INSTITUTE OF ACTUARIAL & QUANTITATIVE STUDIES

Subject: Pricing & Reserving for Life Insurance Products

Chapter:

Category: Assignment Solution

1. (i) • The expression suggests that this is an endowment assurance contract, with benefit on death and maturity being equal to S and term of contract being n years. [1]

• Death benefits are assumed to be payable immediately on death

• Maturity benefit is paid at the end of the contract term

• Premiums are level and are payable continuously

• Initial expenses are incurred on the day the contract is sold and are assumed to be a one-time cost

• Renewal expenses are also assumed to be payable continuously

• Claim related expenses are assumed to be payable immediately on death and on maturity.

[0.5 mark for each valid point] [4]

(ii)

The reserve required to be held at the end of time 2 =

$110{,}000 * (v * q_{49} + v^2 * p_{49} * q_{50} + v^3 * {}_2p_{49} * q_{51}) - 2000 * (1 + v * p_{49} + v^2\, {}_2p_{49})$ [1]

We now need to calculate each of the above probabilities.

$p_{49} = e^{(-0.025)} = 0.97531$

$p_{50} = e^{(-0.03)} = 0.970446$

$p_{51} = e^{(-0.03)} = 0.970446$

${}_2p_{49} = p_{49} * p_{50} = 0.9465$

$v = 0.94340$, $v^2 = 0.89$, $v^3 = 0.83962$ [2]

Reserve =

$110000 * (0.94340 * (1 - 0.97531) + 0.89 * 0.97531 * (1 - 0.970446) + 0.83962 * 0.9465 * (1 - 0.970446) - 2000 * (1 + 0.94340 * 0.97531 + 0.89 * 0.9465)$

$= 110000 * 0.07243 - 2000 * 2.762$

$= 7967.3 - 5525$

$= 2442$ [1]

[4]

[8 Marks]

2. (i) (a) Endowment Assurance
The endowment assurance, in return for a series of regular or single premium, provides following benefits:

- a survival benefit at the end of the term
- a lump sum benefit on death if it occurs before the end of the term.

Cash flows from policyholder's perspective, it will be a series of negative cash flows throughout the specified term or until death if earlier followed by a large positive cash flow at the end of the term (or death if earlier)
Cash flows from insurer's perspective, there is a stream of regular positive cash flows which cease at a specified point (or earlier if the policyholder dies), followed by a large negative cash flow. The negative cash flow is certain to be paid but the timing of that payment depends on whether / when the policyholder dies [2]

(b) Term assurance
A term assurance is an insurance policy which provides a lump sum benefit on death before the end of the specified term usually in return for a series of regular premiums.

Cash flows from policyholder's perspective, it will be a series of negative cash flows throughout the specified term or until death if earlier followed by a large positive cash flow payable on death, if death occurs before the end of the term. If the policyholder survives to the end of the term there is no positive cash flow.

Cash flows from insurer's perspective, there is a stream of regular positive cash flows which cease at a specified point (or earlier, if the policyholder dies), followed by a large negative cash flow, contingent on policyholder death during the term. [2] [4]

(ii) For a fixed purchase amount, the annuity amount is inversely proportional to the expected number of annuity instalments, i.e. if the number of instalments is expected to be more, it reduces the annuity amount.

The option (c) is expected to give the lowest number of annuity instalments as it will commence on the death of the first life and will be payable till the survival of the second life. Further, in other options the number of annuity instalments payable is expected to be more than under option (c). The lowest annuity amount will be for option (b) as the payments are expected to be for longer period. Hence option (c) will give highest annuity amount. [2]

(iii) $a < c < e < d < b$ [2]
[8 Marks]

3.

Let P be the level annual premium. The expected PV of the premium is:

$P^* \ddot{a}_{60:56:5} = P * (\ddot{a}_{60:56} - v^5 *{}_5p_{56} *{}_5p_{60} * \ddot{a}_{65:61})$ [0.5]

Where the 60 year old is subject to male mortality rates and the 56 year old is subject to female mortality rates.

