



Subject: Pricing & Reserving
for Life Insurance
Products

Chapter: Unit 3 & 4

Category: Assignment
Questions

1. i) An insurance contract has the following future loss random variable:

$$S \cdot v^{\min(T_x, n)} + I + e \cdot \overline{a}_{\min(T_x, n)} + f \cdot v^{\min(T_x, n)} - G \cdot \overline{a}_{\min(T_x, n)}$$

Where, I = Initial expenses e = Renewal expenses S = Sum assured G = Gross premium f = Additional expenses when the contract terminates

From the expression above, what can you conclude about the nature of the contract and the timing of the various cash flows? (4)

ii) A 5-year term assurance policy is issued to a life aged 47 exact. Under this policy, annual premiums of INR 2,000 are payable annually in advance. Death benefit of INR 110,000 is payable at the end of the year of death. Calculate the reserve required to be held at the end of the second year using the assumptions below:

Force of mortality equals: $\left\{ \begin{array}{l} 0.02 \text{ for ages } 25 - 39 \\ 0.025 \text{ for age } 40 - 49 \\ 0.03 \text{ for ages } 50 - 59 \end{array} \right.$

Expenses: Nil Interest rate: 6% p.a. effective. (4)
[8]

2. i) Describe the characteristics, including cash flows involved, of

- a) An endowment assurance
- b) A term assurance (4)

ii) Consider a life insurance company which is offering three types of contingent annuity for lives aged 40 years (say). In case of joint life assume that both the lives are of same age

- a) Regular payment till the survival of annuitant.
- b) Joint life annuity- payable till the second life survives
- c) Reversionary annuity- commencing on the death of the first life and payable till the second life survives

Explain which arrangement will give highest annuity amount for an investment of INR 1,00,000. (2)

iii) The company offers different type of options to annuitants in terms of choosing the frequency of the annuity payments:

- a) Annuity payable in arrear
- b) Annuity payable in advance
- c) Annuity payable in arrear in quarterly mode
- d) Annuity payable in advance in quarterly mode
- e) Annuity payable continuously

For interest rate, $i > 0$, arrange the present value of annuities of above options in ascending order. (2)

[8]

3. Priti and Rahul, aged 56 and 60 years exact respectively, take out a policy with the below benefits:

- a) Lump sum of INR 1,00,000 payable at the end of the year of first death if it happens in first 5 years and INR 2,00,000 if the first death happens between 6th and 10th year.
- b) After 10 years, an annuity benefit of INR 20,000 will be payable annually in advance until both are alive and INR 10,000 while only one of them is alive.

Level premiums are payable annually in advance for at most 5 years and will cease on the first death if this occurs earlier.

Calculate the level annual premium payable by the couple on the following basis:

- Interest: 4% p.a.
- Mortality: PMA92C20 for Rahul (male) and PFA92C20 for Priti (female)
- Initial expense: INR 500
- Renewal expense: 2% of each premium excluding the first premium

[8]

4. A male life aged 65 exact purchases a single premium policy with the following benefits:

- a) Annuity of amount INR 10,000 payable monthly in arrears as long as the policyholder is alive.
- b) Return of single premium immediately upon death of the policyholder within 10 years of purchasing the annuity.

Calculate the single premium using the assumptions below assuming the principle of equivalence holds.

Mortality table: PMA92C20

Interest: 4% p.a.

Initial expense of 2% of Single premium

Regular expenses of 0.5% of annual annuity amount

Claim expenses of 0.5% of death benefit payable

(6)

ii) Without undertaking further calculations, briefly describe the impact on the Single Premium of the policy alterations below:

- a) Death benefit is payable at the end of year of death (1)

b) Annuity is payable annually in arrears, with annual annuity amount remaining unchanged (1)

[8]

5. A joint life annuity is issued to a male life now aged 60 years exact and a female life now aged 55 years exact. The annuity is payable monthly in arrears and is subject to following conditions:

- The amount of annuity INR 200,000 per annum is payable while both lives survive.
- If the male life dies first leaving the female life surviving the annuity reduces to INR 100,000 per annum payable until she dies.
- If the female life dies first leaving the male life surviving the annuity reduces to INR 150,000 per annum payable until he dies.
- In addition if either life is alive at the 10th and 20th anniversaries of the policy a cash lump sum of INR 50,000 is paid at each date.

