INSTITUTE OF ACTUARIAL & QUANTITATIVE STUDIES

Subject: Pricing & Reserving for Life Insurance Products

Chapter:

Category: Assignment 1 solutions

1.

i) Constant force of a mortality is a method that enables us to calculate survival probabilities over non-integer time periods and from non-integer ages. It assumes that the force of mortality takes a constant value between consecutive integer ages. [2]

ii) $P55 = e^{-\mu}$

$\mu = -\log(1 - q_{55})$

= -log (1-0.004469)

= 0.004479016

2.

i) Uniform deaths

$${}_{1.75}p_{40.75} = {}_{0.25}p_{40.75} \times p_{41} \times {}_{0.5}p_{42}$$

$$p_{41} = 1 - q_{41} = 1 - 0.001014 = 0.998986$$

$${}_{0.25}p_{40.75} = 1 - {}_{0.25}q_{40.75} = 1 - \left(\frac{0.25q_{40}}{1 - 0.25q_{40}}\right) = 1 - \left(\frac{0.25 \times 0.000937}{1 - 0.25 \times .000937}\right)$$
$$= 0.999766$$

$${}_{0.5}p_{42} = 1 - {}_{0.5}q_{42} = 1 - 0.5q_{42} = 1 - 0.5 \times 0.001104 = 0.999448$$

Based on the above:

$${}_{1.75}p_{40.75} = 0.999766 \times 0.998986 \times 0.999448 = 0.998201$$

ii) Constant force of mortality

$${}_{1.75}p_{40.75} = {}_{0.25}p_{40.75} \times p_{41} \times {}_{0.5}p_{42}$$

$${}_{0.5}p_{42} = (p_{42})^{0.5} = (1 - q_{42})^{0.5} = (1 - 0.001104)^{0.5} = 0.999448$$

$${}_{0.25}p_{40.75} = (p_{40)})^{0.25} = (1 - q_{40})^{0.25} = (1 - 0.000937)^{0.25} = 0.999766$$

So,

$${}_{1.75}p_{40.75} = 0.999766 \times 0.998986 \times 0.999448 = 0.998200$$

3.

$_{1.4}q_{54.5} = 1 - {}_{1.4}p_{54.5}$

Now

$_{1.4}p_{54.5} = {}_{0.5}p_{54.5} * {}_{0.9}p_{55}$

$_{0.5}p_{54.5} = 1 - {}_{0.5}q_{54.5}$

Now by UDD:

$_{0.5}q_{54.5} = 0.5\, q_{54} / (1 - 0.5q_{54})$

$_{0.5}q_{54.5} = 0.5 * 0.00714 / (1-0.5*0.00714)$

$= 0.00358279$

Hence $_{0.5}p_{54.5} = 0.99641721$

Also, $_{0.9}p_{55} = 1 - {}_{0.9}q_{55}$

$= 1 - 0.9\, q_{55}$

$= 1 - 0.9 * 0.00797$

$= 0.992827$

Hence:

$_{1.4}q_{54.5} = 1 - 0.99641721 * 0.992827$

$= 0.01073$

4.

The expected present value of maturity benefit is:

$$EPV = 50{,}000 \times \frac{D_{65}}{D_{[45]}} = 50{,}000 \times \frac{689.23}{1677.42}$$

$= 20544.35$

The expected present value of death benefit is:

$$EPV = 1577(IA)^{1}_{[45]:\overline{20|}} = 1577 \times \left[(IA)_{[45]} - \frac{D_{65}}{D_{[45]}}[(IA)_{65} + 20A_{65}]\right]$$

$$= 1577 \times \left[8.33865 - \frac{689.23}{1677.42}(7.89442 + 20 \times 0.52786)\right]$$

$$= 1193.98$$

Total value of benefits:

$$20544.35 + 1193.98 = 21738.33$$

5.

