Lecture

INSTITUTE OF ACTUARIAL
& QUANTITATIVE STUDIES

Class: FY BSc

Subject : Numerical Methods

Subject Code: PUSAS201

Chapter: Unit 3 Chapter 1

Chapter Name: Algebraic and Quadratic equations

Today's Agenda

1. Algebraic expressions

2. Quadratic equations
    1. Solution by formula
    2. Solving by factorization
    3. Completing the square

1 Algebraic expressions

1. Indices

There are three laws governing indices (or powers):

- $x^a \times x^b = x^{a+b}$

- $x^a / x^b = x^{a-b}$

- $(x^a)^b = x^{ab}$

2. Logarithms

There are three laws of logarithms that can be derived from the laws of indices:

- $\log_a x + \log_a y = \log_a xy$

- $\log_a x - \log_a y = \log_a x/y$

- $\log_a x^n = n \log_a x$

2

Quadratic equations

- An example of a Quadratic Equation:

this makes it Quadratic

$$5x^2 + 3x + 3 = 0$$

The Standard Form of a Quadratic Equation looks like this:

$$ax^2 + bx + c = 0$$

– a, b and c are known values. a can't be 0.
– “x" is the variable, unknown as of now.

The "solutions" to the Quadratic Equation are where it is equal to zero. They are also called "roots“.
There are usually 2 solutions and there are a few different ways to find the solutions: We will look at every method separately.

2.1 Solution by formula

Simply just plug in the values of a, b and c, from the equation and do the calculations.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The ± means there are TWO answers:

$$x = \frac{-b + \sqrt{(b^2 - 4ac)}}{2a}$$

$$x = \frac{-b - \sqrt{(b^2 - 4ac)}}{2a}$$

“$b^2 - 4ac$” in the formula above is called the Discriminant, because it can "discriminate“ between the possible types of answer:

- when $b^2 - 4ac$ is positive, we get two Real solutions
- when it is zero we get just ONE real solution (both answers are the same)
- when it is negative we get a pair of Complex solution

2.1 Example

Solve: $5x^2 + 6x + 1 = 0$

Solution:

Coefficients are: $a = 5, b = 6, c = 1$

Quadratic Formula: $x = \dfrac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$

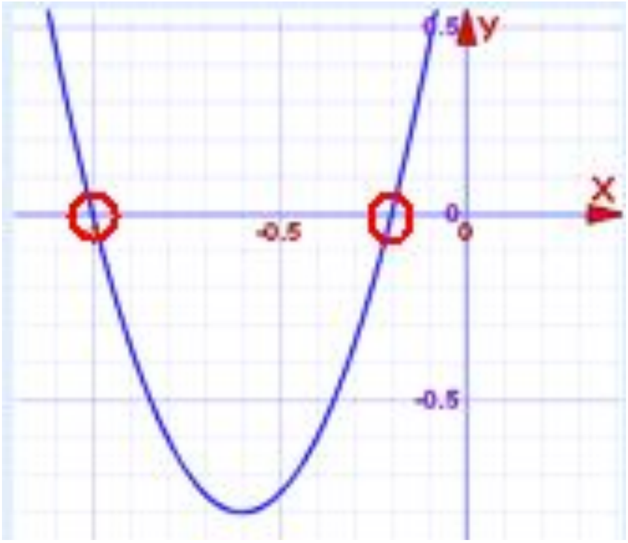
Put in a, b and c: $x = \dfrac{-6 \pm \sqrt{(6^2 - 4\times5\times1)}}{2\times5}$

Solve: $x = \dfrac{-6 \pm \sqrt{(36 - 20)}}{10}$

$x = \dfrac{-6 \pm \sqrt{(16)}}{10}$

$x = \dfrac{-6 \pm 4}{10}$

$x = -0.2$ **or** -1



2.2 Solving by Factorisation

To "Factor" (or "Factorise" in the UK) a Quadratic is to: “find what to multiply to get the Quadratic”.
It is called "Factoring" because we find the factors (a factor is something we multiply by).

