INSTITUTE OF ACTUARIAL & QUANTITATIVE STUDIES

Subject: **Numerical Methods and Algebra**

Assignment-2 Solution

Numerical Methods & Algebra

Assignment 2 - Solutions

1. Determine all values a, b and c such that

$$\begin{bmatrix}1 & 2\\ a & 0\end{bmatrix}\begin{bmatrix}3 & b\\ -4 & 1\end{bmatrix}=\begin{bmatrix}-5 & 6\\ 12 & 16\end{bmatrix}$$

Solution

$$\begin{bmatrix}1 & 2\\ a & 0\end{bmatrix}\begin{bmatrix}3 & b\\ -4 & 1\end{bmatrix}=\begin{bmatrix}3-8 & b+2\\ 3a & ab\end{bmatrix}$$

$$\begin{bmatrix}3-8 & b+2\\ 3a & ab\end{bmatrix}=\begin{bmatrix}-5 & 6\\ 12 & 16\end{bmatrix}$$

We get

$$b+2=6$$

$$3a=12$$

$$ab=16$$

Therefore

$$a=4, b=4$$

2. Let

$$A=\begin{bmatrix}1 & 0 & 0\\ 0 & -1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

Find A^{20}

Solution

$$A^2=\begin{bmatrix}1 & 0 & 0\\ 0 & -1 & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0 & 0\\ 0 & -1 & 0\\ 0 & 0 & 1\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}=I$$

$$A^{20}=(A^2)^{10}=I^{10}=I$$

3. Determine those values of λ for which the matrix

$$\begin{bmatrix} 1 & \lambda & 0 \\ 3 & 2 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

Is not invertible

Solution

The matrix is not invertible if the determinant is zero.

$$\begin{vmatrix} 1 & \lambda & 0 \\ 3 & 2 & 0 \\ 1 & 1 & 1 \end{vmatrix} = 1(2) - \lambda(3-1) = 0$$

$$2 - 2\lambda = 0$$

$$\lambda = 1$$

4. January sales at the A-Plus auto parts stores in Mumbai and Pune are given in the following table.

	Mumbai	Pune
Wiper blades	20	15
Cleaning fluid (bottles)	10	12
Floor mats	8	4

The usual selling prices for these items are Rs700.00 each for wiper blades, Rs 300.00 per bottle for cleaning fluid, and Rs 1200.00 each for floor mats. The discount prices for A-Plus Club members are Rs 600.00 each for wiper blades, Rs 200.00 per bottle for cleaning fluid, and Rs 1000.00 each for floor mats. Use matrix multiplication to compute the total revenue at each store, assuming first that all items were sold at the usual prices, and then that they were all sold at the discount prices.

Solution

Let us find the sales matrix for the parts be S

$$\begin{array}{cc} & \begin{array}{ccc} W.B. & C.F. & F.M. \end{array} \\ \begin{array}{c} \text{Mumbai} \\ \text{Pune} \end{array} & \begin{bmatrix} 20 & 10 & 8 \\ 15 & 12 & 4 \end{bmatrix} \end{array}$$

Let the price matrix for the parts be P

$$P = \begin{array}{r|cc} & \text{Undiscounted} & \text{Discounted} \\ \text{Wiper blades} & 700 & 600 \\ \text{Cleaning fluid} & 300 & 200 \\ \text{Floor Mats} & 1200 & 1000 \end{array}$$

Revenue Matrix = $S \times P$

$$= \begin{bmatrix} 20 & 10 & 8 \\ 15 & 12 & 4 \end{bmatrix} \times \begin{bmatrix} 700 & 600 \\ 300 & 200 \\ 1200 & 1000 \end{bmatrix}$$

$$= \begin{bmatrix} 20 \times 700 + 10 \times 300 + 8 \times 1200 & 20 \times 600 + 10 \times 200 + 8 \times 1000 \\ 15 \times 700 + 12 \times 300 + 4 \times 1200 & 15 \times 600 + 12 \times 200 + 4 \times 1000 \end{bmatrix}$$

$$= \begin{bmatrix} 26600 & 22000 \\ 18900 & 14400 \end{bmatrix}$$

The revenue matrix is

$$\begin{array}{r|cc} & \text{Undiscounted} & \text{Discounted} \\ \text{Mumbai} & 26000 & 22000 \\ \text{Pune} & 18900 & 14400 \end{array}$$

5. The fourth term of an arithmetic progression is equal to 3 times the first term and the seventh term exceeds twice the third term by 1. Find its first term and the common difference.

