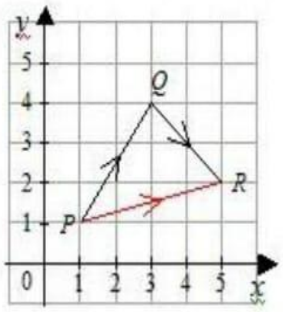
INSTITUTE OF ACTUARIAL & QUANTITATIVE STUDIES

Subject: Numerical methods

Chapter: Vectors

Category: Practice questions solutions

1. i)



The sum of vectors $\overline{PQ}$ and $\overline{QR}$ is the same as the vector $\overline{PR}$ i.e. $\overline{PQ}+\overline{QR}=\overline{PR}$.
In column vector form, we add the corresponding components of the vectors $\overline{PQ}+\overline{QR}=\begin{pmatrix}2\\3\end{pmatrix}+\begin{pmatrix}2\\-2\end{pmatrix}=\begin{pmatrix}4\\1\end{pmatrix}=\overline{PR}$

ii)

$(\vec{a}+\vec{b}+\vec{c}) = (1-2+1)\,\hat{\imath} + (-2+4-6)\,\hat{\jmath} + (1+5-7)\,\hat{k}$

$= 0\hat{\imath} - 4\hat{\jmath} - \hat{k}$

$= -4\hat{\jmath} - \hat{k}$

2. (a) C =-A+-B

(b) C=A+B

(c) C=-B+A

(d) C-A+D+B =0

3.

Given, $(\vec{a}-\vec{b}) \times (\vec{a}+\vec{b})$

$\Rightarrow (\vec{a}\times\vec{a}) + (\vec{a}\times\vec{b}) - (\vec{b}\times\vec{a}) - (\vec{b}\times\vec{b})$

we know that,
$\vec{a}\times\vec{a} = 0\ ,\vec{b}\times\vec{b} = 0\ and, \vec{a}\times\vec{b} = -(\vec{b}\times\vec{a})$

$\Rightarrow (\vec{a}\times\vec{b}) - (\vec{b}\times\vec{a})$

$\Rightarrow (\vec{a}\times\vec{b}) - [-(\vec{a}\times\vec{b})]$

$= (\vec{a}\times\vec{b}) + (\vec{a}\times\vec{b}) = 2(\vec{a}\times\vec{b})$

4. (i+2j-3k). (2i-j+k) = 1(2)+2(-1)-3(1) =2-2-3 = -3

5. Solution = (2, 7, -7)

6. Solution: c = (4 – 2, 5 – 4, 6 – 6) = (2 ,1, 0)

Magnitude of c =√2 2 + 1 2 + 0 = √5

7. AB = (-2-4)I +(-2-(-4))j +(0-0)k

= -6i+2j

|AB| =$\sqrt{(-6)^2+(2)^2}$

=√40

=2√10

8.

a) $\overrightarrow{DC} + \overrightarrow{CA} = \overrightarrow{DA}$ triangle law of vector addition

b) $\overrightarrow{BD} + \overrightarrow{DC} + \overrightarrow{CA} = (\overrightarrow{BD} + \overrightarrow{DC}) + \overrightarrow{CA}$ associative law

$= \overrightarrow{BC} + \overrightarrow{CA}$ triangle law of vector addition

$= \overrightarrow{BA}$ triangle law of vector addition

9. Solution: D

Explanation: As the magnitude of vector d is not 1

10. The vector with initial point P(1,3,2) and terminal point Q(-1,0,8) is given by

$$\overrightarrow{PQ} = (-1-1)\hat{i} + (0-3)\hat{j} + (8-2)\hat{k} = -2\hat{i} - 3\hat{j} + 6\hat{k}$$

Thus $\overrightarrow{QP} = -\overrightarrow{PQ} = 2\hat{i} + 3\hat{j} - 6\hat{k}$

$$\Rightarrow |\overrightarrow{QP}| = \sqrt{2^2 + 3^2 + (-6)^2} = \sqrt{4+9+36} = \sqrt{49} = 7$$

Therefore, unit vector in the direction of QP is given by

$$\widehat{QP} = \frac{\overrightarrow{QP}}{|\overrightarrow{QP}|} = \frac{2\hat{i} + 3\hat{j} - 6\hat{k}}{7}$$

Hence, the required vector of magnitude 11 in direction of QP is

$$11\,\widehat{QP} = 11\left(\frac{2\hat{i} + 3\hat{j} - 6\hat{k}}{7}\right) = \frac{22}{7}\hat{i} + \frac{33}{7}\hat{j} - \frac{66}{7}\hat{k}.$$

11.

$$\text{Let } \vec{a} = i - 2j$$

$$|\vec{a}| = \sqrt{1^2 + (-2)^2}$$

$$\therefore |\vec{a}| = \sqrt{5}$$

$$\therefore \hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{i - 2j}{\sqrt{5}}$$

Let $\vec{b}$ be the vectors parallel to $\vec{a}$ having magnitude 10 units

$$\therefore |\vec{b}| = 10$$

$\because$ If vectors are parallel then unit vector along & parallel vectors are same

$$\text{Now } \vec{b} = |\vec{b}|\,\hat{a}$$

$$= 10 \cdot \frac{i - 2j}{\sqrt{5}}$$

$$\therefore \text{Reqd vector } \vec{b} = \frac{10i}{\sqrt{5}} - \frac{20j}{\sqrt{5}}$$

12. Solution = 6n

13. The derivation is the same as in the PPt for Vectors.

14.

