ 0 & 1 & 3 \end{bmatrix} A^{-1} = \begin{bmatrix} -2 & 1 & 0 \\ 5 & -2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Applying $R_1 \to R_1 - R_2, R_3 \to R_3 - R_2$,we get

$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 4 \\ 0 & 0 & -1 \end{bmatrix} A^{-1} = \begin{bmatrix} -7 & 3 & -3 \\ 5 & -2 & 3 \\ -5 & 2 & -2 \end{bmatrix}$$

Applying $R_1 \to (-1) R_3$, we get

$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 4 \\ 0 & 0 & -1 \end{bmatrix} A^{-1} = \begin{bmatrix} -7 & 3 & -3 \\ 5 & -2 & 3 \\ -5 & 2 & -2 \end{bmatrix}$$

Applying $R_1 \to R_1 + 2R_3, R_2 \to R_2 - 4R_3$, we get

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$$

$$\therefore A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}.$$