Calculate the present value of this annuity assuming the basis below:

Mortality PMA92C20 and PFA92C20

Interest 4% per annum

Expenses Nil

6. A life insurance company has launched a protection product, which in case of death of the life assured during the policy term, pays the beneficiary the Basic Sum Assured immediately upon death. Additionally, the beneficiary will also receive monthly income of 1% of the Basic Sum Assured, for a fixed period of 10 years. The monthly income shall be payable in arrears and commence one month after the death of the life assured. The policy term extends till the assured attains age of 70 years exact.

Premiums are payable annually in advance over the policy term.

Basic Sum Assured is INR 5,000,000.

Other details regarding the basis are provided below:

Age of policyholder – 40 years exact

Mortality: AM92 Ultimate

Initial Expense = INR 2000 plus 20% of first year premium

Renewal expense = INR 500 per annum (assume no inflation), incurred from start of the second policy year and until the policy remains in-force.

Claim expenses are 0.5% of benefit payout.

Commission = first year commission is 15% of first year premium; and renewal commissions are 3% of premium

Interest rate = 4% p.a.

- i) Using the principle of equivalence, calculate the gross premium.
- ii) The insurance company is looking to launch another variant of the same product. Under this variant, death benefit will be payable until the policyholder attains the age of 99 and monthly income benefits upon death are no longer payable. All other details remain the same as above.

Calculate the gross premium, using the principle of equivalence

- iii) Without performing further calculations, comment on how premium in part (ii) will change in the following scenarios
- Premiums are paid quarterly in advance
 - Benefits are payable whole of life

7. Mr. Sharma (exact age 65) purchases a Joint Life Annuity with his spouse Mrs. Sharma (exact age 64). The annuity provides following income benefits:

- Definite benefits of INR 50,000 per month payable for first 5 years, irrespective of survival status of any or both lives.
- Post completion of 5 years, if both lives are alive, the benefits increase to INR 75,000 per month until the death of the first life, post which it continues at INR 50,000 per month till the death of remaining life.
- However if at the end of 5th year If Mr. Sharma is alive and Mrs. Sharma is dead it will pay a lump sum benefit of INR 2,500,000 and the contract would cease. But, if Mr. Sharma is dead and Mrs. Sharma is alive, the benefit would continue at INR 50,000 per month till she dies.

All benefits are paid at the end of month.

Calculate the expected present value of such a contract.

Basis: Mortality: PMA92C20 (male life), PFA92C20 (female life)

Rate of interest: 4% per annum

Expenses: Nil

[10]

8. i) Define gross premium prospective reserve and gross premium retrospective reserve. (2)

ii) State the conditions under which, in general, the prospective reserve is equal to the retrospective reserve, allowing for expenses. (2)

iii) Prove that, under the above conditions, the prospective reserve is equal to the retrospective reserve at time t for an immediate annuity contract with annual annuity amount of B payable in arrears, initial expenses of I at the outset of the contract and renewal expenses of R at the start of every year from the second year onwards. (4)
[8]

9. At the beginning of 2005, a life insurance company issued a number of immediate annuity policies to male lives then aged 50 exact. Each policy provides an annual annuity amount of Rs. 1,000 in advance and a death benefit of the single premium amount payable at the end of year of death.

i) Calculate the single premium for each policy using the following premium basis:

Mortality: AM92 Ultimate

Interest: 4% per annum

Commission: 2% of the single premium

Initial expenses: Rs. 100

Renewal expenses: Rs. 10 per annum at the start of the second and subsequent policy years (2)

ii) Calculate the gross premium reserve for each policy in force at the end of the tenth policy year using the same basis as above. (2)

iii) Using the recursive relationship between reserves at successive intervals, calculate the gross premium reserve for each policy in force at the end of the ninth policy year. (2)

iv) At the beginning of 2014, there were 500 policies in force.