Using the tables:

$P^* \ddot{a}_{60:56:5} = P * (14.646 - 1.04\text{^}{-5} * 9828.163/ 9907.249 * 9647.797/9826.131 * 12.560)$

$= 4.59087\ P$ [1]

The expected present value of the expenses is:

$500 + 0.02 * P * (\ddot{a}_{60:56:5} -1) = 500 + 0.02*P*3.59087 = 500 + 0.07182*P$ [0.5]

The expected present value of the benefit payable on death is 100,000 for the first 5 years and 200,000 between 6 to 10 years.

$100{,}000 * A^1_{60:56:10} + 100{,}000\ A^1_{65:61:5} * v^5 *{}_5p_{56} *{}_5p_{60}$ [1]

where

$A^1_{60:56:10} = (A_{60:56} - v^{10} *{}_{10}p_{56} *{}_{10}p_{60}\ A_{70:66})$ [0.5]

$= (1\text{-}d^* \ddot{a}_{60:56} - v^{10} *{}_{10}p_{56} *{}_{10}p_{60} * (1\text{-}d^* \ddot{a}_{70:66}))$

$=(1\text{-}\ 0.04/1.04 * 14.646 - 1.04\text{^}{-10} * 9658.285/9907.249 * 9238.134/9826.131*(1\text{-}0.04/1.04 * 10.368))$

$=0.43669 - 0.61918 * 0.60123$

$=0.06442$ [0.5]

Similarly,

$A^1_{65:61:5}$ = = ($A_{65:61} - v^5 * {}_5p_{61} * {}_5p_{65}\ A_{70:66}$) [0.5]

= (1-d* $ä_{65:61} - v^5 * {}_5p_{61} * {}_5p_{65}$ * (1-d* $ä_{70:66}$))

=(1- 0.04/1.04 *12.560 – 1.04^-5 * 9658.285/9828.163 * 9238.134/9647.797*(1-0.04/1.04 * 10.368))

= 0.51692 – 0.77342 *0.60123 [0.5]

= 0.05192

Total EPV of lumpsum benefit:

100,000 * $A^1_{60:56:10}$ + 100,000 $A^1_{65:61:5}$ * $v^5 * {}_5p_{56} * {}_5p_{60}$ [0.5]

= 100,000 * 0.06442 + 100,000 * 0.05192* 1.04^-5 * 9828.163/ 9907.249 * 9647.797/9826.131

=INR 10,598.55

The deferred annuity of INR 20,000 p.a. while both lives are alive or INR 10,000 if only one of them is alive. This is equivalent to each life receiving a single life annuity of INR 10,000 (since 20,000 will be paid in total if both are alive) [0.5]

10,000 * ($v^{10} * {}_{10}p_{56}$ * $ä_{66}$+ $v^{10} * {}_{10}p_{60}$* $ä_{70}$)

= 10,000 * 1.04^-10 * (9658.285/9907.249*14.494 + 9238.134/9826.131*11.562)

= INR 168,890.38 [1]

Setting the EPV of premiums equal to the EPV of benefits and expenses gives

4.59087 P = 168,890.38 + 10,598.55 + 500 + 0.07182*P [0.5]

4.51905P = 179,988.93

P = INR 39,828.93 [0.5]

[8 Marks]

4. Let Single premium = P

Monthly Annuity amount (S) = 10,000

Initial expense (f) = 2% of Single Premium
Regular expense (e)= 0.5% * 12 * S
Claim expenses = 0.5% * P at the time of death
Age of policyholder (x) = 65
Annuity payments and regular expenses are assumed to be made in arrears. Claim expenses are paid immediately upon death.

By principle of equivalence:
$P = 12 * S * a_{65}^{(12)} + 2\% * P + 0.5\% * 12 * S * a_{65} + P\ \bar{A}^1_{65:10} + 0.5\% * P * \bar{A}^1_{65:10}$

[1.5]

(Full marks can be awarded to the candidate if regular expenses are considered as monthly in the above formula)

In the equation above:
$a_{65}^{(12)} = \ddot{a}_{65}^{(12)} - 1/12 = \ddot{a}_{65} - (12\text{-}1)/(2*12) - 1/12 = 13.666 - 11/24 - 1/12 = 13.124$
$a_{65} = \ddot{a}_{65} - 1 = 13.666 - 1 = 12.666$

[1]