$\bar{A}^1_{x:\overline{n|}} = \sum_{t=0}^{n-1} t|\bar{A}^1_{x:\overline{1|}}$

$$\bar{A}^1_{x:\overline{n|}} = \sum_{x=0}^{n-1} v^t \, tp_x \, \bar{A}^1_{x+t:\overline{1|}}$$

$\bar{A}^1_{x+t:\overline{1|}} = \int_0^1 v^s \, sp_{x+t} \, \mu_{x+t+s} \, ds$

Assuming a uniform distribution of deaths, then $sp_{x+t} \, \mu_{x+t+s} = q_{x+t}$

$$\bar{A}^1_{x+t:\overline{1|}} = \int_0^1 v^s \, q_{x+t} \, ds \quad = q_{x+t} \int_0^1 v^s ds$$

$= q_{x+t} \frac{iv}{\delta}$

$$\bar{A}^1_{x:\overline{n|}} = \sum_{x=0}^{n-1} v^t \, tp_x \, q_{x+t} \frac{iv}{\delta}$$

$$\bar{A}^1_{x:\overline{n|}} = \frac{i}{\delta} \sum_{x=0}^{n-1} v^{t+1} \, tp_x \, q_{x+t}$$

$\bar{A}^1_{x:\overline{n|}} = \frac{i}{\delta} A^1_{x:\overline{n|}}$

6.

i)

P = 50,00,000 * (A_{45} - v^{20} * ${}_{20}p_{45}$ * A_{65})

P = 50,00,000 * (0.27605 – 1.04^(-20) * 8821.2612*0.52786/9801.3123)

P = INR 2,96,152

For 10,000 policies

= 10,000 * P

= INR 2,96,15,20,000 [2]

ii) Per policy reserves at the end of 1st year:

V_1 = 50,00,000 * (A_{44} - v^{19} * ${}_{19}p_{44}$ * A_{63}) @ 6%

= 50,00,000*(0.15146 - (1.06^(-19) * (9037.3973/9814.3359)*0.37091)

= 50,00,000 * 0.0385741

= INR 1, 92,870

Mortality Profit = EDS – ADS

Here DSAR = SA – V_1 = 50, 00,000 - 1, 92,870 = 48, 07,130

EDS = 10,000 * q_{45} * 48,07,130

= 10,000 * 0.001465 * 48,07,130

= 7, 04, 24,400

ADS = 10 * 48, 07,130

Mortality Profit = 2, 23, 53,150 INR [5]

[7 Marks]

7.

The expected present value is

$$50000\ {}_{10|}A_{50} = 50000\ v^{10}\ {}_{10}p_{50}\ A_{60}$$

$$= 50000\ 1.06^{-10} \times \frac{l_{60}}{l_{50}} \times A_{60}$$

$$= 50000 * 1.06^{-10} \times \frac{9287.2164}{9712.0728} \times 0.32692$$

= Rs.8728

8.

Given age = 65 years

1 year select mortality

Int rate = 8% P.A

$q_{[65]} = 0.75*q_{65}$

$A_{65} = 0.5412$ and $A_{66} = 0.5591$

SA = Rs. 100,000

Single premium = ?

Now:

Single Premium = 100,000 * $A_{[65]}$

$= 100{,}000 * (v * q_{[65]} + v * p_{[65]} * A_{66})$

$= 100{,}000 * (v * q_{[65]} + v * (1-q_{[65]}) * A_{66})$

We have to calculate $q_{[65]}$ from given data

Now:

$A_{65} = v * q_{65} + v * p_{65} * A_{66}$

$A_{65} = v * q_{65} + v * (1-q_{65}) * A_{66}$

$0.5412 = 1.08\text{^}(-1) * q_{65} + 1.08\text{^}(-1) * (1- q_{65}) * 0.5591$

Solving

$q_{65} = 0.0576$

Now as given:

$q_{[65]} = 0.75 * q_{65}$

$q_{[65]} = 0.75 * 0.0576$

$q_{[65]} = 0.0432$

Hence:

Single Premium = $100{,}000 * (v * q_{[65]} + v * (1-q_{[65]}) * A_{66})$

$= 100{,}000 * (1.08\text{^}(-1) * 0.0432 + 1.08\text{^}(-1) * (1-0.0432) * 0.5591)$

= Rs. 53,532

9.