The actual experience for this portfolio of business during 2014 was as follows:

Number of deaths: 1

Interest earned: 4%

Expense incurred per policy in force at beginning of policy year: Rs. 9

Calculate the profit or loss from this portfolio of business in 2014 separately from mortality, interest and expenses. (6)

[12]

10. Using the PMA92C20 and PFA92C20 tables and 4% pa interest, calculate $\ddot{a}_{50:50:20|}$, assuming that one life is male and the other is female. [6]

11. A life insurance company issues a 35-year endowment assurance contract to a life aged 30 exact. The sum assured of Rs 500,000 is payable at maturity or at the end of the year of death if earlier. Level premiums are payable annually in advance for the duration of the contract.

i) Calculate the annual premium using the following basis:

- Interest: 6% per annum
- Mortality: AM92 Ultimate
- Expenses:
 - o Initial: Rs 300 plus 50% of the annual premium
 - o Renewal: 2% of the second and subsequent annual premiums
 - o Claim: Rs 600 on death; Rs 200 on maturity (6)

ii) Calculate the retrospective policy reserve after 25 years, using the same basis as in (i), but with 4% per annum interest. (4)

[10]

12. i) On 1st April 2014, a life insurance company issued joint whole life assurance policies to couples. Each couple comprised one male and one female life and both were aged 50 exact on 1st April 2014. Under each policy, a sum assured of Rs. 2,00,000 is payable immediately on the death of the second of the lives to die. Premiums under each policy are payable annually in advance while at least one of the lives is alive.

You are required to calculate: The annual premium payable under the policy.

Basis:

- Mortality: PMA92C20 for the male and PFA92C20 for the female
- Interest: 4% per annum
- Initial Expenses: Rs 1,000
- Expenses: 5% of each premium payment (7)

ii) On 1st April 2018, 5,000 of these policies were still in force. Under 100 of these policies only the female life was alive. Both lives were alive under the other 4,900 policies.

The company calculates provisions for the policies on a net premium basis, using PMA92C20 and PFA92C20 mortality for the male and female lives respectively and 4% per annum interest.

During the calendar year 2018, there was one claim for death benefit, in respect of a policy where the female life only was alive at the start of the year.

In addition, one male life died during the year under a policy where both lives were alive at the start of the year. 4,999 of the policies were in force at the end of the year.

Calculate the mortality profit or loss for the group of 5,000 policies for the calendar year 2018. (13)
[20]

13. A life insurance company issues an annuity policy to two lives each aged 60 males and 55 females exact in return for a single premium. Under the policy, an annuity is payable annually in advance while at least one of the lives is alive. The premium is Rs 1,00,000.

i) Write down an expression for the net future loss random variable at the outset for this policy. (3) ii) Derive the annuity in respect of the premium payable.

Basis:

- Mortality: PMA92C20 for the first life, PFA92C20 for the second life
- Interest: 4% per annum (6)

iii) Calculate the standard deviation of the net future loss random variable at the outset for this policy, using the basis in part (ii) above.

You are given that $\ddot{a}_{\overline{60m:60f}|} = 11.957$ at a rate of interest 8.16% pa. (6)
[15]

14. A joint life whole life assurance with sum assured INR 100,000, payable at the end of the year of death of the first life to die, is sold to male lives aged 58 and female lives aged 56 both exact.

i) Calculate the annual premium payable for this policy assuming PMA92C20/PFA92C20 mortality rates, 4% per annum interest and zero expenses. (3)

A number of such policies were sold on 1st April 2010. On 1st April 2017, 5000 such policies were still in-force. During the financial year 2017-18 ten claims were reported.

ii) Calculate the mortality surplus or deficit for the financial year 2017-18 assuming the reserving basis is the same as the pricing basis. (11)
[14]

15. A life insurance company issues whole life insurance policies to lives age 40 years exact for a sum assured of Rs. 1,50,000 payable at the end of year of death. Premiums are payable annually in advance.

i) Calculate the annual premium for each policy using the basis below.

Basis: Mortality AM92 Select

Interest 6% per annum

Initial Commission 75% of annual premium

Initial Expense Rs.500

Renewal Commission 3% of each annual premium excluding the first

Renewal expense Rs.100 per annum at the start of second and subsequent policy years

ii) Calculate the minimum annual premium the company should charge in order that the probability of making a loss on any policy would be 5% or less.



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