$\bar{A}^1_{65:10} = 1.04^{(1/2)} * A^1_{65:10}$
$A^1_{65:10} = A_{65} - 1.04^{(-10)} * {}_{10}p_{65} A_{75}$
$A_{65} = 1 - 0.04/1.04 * \ddot{a}_{65} = 1 - 0.04/1.04 * 13.666 = 0.474385$
$A_{75} = 1 - 0.04/1.04 * \ddot{a}_{75} = 1 - 0.04/1.04 * 9.456 = 0.636308$
${}_{10}p_{65} = l_{75} / l_{65} = 8405.160 / 9647.797 = 0.8712$

So $\bar{A}^1_{65:10} = 1.04^{(1/2)} * (0.474385 - 1.04^{(-10)} * 0.8712 * 0.636308) = 0.101863$ [2.5]
Therefore,
$P = 120000 * 13.124 + 0.02 * P + 600 * 12.666 + P * (0.101863 + 0.005 * 0.101863)$
$P\ (1 - 0.02 - 0.102372) = 1{,}582{,}480$
$P = 1{,}803{,}133$

[1]
[6]

ii)

a. If death benefit payable at end of year of death, this will reduce the present value of death benefit to be paid out. This will also reduce the present value of claim related expenses which are assumed to be incurred at the time of death payment. Therefore, the single premium will reduce. [1]

b. If annuity is payable annually in arrears then PV of annuity outgo will reduce, as roughly
$a_{65}^{(12)} = a_{65} + 11/24$, which means that $a_{65}^{(12)}$ is higher than a_{65}. Therefore, single premium will reduce. [1]

5.

The present value of the annuity is given by

$$100000\, a_{\overline{60:55}}^{(12)} + 50000\, a_{60:55}^{(12)} + 50000\, a_{60}^{(12)} + 50000\,(v^{10}10p_{\overline{60:55}} + v^{20}20p_{\overline{60:55}})$$

$$a_{60}^{(12)} = \ddot{a}_{60} - \frac{13}{24} = 15.632 - \frac{13}{24} = 15.0903$$

$$a_{60:55}^{(12)} = \ddot{a}_{60:55} - \frac{13}{24} = 14.756 - \frac{13}{24} = 14.214$$

$$a_{\overline{60:55}}^{(12)} = \ddot{a}_{60} + \ddot{a}_{55} - \ddot{a}_{60:55} - \frac{13}{24} = 15.632 + 18.210 - 14.214 - \frac{13}{24} = 18.544$$

$$v^{10}10p_{\overline{60:55}} = \frac{1-\left(1-\frac{l_{70}}{l_{60}}\right)\left(1-\frac{l_{65}}{l_{55}}\right)}{1.04^{10}}$$

$$= \frac{1-\left(1-\frac{9238.134}{9826.131}\right)\left(1-\frac{9703.708}{9917.623}\right)}{1.48024} = 0.67469$$

$$v^{20}20p_{\overline{60:55}} = \frac{1-\left(1-\frac{l_{80}}{l_{60}}\right)\left(1-\frac{l_{75}}{l_{55}}\right)}{1.04^{20}}$$

$$= \frac{1-\left(1-\frac{6953.536}{9826.131}\right)\left(1-\frac{8784.955}{9917.623}\right)}{2.19112}$$

$$= 0.44115$$

So the value is:

$$100000 \times 18.544 + 50000 \times 14.214 + 50000 \times 15.0903 + 50000 \times (0.67469 + 0.44115)$$

=3375407

6. Age = 40. Policy term = 30.

i) Based on principle of equivalence,

PV Premium = PV Claims + PV Expenses + PV Commissions

(0.5)

PV Premium = P x $a^{due}_{40:30}$

$a^{due}_{40:30} = a^{due}_{40} - v^{30} \times l_{70}/l_{40}\ a^{due}_{70} = 20.005 - 1.04^{-30} \times 8054.0544/9856.286 \times 10.375$

=17.3911

PV Premium = 17.3911x P

(2)

PV Commissions

P x $(0.15 + v \times 0.03 \times l_{41}/l_{40} \times a^{due}_{41:29})$

$a^{due}_{41:29} = a^{due}_{41} - v^{29} \times l_{70}/l_{41}\ a^{due}_{70} = 19.784 - 1.04^{-29} \times 8054.0544/9847.051 \times 10.375$

=17.06299

$l_{41}/l_{40} = 9847.051/9856.286 = 0.999063$

So PV Commission = 0.641741 x P

(2.5)