(i) The loss function is given by:

$$L = 120{,}000 a_{\overline{K_{70}}|} - P \text{ where } P \text{ is the single premium.}$$

$$E(L) = 0$$

$$\Rightarrow P = 120{,}000 E(a_{\overline{K_{70}}|}) = 120{,}000 a_{70} = 120{,}000(11.562 - 1) = 1{,}267{,}440$$

(ii) $\Pr(L > 0) = \Pr(120{,}000 a_{\overline{K_{70}}|} > 1{,}267{,}440) = \Pr(a_{\overline{K_{70}}|} > 10.562)$

Note that $a_{\overline{13}|} = 9.9856$ and $a_{\overline{14}|} = 10.5631$

$$\therefore \Pr(L > 0) = \Pr(K_{70} \geq 14) = {}_{14}p_{70}$$

$$\Rightarrow \Pr(L > 0) = \frac{l_{84}}{l_{70}} = \frac{5{,}339.057}{9{,}238.134} = 57.79\%$$

(iii) If the single premium $P_{(k)}$ is set such that:

$L_k = 120{,}000 a_{\overline{k}|} - P_{(k)} = 0$ i.e. $P_{(k)} = 120{,}000 a_{\overline{k}|}$
then, the loss will be positive only if $K_{70} > k$ since for annuities, the loss will increase the longer the person lives.

$$\therefore \Pr(L_k > 0) = \Pr(K_{70} > k) = \Pr(K_{70} \geq k+1) = {}_{k+1}p_{70} = \frac{l_{k+71}}{l_{70}}$$

$$\therefore \Pr(L_k > 0) \leq 0.05 \Rightarrow l_{k+71} \leq 0.05 l_{70} \text{ i.e. } l_{k+71} \leq 461.9067$$

Note that $l_{97} = 611.905$ and $l_{98} = 459.696$

$$\therefore \Pr(L_k > 0) \leq 0.05 \Rightarrow k + 71 = 98 \Rightarrow k = 27$$

Hence the required premium is given by:

$120{,}000 a_{\overline{27}|} = 120{,}000 \times 16.3296 = 1{,}959{,}552$ **[5]**

10.

A continuous whole life annuity is issued to a Life aged X.

i) Continuous whole life annuity issued to life aged X is denoted by $\bar{a}_{\overline{Tx}|}$

It's expected present value is denoted by $\bar{a}_x$

ii) Given the force of mortality is constant we could say that:

$\bar{a}_x = \int_0^\infty v^t * tpx * dt$

$= \int_0^\infty e^{-\delta t} * e^{-\mu t} * dt$

$= \int_0^\infty e^{-(\delta+\mu)t} * dt =$ 16 (Given in Question)

= (1/(μ+δ)) = 16

= (μ+δ) = 0.0625

Give δ = 0.04 gives μ = 0.0225

Now:

$\bar{A}_x = \int_0^\infty v^t * tpx * \mu_{x+t} * dt$

$\bar{A}_x = \int_0^\infty e^{-\delta t} * e^{-\mu t} * \mu * dt$

$\bar{A}_x$= μ / (μ+δ)

= 0.0225 * 16

= 0.36

Also

${}^2\bar{A}_x = \bar{A}_x$ with twice force of interest. Hence

${}^2\bar{A}_x$= μ / (μ+2δ)

= 0.0225/(0.0225 + 2*0.04)

= 0.219512

Now Var($\bar{a}_{\overline{Tx}|}$) = (${}^2\bar{A}_x$ - ($\bar{A}_x$)^2) / δ^2

= (0.219512 – 0.36^2)/0.04^2

= 56.195 → SD = 7.496332

11.

i) Following could be the reasons:

- Term assurance portfolio must have witnessed huge number of lapses during these years. These could be because of many reasons like, policyholders switching to other companies which might offer better rates, changes in the personal situation of individuals, leading to no further need of term assurance, in such scenarios the policyholder could lapse the policy specially given that it's a regular premium contract and each year of premium is like a cost of cover for one year. This would not be the case with immediate annuity policies, as the companies will not generally offer surrender values and it would not make sense for policyholders in such case to lapse and forfeit the future benefits.
- Small differences could also be on account of difference in death rates for two portfolios. [2]

ii) Reserve for Annuity portfolio:

$= 9{,}900 * 250{,}000 * a_{60}$
$= 9{,}900 * 250{,}000 * (15.632 – 1)$
$= 9{,}900 * 3{,}658{,}000$
= Rs. 36,214.2 Million

Calculation of reserves for term insurance:

First calculate per policy annual net premium = P

$P * \ddot{a}_{40:\overline{25|}} = 2{,}500{,}000 * A^{1}_{40:\overline{25|}}$

$P * \ddot{a}_{40:\overline{25|}} = 2{,}500{,}000 * (A_{40} - 1.04^{(-25)} * (l_{65}/l_{40}) * A_{65})$

P = 2,500,000*(0.23056-1.04^(-25)*(8821.2612/9856.2863) * 0.52786)/15.884

P = Rs. 8,396

Reserves for Term Insurance portfolio

$= 8{,}500 * (2{,}500{,}000 * A^{1}_{50:\overline{15|}} - P * \ddot{a}_{50:\overline{15|}})$

$= 8{,}500*(2{,}500{,}000*(A_{50}-1.04^{(-15)}* (l_{65}/l_{50}) * A_{65}) – 8{,}396 * 11.253)$

= 8,500*(2,500,000*(0.32907-1.04^(-15)*(8821.2612/9712.0728)*0.52786)
- 8,396 * 11.253)

= Rs. 532.5 Million

Total Reserve = 36,214.2 + 532.5 = Rs. 36,746.7 Million [6]

iii) Following could be possible sources of surplus for a life insurance company:

- Mortality surplus
- Lapse/surrender surplus
- Expense surplus
- Interest surplus [2]

iv) Calculation of mortality profit:

Annuity portfolio:

Death strain for each policy surviving till 31st Dec 2016

DSAR = 0 – (250,000 + 3,658,000) = - 3,908,000

Actual death strain (ADS) = Actual deaths * DSAR = 4 * - 3,908,000
= -15,632,000

Expected death strain (EDS) = Expected deaths * DSAR
= Policies at start of 2016 * q_{59} * DSAR
= (9900+4)* 0.002110 * (-3,908,000)
= - 81,667,196

Mortality Profit= EDS – ADS
= Rs - 66,035,196 (i.e a loss)

Term assurance portfolio:

Death strain for each policy surviving till 31st Dec 2016

DSAR = 2,500,000 – 62,650 = 2,437,350

Actual death strain (ADS) = Actual deaths * DSAR = 5 * 2,437,350
= 12,186,751

Expected death strain (EDS) = Expected deaths * DSAR
= Policies at start of 2016 * q_{49} * DSAR
= (8500+5)* 0.002241 * 2,437,350
= 46,455,172

Mortality Profit= EDS – ADS
= Rs. 34,268,421 (i.e a profit)

Total mortality profit = 34,268,421 - 66,035,196
= Rs. -31,766,774 (loss of 31,766,774) [6]

[16 Marks]

12.

i) $s_{55:\overline{10|}}$

$= 1.06\text{^}10 \ * \ a_{55:\overline{10|}} \ / \ {}_{10}p_{55}$

$= 1.06\text{^}10 \ * ((\ddot{a}_{55} - 1) - 1.06\text{^}(-10) \ *(l_{65}/l_{55}) * (\ddot{a}_{65} - 1)) \ / \ (l_{65}/l_{55})$

$= 1.06\text{^}10 \ * ((13.057 - 1) - 1.06\text{^}(-10) \ *(8821.2612/9557.8179) * (10.569 - 1)) \ / \ (8821.2612/9557.8179)$

$= 13.82616$ [2]

ii) $\ddot{a}^{(4)}_{[40]:\overline{20|}}$

$= \ddot{a}^{(4)}_{[40]} - v^{20} * (l_{60}/l_{[40]}) * \ddot{a}^{(4)}_{60}$

$= (\ddot{a}_{[40]} - 3/8) - 1.06\text{^}(-20) \ * \ (l_{60}/l_{[40]}) \ * \ (\ddot{a}_{60} - 3/8)$

$= (15.494 - 3/8) - 1.06\text{^}(-20) \ * \ (9287.2164/9854.3036) \ * \ (11.891 - 3/8)$

$= 11.735$ (rounded) [2]

[4 Marks]

13.