Given $P \equiv (2,3,0)$ $\quad Q \equiv (-1,-2,4)$

Let i, j, k be unit vectors along axes

then $\overrightarrow{OP} = 2i + 3j$

$$\overrightarrow{OQ} = -i - 2j + 4k$$

$$\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP}$$

$$= -i - 2j + 4k - (2i + 3j)$$

$$= -i - 2i + 4j - 2i - 3j$$

$$\therefore \overrightarrow{PQ} = -3i - 5j + 4k$$

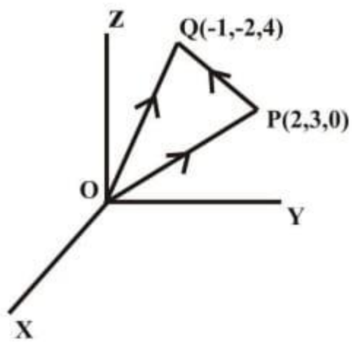
$$\left|\overrightarrow{PQ}\right| = \sqrt{(-3)^2 + (-5)^2 + (4)^2} = \sqrt{9+25+16} = \sqrt{50}$$

$$\therefore \left|\overrightarrow{PQ}\right| = 5\sqrt{2}$$

$$\hat{\overrightarrow{PQ}} = \frac{\overrightarrow{PQ}}{\left|\overrightarrow{PQ}\right|} = \frac{-3i - 5j + 4k}{5\sqrt{2}}$$

$$\therefore \hat{\overrightarrow{PQ}} = \frac{-3}{5\sqrt{2}} i - \frac{5}{5\sqrt{2}} j + \frac{4}{5\sqrt{2}} k$$

$$\therefore \text{direction cosines are} \left(\frac{-3}{5\sqrt{2}}, \frac{1}{\sqrt{2}}, \frac{4}{5\sqrt{2}} \right)$$



15.

Internally,

$$\overrightarrow{OR} = \frac{m\overrightarrow{OQ} + n\overrightarrow{OP}}{m+n}$$

$$= \frac{2(\vec{a}+\vec{b}) + 1(3\vec{a} - 2\vec{b})}{2+1}$$

$$= \frac{2\vec{a} + 3\vec{a} + 2\vec{b} - 2\vec{b}}{2+1}$$

$$\overrightarrow{OR} = \frac{5\vec{a}}{3}$$

externally $\overrightarrow{OR} = \dfrac{m\overrightarrow{OQ} - n\overrightarrow{OP}}{m-n}$

$$= \frac{2(\vec{a}+\vec{b}) - 1(3\vec{a} - 2\vec{b})}{2-1}$$

$$= \frac{2\vec{a} - 3\vec{a} + 2\vec{b} + 2\vec{b}}{1}$$

$$\overrightarrow{OR} = 4\vec{b} - \vec{a}$$

16. First we can find the vector BC

$$\overrightarrow{BC} = -\overrightarrow{AB} + \overrightarrow{AC} = -\mathbf{a} + 2\mathbf{b}$$

Then we can find,

$$\overrightarrow{BD} = \frac{3}{5}\overrightarrow{BC} = -\frac{3}{5}\mathbf{a} + \frac{6}{5}\mathbf{b}$$

Next,

$$\begin{aligned}\overrightarrow{AD} &= \overrightarrow{AB} + \overrightarrow{BD} \\ &= \mathbf{a} + \left(-\frac{3}{5}\mathbf{a} + \frac{6}{5}\mathbf{b}\right) \\ &= \frac{2}{5}\mathbf{a} + \frac{6}{5}\mathbf{b}\end{aligned}$$

Finally,

$$\begin{aligned}\overrightarrow{AE} &= \frac{1}{3}\overrightarrow{AD} \\ &= \frac{1}{3}\left(\frac{2}{5}\mathbf{a} + \frac{6}{5}\mathbf{b}\right) \\ &= \frac{2}{15}\mathbf{a} + \frac{2}{5}\mathbf{b}\end{aligned}$$

17. Let

$\vec{a}$ = 3i + 4j – k and $\vec{b}$ = 2i – j + k

The dot product is defined as

$\vec{a}$. $\vec{b}$ = (3i + 4j – k).(2i – j + k) = (3)(2) + (4)(-1) + (-1)(1) = 6-4-1 = 1

Thus, $\vec{a}$.$\vec{b}$ = 1

The Magnitude of vectors is given by

$$|\vec{a}| = \sqrt{(3^2 + 4^2 + (-1)^2)} = \sqrt{26} = 5.09$$

$$|\vec{b}| = \sqrt{(2^2 + (-1)^2 + 1^2)} = \sqrt{6} = 2.45$$

The angle between the two vectors is

$$\theta = cos^{-1} \frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|}$$

$$\theta = cos^{-1} \frac{1}{(5.09)(2.45)}$$

$$\theta = cos^{-1} \frac{1}{12.47}$$

$$\theta = cos^{-1}(0.0802)$$

$$\theta = 85.39^{\circ}$$