PV Initial Expenses = 2000 + 0.2 x P

(1)

PV Renewal Expense = 500 x v x $l_{41}/l_{40} \times a^{due}_{41:29}$ = 500 x 16.39135= 8195.676 (based on calculations performed for PV Commissions already)

(1)

PV Claims = $(5{,}000{,}000 + 12 \times 0.01 \times 5{,}000{,}000 \times a^{(12)}_{10}) \times A^{1}_{40:30}$ (continuous)

$a^{(12)}_{10} = (1- v^{10}) / i^{(12)} = 8.258599$

(1)

$A^1_{40:30} = A_{40} - A_{70} \times v^{30} \times l_{70}/l_{40} = 0.23056 – 0.60097 \times 1.04^{-30} \times 8054.0544/ 9856.286 = 0.07915$

$A^1_{40:30}{}^{(continuous)} = 1.04^{0.5} \times 0.07915= 0.080718$

(1)

PV Claims = (5,000,000 + 12 x 0.01 x 5,000,000 x 8.258599) x 0.080718
= 803,561

(1)

PV Claim expenses = 0.5% x PV Claims = 0.005 x 803561 = 4017.8

(1)

So based on principle of equivalence we have,
17.3911 x P = 0.641741 x P + 2000 + 0.2 x P + 8195.676 + 803561 + 4017.8
16.54936 x P = 817774
P = 49414

ii) Under this variant,

PV Claims = (5,000,000) x $A^1_{40:59}$ (continuous)

(0.5)

$A^1_{40:59} = A_{40} - A_{99} \times v^{59} \times l_{99}/l_{40} =$ 0.23056 – 0.90139 x $(1.04)^{(-59)}$ x 143.712 / 9856.286= 0.229261

$A^1_{40:59}$ (continuous) = $1.04^{0.5}$ x 0.229261 = 0.233801

(1.5)

PV Claims = 1,169,005

(0.5)

PV claim expenses = 0.005 x 1,169,005 = 5845.023

(0.5)

So, using elements from premium equation in part a)

16.54936 x P = 2000 + 8195.676 + 1,169,005 + 5845.023 = 1,185,045

P = 71,606.7

(1)

iii)

a) Premium payable will be higher. This is because company is able to earn higher investment income when full year's premium is received at start of year rather than spread over the year.

b) Premium payable will be higher. Under whole of life, probability of claim payment will increase resulting in higher expected claims and claims expenses.

7.

This can be divided into sub benefits:

Working in '000 (thousands)

We can break the benefits into three components:

A. A = Guaranteed annuity for first 5 years = $12*50* a_{\overline{5|}}^{(12)}$ at 4%

B. B = Benefit if both alive at end of 5th year =

$${}_5p_{65}(M) * {}_5p_{64}(F) * v^5 * [50 * 12 * a^{(12)}_{\overline{70(m):69(f)}} + 25 * 12 * a^{(12)}_{70(m):69(f)}]$$

C. C = Benefit if only one is alive at the end of 5th year =

$${}_5p_{65}(M) * {}_5q_{64}(F) * v^5 * 2{,}500 + {}_5q_{65}(M) * {}_5p_{64}(F) * v^5 * 50 * 12 * a^{(12)}_{69(f)}$$

Now:

$a_{\overline{5|}}^{(12)}$ @ 4% = $i / i^{(12)} * a_{\overline{5|}}$ @ 4%

= [0.04/{(1.04^(1/12)-1)*12}] * 4.4518

= (0.04/0.039285) * 4.4518

= 4.53283

$a^{(12)}_{69(f)}$ = ($\ddot{a}_{69(f)}$ - 13/24)

= (13.33-13/24)

= 12.78833

$$a^{(12)}_{\overline{70(m):69(f)}} = [\ (\ddot{a}_{70(m)} - 13/24) + (\ddot{a}_{69(f)} - 13/24) - (\ddot{a}_{70(m):69(f)} - 13/24)]$$
$$= (11.562 + 13.33 - 9.932 - (13/24))$$
$$= 14.41833$$

(Where $\ddot{a}_{70(m):69(f)}$ = 9.932)

$$a^{(12)}_{70(m):69(f)} = (\ddot{a}_{70(m):69(f)} - 13/24)$$
$$= (9.932-13/24)$$
$$= 9.390333$$