Let P be the net premium for the policy payable annually in advance. Then, equation of value becomes:

$$P\ddot{a}_{45:\overline{15}|} = 100000 * (A_{45:\overline{20}|} + v^{20}\ {}_{20}p_{45})$$

11.386 P =100000 * (0.46998 + 0.41075)

P = Rs.7735.20

Net premium reserve at the end of the 13th policy year is

$${}_{13}V = 100000 * (A_{58:\overline{7}|} + v^{7}\ {}_{7}p_{58}) - P\ddot{a}_{58:\overline{2}|}$$

= 100000 * (0.76516+ 0.71209) - 7735.20 * 1.955

=147724.80 - 15122.30
= 132602.50

Death strain at risk per policy = 100000 - 132602.50= - 32602.50

$EDS = 199q_{57}$ x (- 32602.50)

=199 x0.00565 x (-32602.50)
= - 36656.62

ADS= 4x -32602.50= -130410

Mortality Profit = - 36656.62 - (- 130410.00) = Rs. 93753.38

14.

(i) $g(T) = \begin{cases} 5{,}000v^2\bar{a}_{\overline{T_{63}-2}|} & \text{if } T_{63} \geq 2 \quad (\text{or } 5{,}000(\bar{a}_{\overline{T_{63}}|} - \bar{a}_{\overline{2}|}) \\ 0 & \text{if } T_{63} < 2 \end{cases}$

(ii)

$$E[g(T)] = (100)(5{,}000)v^2\ {}_2p_{63}\bar{a}_{65} = (500{,}000)(0.92456)(0.992617)(14.871 - 0.5)$$
$$= (500{,}000)(13.1887) = 6{,}594{,}350$$

(iii) $Var[g(T)] = E[g(T)^2] - E[g(T)]^2$

For Re 1 of annuity:

$$E[g(T)^2] = \int_2^{\infty} {}_tp_{63}\mu_{63+t}[v^2\bar{a}_{\overline{t-2}|}]^2\,dt$$

Let $t = r + 2 \Rightarrow$

$$E[g(T)^2] = \int_0^{\infty} {}_{r+2}p_{63}\mu_{63+r+2}[v^2\bar{a}_{\overline{r}|}]^2\,dr$$

$$= \int_0^{\infty} {}_rp_{65}\,{}_2p_{63}\mu_{65+r}v^4\left[\frac{1-v^r}{\delta}\right]^2 dr$$

$$= \frac{{}_2p_{63}v^4}{\delta^2}\int_0^{\infty} {}_rp_{65}\mu_{65+r}[1 - 2v^r + v^{2r}]\,dr$$

$$= \frac{{}_2p_{63}v^4}{\delta^2}[1 - 2\bar{A}_{65} + {}^2\bar{A}_{65}]$$

where

$$\bar{A}_{65} = (1.04)^{0.5}(1 - d\ddot{a}_{65}) = 1.019804\{1 - \left(\frac{0.04}{1.04}\right)(14.871)\} = 0.436515$$

and ${}^2\bar{A}_{65} = (1.04)({}^2A_{65}) = (1.04)(0.20847) = 0.21681$

$$\therefore E[g(T)^2] = \frac{(0.992617)(0.85480)}{(0.039221)^2}[1 - (2)(0.436515) + (0.21681)] = 189.622$$

$$Var[g(T)] = 189.622 - (13.1887)^2 = 15.680$$

For annuity of 5,000 we need to increase by $5{,}000^2$ and for 100 (independent) lives we need to multiply by 100.

Total variance = $(15.680)(5{,}000^2)(100) = 39{,}200{,}000{,}000 = (197{,}999)^2$

15.

V_t = Reserve at the start of year
V_{t+1} = Reserve at the end of the year
i = interest rate
P = Annual premium
q_{x+t} = Probability of death at age x+t

a.

The guaranteed benefit is payable at the end of every month and doubles from the second month therefore the accumulated value of this benefit at the end of the year would be

$$= X\,(1+i)^{11/12} + 2X\,(1+i)^{10/12} + \ldots\ldots + (2)^{11} X\,(1+i)^{0/12}$$

$$= \sum_{j=0}^{11} 2^j X\,(1+i)^{(11-j)/12}$$

Therefore the recursive relationship will be:

$$(V_t + P)(1+i) = q_{x+t}\,S + \sum_{j=0}^{11} 2^j X\,(1+i)^{(11-j)/12} + p_{x+t}\,V_{t+1}\,(2) -$$

b.

The survival benefit of R is payable at the middle of the year, therefore,

$$(V_t + P)(1+i) = q_{x+t}\,S + p_{x+t-0.5}\,R(1+i)^{1/2} + p_{x+t}\,V_{t+1}$$