The basic form is:

$$ax^2 + bx + c = 0$$

Step 1: Find two numbers that multiply to give ac (in other words a times c), and add to give b.
Step 2: Rewrite the middle with those numbers:
Step 3: Factor the first two and last two terms separately:
Step 4: If we've done this correctly, our two new terms should have a clearly visible common factor.

2.2 Example

Solve: $2x^2 + 7x + 3 = 0$

Solution:

$2x^2 + 6x + x + 3 = 0$

$2x (x + 3) + (x + 3) = 0$

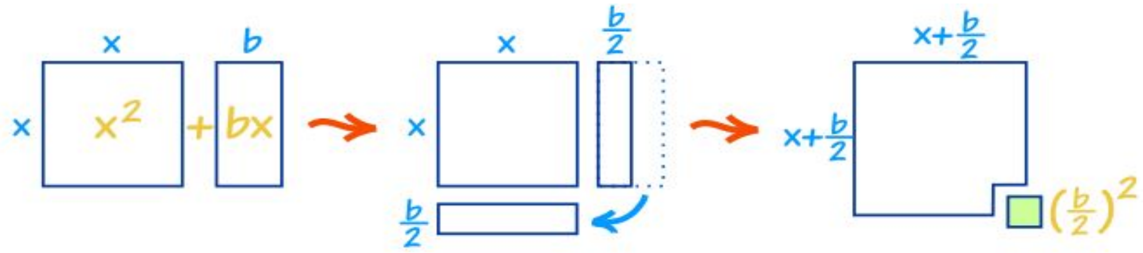
$2x (x + 3) + 1 (x + 3) = 0$

$(2x + 1) (x + 3) = 0$

$x = -1/2$ or -3

2.3 Completing the Square

First lets understand what it means to complete the square, then we learn how to apply the method to solve quadratic equations.
Well, with a little inspiration from Geometry we can explain it, like this:



As you can see $x^2 + bx$ can be rearranged nearly into a square and we can complete the square with $(b/2)^2$.
In Algebra it looks like this:

$$x^2 + bx \quad + (b/2)^2 \quad = \quad (x+b/2)^2$$

"Complete the Square"

So, by adding $(b/2)^2$ we can complete the square and $(x+b/2)^2$ has x only once, which is easier to use.

We can't just add $(b/2)^2$ without also subtracting it too! Otherwise the whole value changes. hence we add and subtract the same value. We look further how this works.

2.3 Completing the Square

We can complete the square to solve a Quadratic Equation (find where it is equal to zero).

But a general Quadratic Equation can have a coefficient of a in front of x^2: $ax^2 + bx + c = 0$

It is easy to deal with, just divide the whole equation by "a" first, then carry on: $x^2 + (b/a)x + c/a = 0$

Steps > Now we can solve a Quadratic Equation in 5 steps:
Step 1 – Divide all terms by a (the coefficient of x^2).
Step 2 – Move the number term (c/a) to the right side of the equation.
Step 3 – Complete the square on the left side of the equation and balance this by adding the same value to the right side of the equation.
We now have something that looks like $(x + p)^2 = q$, which can be solved rather easily:
Step 4 – Take the square root on both sides of the equation.
Step 5 – Subtract the number that remains on the left side of the equation to find x.

2.3 Example

Solve: $x^2 + 4x + 1 = 0$

Solution:

Step 1 can be skipped in this example since the coefficient of x^2 is 1

Step 2 Move the number term to the right side of the equation:

$$x^2 + 4x = -1$$

Step 3 Complete the square on the left side of the equation and balance this by adding the same number to the right side of the equation.

$(b/2)^2 = (4/2)^2 = 2^2 = 4$

$$x^2 + 4x + 4 = -1 + 4$$

$$(x + 2)^2 = 3$$

Step 4 Take the square root on both sides of the equation:

$$x + 2 = \pm\sqrt{3} = \pm 1.73 \text{ (to 2 decimals)}$$

Step 5 Subtract 2 from both sides:

$$x = \pm 1.73 - 2 = -3.73 \text{ or } -0.27$$