Putting all these in above benefits and looking up other probabilities from given mortality tables we get:

A = 600*4.53283 = 2719.698

B = (9238.134/9647.797) * (9473.745/9742.64) *
(50*12*14.41833 + 25*12*9.390333)/1.04^5
= 0.95753 * 0.9724 * (600*14.41833 + 300*9.390333) / 1.04^5
= 8776.514

C = 0.95753 * (1-0.9724)*2500 / 1.04^5
+ (1-0.95753) * 0.9724*50*12*12.788 / 1.04^5

= 314.74839

PV of total benefits = (A+ B+ C)*1000 = INR 1, 18, 10,960

[10 Marks]

8.

(i) Gross premium prospective reserve is the expected present value of future benefits and future expenses less the expected present value of future gross premiums.

Gross premium retrospective reserve is the expected accumulation of past gross premiums received, less past expected expenses and benefits.

[2]

(ii) Gross premium retrospective and prospective reserves will be equal if:

- the mortality, interest rate and expense basis used is the same as used to determine the original gross premium; and
- the gross premium is that determined on the original basis (mortality, interest, expenses) using the equivalence principle

[2]

(iii) Let P be the single premium. Then, by equivalence principle:

$$P = (B+R)a_x + I$$
$$\Rightarrow P = (B+R)(a_{x:\overline{t}|} + v^t\,{}_tp_x a_{x+t}) + I$$
$$\Rightarrow P - I - (B+R)a_{x:\overline{t}|} = (B+R)v^t\,{}_tp_x a_{x+t}$$
$$\Rightarrow [P - I - (B+R)a_{x:\overline{t}|}]\frac{(1+i)^t}{{}_tp_x} = (B+R)a_{x+t}$$
$$i.e.\ {}_tV^{retro} = {}_tV^{pro}$$

[4]

[8 Marks]

9.

(i) Let P be the single premium. Then, the equation of value is:

$$P = 1{,}000\ddot{a}_{50} + 90 + 10\ddot{a}_{50} + 0.02P + P \leftrightarrow A_{50}$$
$$\Rightarrow P = (1{,}000+10)\times 17.444 + 90 + P(0.02 + 0.32907) \Rightarrow P = 27{,}204.83$$

[2]

(ii) $_{60}V = 1{,}000\ddot{a}_{60} + 10\ddot{a}_{60} + P \leftrightarrow A_{60}$

$$\Rightarrow {}_{60}V = (1{,}000+10)\times 14.134 + 27{,}204.83 \times 0.45640 = 26{,}691.62$$

[2]

(iii) $_{59}V = \dfrac{{}_{60}V(1-q_{59}) + Pq_{59}}{1+i} + 1{,}000 + 10$

$$Þ\ {}_{59}V = \frac{26{,}691.62 \leftrightarrow (1-0.007140) + 27{,}204.83 \leftrightarrow 0.007140}{1.04} + 1{,}010 = 26{,}678.54$$

[2]

(iv) The death strain at risk per policy in the 10th year is given by:

$$DSAR = P - {}_{60}V = 513.21$$

$$\therefore EDS = DSAR \times 500 \times q_{59} = 1{,}832.16 \text{ and}$$
$$ADS = DSAR \times 1 = 513.21$$

Therefore, the mortality profit in the 10th year is given by:

$$EDS - ADS = 1{,}318.95$$

The total profit (from all sources) in the 10th year is given by:

$$500 \times ({}_{59}V - 1{,}000 - 9)(1+4\%) - P \times 1 - (500-1) \times {}_{60}V$$
$$= 1{,}837.59$$

Note that the interest rate profit in the 10th year is 0 since the actual interest rate earned is same as that expected on the valuation basis.

The expense profit in the 10th year is therefore the difference between the total profit and the mortality profit i.e. 1,837.59 – 1,318.95 = 518.64 **[6]**

[12 Marks]

10.