<u>Solution</u>

Let the first term be a and the common difference be d

The nth term $=a + (n-1)d$

According to the given conditions we have

$$a + 3d = 3a$$

$$a + 6d = 2(a + 2d) + 1$$

Simplifying we have

$$2a = 3d$$

And

$$a = 2d - 1$$

Eliminating a we get

$$4d - 2 = 3d \text{ or } d = 2$$

$$a = 3$$

6. If 7 times the 7th term of an arithmetic progression is equal to 11 times its 11th term, show that the 18th term of an arithmetic progression is zero.

Solution

Let the first term be a and the common difference be d

We have

$$7(a + 6d) = 11(a + 10d)$$

$$4a + 68d = 0$$

$$a + 17d = 0$$

$a + 17d$ is nothing but the 18^{th} term

Hence proved.

7. The sum of the first 20 terms of an arithmetic series is identical to the sum of the first 22 terms. If the common difference is −2, find the first term.

Solution

The sum to n terms is given be $\frac{n}{2}(2a + (n - 1)d)$

Therefore we have

$$10(2a + 19d) = 11(2a + 21d)$$

$$20a + 190d = 22a + 231d$$

$$-2a = 41d$$

Given d=-2

$$-2a = 41(-2)$$

$$a = 41$$

8. If the sum of the first n terms of an A.P. is given by $S_n = 2n^2 + 5n$, Find the nth term of the A.P.

Solution

The sum to n terms is given be $\frac{n}{2}(2a + (n-1)d)$

$$= na + \frac{n^2 d}{2} - \frac{nd}{2}$$
$$= n^2\left(\frac{d}{2}\right) + n\left(a - \frac{d}{2}\right)$$
$$Comparing\ coefficients\ of\ n\ and\ n^2\ we\ get$$
$$\frac{d}{2} = 2\ and\ a - \frac{d}{2} = 5$$

Solving, we get $d = 4\ and\ a = 7$

9. Show that $111 \ldots \ldots \ldots \ldots .1\ \}$ (91 times) is not a prime number.

Solution

$111 \ldots \ldots \ldots \ldots .1\ \}$ (91 times)$=1 + 10 + 100 + 1000 + \cdots .10^{90}$

$$This\ is\ the\ sum\ of\ a\ GP\ with\ a = 1, n = 91, r = 10$$
$$Sum = \frac{1(10^{91} - 1)}{10 - 1}$$
$$= \frac{(10^7)^{13} - 1}{10^7 - 1} \times \frac{10^7 - 1}{10 - 1}$$
$$= (\ 1 + 10^7 + 10^{14} + \cdots 10^{91}) \times (1 + 10 + 100 + \cdots 10^7)$$

Since both the terms are natural numbers, the given number is a multiple of two numbers Therefore the given number is composite and not a prime.

10. The sum of three numbers in G.P. is 31 and sum of their squares is 651. Find the numbers.

Solution

Let the numbers be a, ar and ar^2

We have

$a + ar + ar^2 = 31$

$a(1 + r + r^2) = 31$ (1)

We also have

$a^2 + (ar)^2 + (ar^2)^2 = 651$

$a^2(1+r^2+r^4) = 651$ (2)

Squaring equation (1) we get

$a^2(1+r^2+r^4+2r+2r^2+2r^3) = 961$

Or

$a^2(1+r^2+r^4) + a^2(2r+2r^2+2r^3) = 961$

$a^2(1+r^2+r^4) + (2ar)(a)(1+r+r^2) = 961$

Substituting from (1) & (2) we get

$$651 + (2ar)(31) = 961$$

Or

$$2ar(31) = 310$$

$$ar = 5$$

$$a = \frac{5}{r}$$

Substituting the value of a in (1), we get

$$\frac{5}{r}(1+r+r^2) = 31$$

$$\frac{5}{r} + 5 + 5r = 31$$

$$5r + \frac{5}{r} = 26$$

$$5r^2 - 26r + 5 = 0$$

$$5r^2 - 25r - r + 5 = 0$$

$$5r(r-5) - (r-5) = 0$$

$$(5r-1)(r-5) = 0$$

$$r = \frac{1}{5} or\ r = 5$$

$$a = 25\ or\ a = 1$$

The numbers are 1,5,25.