$\ddot{a}_{\overline{50:50:20|}} = \ddot{a}^{(m)}_{50:\overline{20|}} + \ddot{a}^{(f)}_{50:\overline{20|}} - \ddot{a}_{50:50:\overline{20|}}$

$\ddot{a}_{50:\overline{20|}} = \ddot{a}_{50}$ - v^{20} * l_{70} / l_{20} * $\ddot{a}_{70}$

For males, this is

$\ddot{a}^{(m)}_{50:\overline{20|}}$ = 18.843 - 1.04^(-20) * 9,238.134 / 9941.923 * 11.562 = 13.940

For females this is

$\ddot{a}^{(f)}_{50:\overline{20|}}$ = 19.539 - 1.04^(-20) * 9,392.621 / 9952.697 * 12.934 = 13.968

$\ddot{a}_{50:50:\overline{20|}}$ = $\ddot{a}_{50:50}$ - 1.04^(-20) * 9, 238.134/9941.923 * 9,392.621/9952.697 * $\ddot{a}_{70:70}$

= 17.688 - 1.04^(-20) * 9, 238.134/9941.923 * 9,392.621/9952.697 * 9.7666

= 13.780

Therefore, $\ddot{a}_{\overline{50:50:20|}}$ = 13.940 + 13.968 – 13.780 = 14.128

11.

i) P $\ddot{a}_{30:35}$ =500600 $A_{30:35}$ – 400 $A_{30:35}1$ + 0.02 P $\ddot{a}_{30:35}$ – 0.02 P + 300 + 0.5P

Expected Present Value of premium

P $\ddot{a}_{30:35}$ = 15.150 P

EPV of benefits and claim expenses

$A_{30:35}$ =0.14246

$A_{30:35}1$ = v^{35} X $_{35}P_{\ 40}$ = 0.13011X0.88877 = 0.11563

EPV of benefits

500600 X .14246 – 400 X 0.11563

= 71269.22

EVP of remaining Expense

0.02 X 15.150 P – 0.02 P +0.5P + 300 = .783 P +300

Equation of value

15.150 P = 71269.22 + 300 +.783 P

14.367 P = 71569.22

P = 4981.5 pa

ii) **Retrospective reserve**

$_{25}V^{Retro} = \frac{1}{v^{25}\ X\ _{25}P_{\ 30}}$ X 0.98 P $\ddot{a}_{30:25}$ – 0.48P-300 -500600 $A^{\ \ 1}_{30:35}$

v^{25} X $_{25}P_{\ 30}$ = .37512 X 0.96298 = 0.36123

$\ddot{a}_{30:25}$=$\ddot{a}_{30}$ - $v^{25}\ _{25}P_{\ 30}\ddot{a}_{55}$ =21.834 - .36123 X 15.873 = 16.1

$A^{\ \ 1}_{30:35}$ = A_{30} -$v^{25}\ _{25}P_{\ 30}A_{55}$ =.16023 - .36123X.38950 = 0.01953

$_{25}V^{Retro}$= (1/.36123) [4981.5((0.98)*(16.100)-(0.48)]-300-(500600X0.01953)

=1,83,069.7 [4]

[10 Marks]

12.

i) Let P be the annual premium

Premium = P $\ddot{a}_{\overline{50^m:50^f}}$

$= P\left[\ddot{a}_{50^m} + \ddot{a}_{50^f} - \ddot{a}_{50^m:50^f}\right]$

= P [18.843+19.539-17.688]

= 20.694 P

Initial expense = 1,000

Recurring expense = 0.05 P x $\ddot{a}_{\overline{50^m:50^f}}$

= 0.05P x 20.694

= 1.0347 P

Claim cost = 2,00,000 $\bar{A}_{\overline{50^m:50^f}}$

= 2,00,000 x $1.04^{0.5}$ - $A_{\overline{50^m:50^f}}$

=2,00,000 x $1.04^{0.5}$ x $\left(1 - d\ddot{a}_{\overline{50^m:50^f}}\right)$

=2,00,000 x $1.04^{0.5}$ x $\left(1 - \frac{0.04}{1.04} \text{ x } 20.694\right)$

= 41,623. 68852

=20.694 P = 1000 + 1.0347 P + 41,623.68852

P = 2,168. 02

ii) Net Premium

$= 2{,}00{,}000 \times 1.04^{0.5} \times \left(\frac{1}{\ddot{a}_{50m:\,50f}} - d\right)$

$=2{,}00{,}000 \times 1.04^{0.5} \times \left(\frac{1}{20.694} - \frac{0.04}{1.04}\right)$

= 2,011.39

We will require three provisions at the end of 5th policy year.

- **Both lives are alive**

$= 2{,}00{,}000 \times 1.04^{0.5} \times \left(1 - \frac{\ddot{a}_{\overline{55:55}}}{\ddot{a}_{\overline{50:50}}}\right)$

$= 2{,}00{,}000 \times 1.04^{0.5} \times \left(1 - \frac{17.364+18.210-16.016}{18.843+19.539-17.688}\right)$

= 11,196.46

- **Only Male alive**

$= 2{,}00{,}000 \times \bar{A}_{55}$ $-2{,}011.39\ \ddot{a}_{55}$

$= 2{,}00{,}000 \times 1.04^{0.5} \times \left(1 - \frac{0.04}{1.04}\,x17.364\right)$ - 2,011.39 x 17.364

= 32,820.60

- **Only Female alive**

  = 2,00,000 x $\bar{A}_{55}$ -2,011.39 $\ddot{a}_{55}$

  = 2,00,000 x $1.04^{0.5}$ x $\left(1 - \frac{0.04}{1.04}\ x\ 18.210\right)$ - 2,011.39 x 18.21

  = 24, 482.39

Mortality Profit/ loss = Expected Death Strain – Actual Death Strain

There are four components

(a) **Both lives die during 2018: no claims reported**

= (4900 x q_{54^m} x q_{54^f} -0) x (2,00,000 x $1.04^{0.5}$ -11,196.46)

= 849.37

(b) **Female alive at the start of the year but dies during the year: one actual claim**

=(100 x q_{54^f} -1) x (2,00,000 x $1.04^{0.5}$ -24,482.39)

= -1,63,109.96

(c) **Both lives alive at start of the year, only male dies during the year: one actual claim**

=(4900 x q_{54^m} x p_{54^f} -1) x (24,482.39 – 11,196.46)

=50,845.17

(d) **Both lives alive at start of the year, only** female **dies during the year: No actual claims**

=(4900 x p_{54^m} x q_{54^f} -0) x (32,820.611 – 11,196.46)

=96,538.66

Total mortality loss = 849.37 - 1,63,109.96 + 50,845.17+ 96,538.66 **= (14,876.77)**

[13]

[20 Marks]

13.

i) Net future loss random variable

Let x be the annual annuity payment amount.

The net future loss random variable at the outset for this policy is

$L = X\ \ddot{a}_{\overline{\max(k_{60^m}, k_{55^f}) + 1}} - P$

Where,

P is the single premium i.e. 1,00,000

K_{60} is the curtate future lifetime of a male life aged 60

K_{55} is the curtate future lifetime of a female life aged 55

ii) Annual annuity payment amount

$P = x\ \ddot{a}_{\overline{60^m : 55^f}}$ @ 4%

$= 1{,}00{,}000 = x \left[\ddot{a}_{60^m} + \ddot{a}_{55^f} - \ddot{a}_{\overline{60^m : 55^f}}\right]$

=1,00,000 = x (15,632 + 18. 210 – 14.756) x = 5,239. 442523

Annual annuity payment is 5,239.44

iii) Standard deviation

The variance of the net future loss random variable is

Var (L) = $5{,}239.44^2 \left[\left\{ 2_{A_{\overline{60^m:55^f}}} - \left(A_{\overline{60^m:55^f}}\right)2\right\} ./\ d\right]$

Using premium conversion and the result of 11 (b) we have,

$A_{\overline{60^m:55^f}}$ = $\left(1 - d\ \ddot{a}_{\overline{60:55}}\right)$ @ 4%

= 1- $\frac{0.04}{1.04}$ x 19.086

= 0.26592307

And,

$A_{\overline{60^m:55^f}}$ = $\left(1 - d\ \ddot{a}_{\overline{60:55}}\right)$ @ 8.16%

= 1- $\frac{0.0816}{1.0816}$ x $\ddot{a}_{\overline{60:55}}$

So, the standard deviation of L is

$$\sqrt{5{,}239.44\text{^}2 + \left[\left\{ 2_{A_{\overline{60^m:55^f}}} - \left(A_{\overline{60^m:55^f}}\right)\text{^}2\right\} ./\ d^2\right]}$$

14.

i) The premium equation assuming that the annual premium equals P is

$P * \ddot{a}_{58:56} = 100000 * A_{58:56}$

Or $P = 100000 * A_{58:56} / \ddot{a}_{58:56}$

Now the annuity value from the table is $\ddot{a}_{58:56} = 15.180$

Also we know, $A_{58:56} = 1 - d * \ddot{a}_{58:56} = 1 - 0.04/1.04*15.180 = 1 - 0.38462 * 15.180$

Or $A_{58:56} = .416154$

Therefore $P = 100000*.416154/15.180$

Or $P = 2741.46$

ii) The reserve for a policy as at 31st March 2018 can be expressed as

${}_8V = 100000 * A_{66:64} - 2741.46 * \ddot{a}_{66:64}$

From the tables we have $\ddot{a}_{66:64} = 11.845$

Similarly we know that,

$A_{66:64} = 1 - d * \ddot{a}_{66:64} = 1 - 0.04/1.04*11.845$

Or $A_{66:64} = 0.544423$

Thus, ${}_8V = 100000 * 0.544423 - 2741.46 * 11.845$

Or ${}_8V = 21969.7$

So the death strain at risk is $SA - {}_8V = 100000 - 21969.7 = 78030.30$

The probability that a claim will be made in the FY 17-18 requires the failure of the joint life status xy during the FY 17-18. In other words at least one of the lives should die within a year.

The probability that at least one of the lives dies during the FY 17-18 can be calculated as
(1 – probability that both lives survive the FY 17-18)

Therefore, Probability of a claim in FY 17-18 = 1 – prob both lives survive in FY 17-18

= 1 – prob male survives * prob female survives

= $1 - p^{m}_{65} * p^{f}_{63}$

= 1 – (1-0.006032)*(1-0.003401)

= 0.009402

Since 5000 policies were in force as at 1st April 2017, the expected number of claims over the FY 17-18 = 5000 * 0.009402 = 47.06243

Expected death strain = Expected number of claims over FY 17-18 * DSAR

= 47.06243 * 78030.30

= 3672295.35

Actual death strain = Actual number of deaths * DSAR

= 10 * 78030.30

= 780303.03

Thus, Mortality profit over the FY 17-18 = Expected death strain – Actual death strain

= 3672295.35 - 780303.03

= 2891992.32

[11]

[14 Marks]

15.

i) Let P be the annual premium

Expected present value of premiums

$$P\ddot{a}_{[40]} = 15.494\ P$$

EPV of benefits = 150000 x $A_{[40]}$ = 150000 x 0.12296
= 18,444

EPV of expenses

$0.75P + 500 + (100 + 0.03P)a_{[40]}$
= 0.75P + 500 + (100 + 0.03P) x 14.494
= 1949.4 + 1.18482 x P

So,

15.494P = 18444 + 1949.4 + 1.18482 P

P = 1425

ii) Let P' be the required minimum office premium

Then the insurer's loss random variable for this policy is given by:

$$L = 150000\ v^{K_{[40]}+1} + 500 + 0.75P' + (0.03P' + 100)a_{\overline{K_{[40]}}|} - P'\ddot{a}_{\overline{K_{[40]}+1}|}$$

We need to find the value of t such that

$$P\ (L > 0) = P(T < t) = \ 0.05 \Rightarrow P(T > t) = 0.95$$

Using AM92 Select, we require:

$$\frac{l_{[40]+t}}{l_{[40]}} \geq 0.95 \Rightarrow l_{[40]+t} \geq 0.95 l_{[40]} = 0.95 \times 9854.3036 = 9361.5884$$

As $l_{58} = 9413.8004\ and\ l_{59} = 9354.004\ then\ t\ lies\ between\ 18\ \&\ 19$

$$so\ K_{[40]} = 18$$

We therefore, need the minimum premium such that L=0

$$L = 0 = 150000v^{19} + 500 + 0.75P' + (.03P' + 100)a_{\overline{18}|} - P'\ddot{a}_{\overline{19}|}$$

$$\Rightarrow 0 = 150000 \times 0.33051 + 500 + 0.75P' + (.03P' + 100) \times 10.8276 - 11.8276P'$$

$$\Rightarrow P' = \frac{51159.71}{10.75277} = 4757.82$$