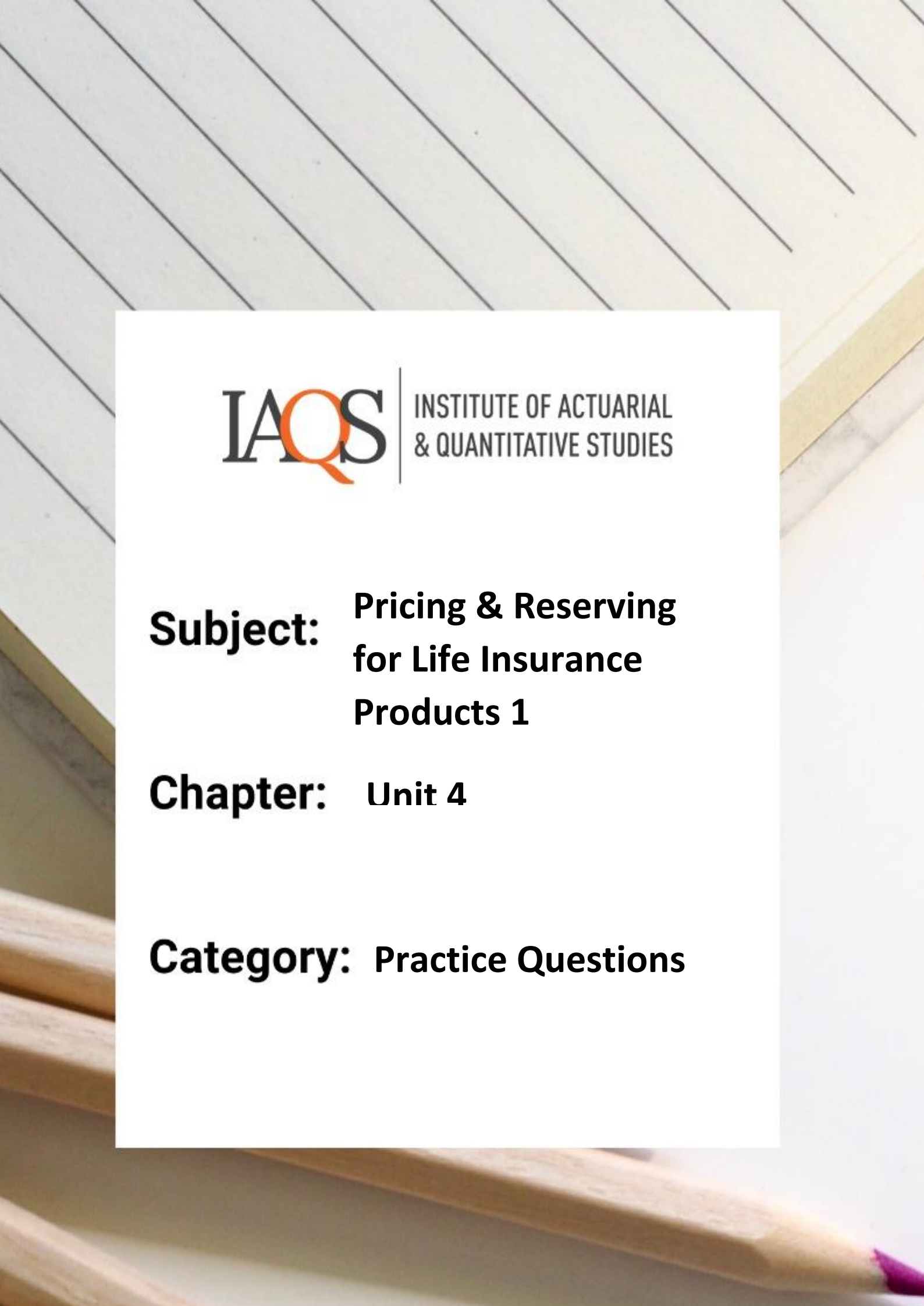

Subject: Pricing & Reserving for Life Insurance Products 1

Chapter: Unit 4

Category: Practice Questions

1. CT5 September 2010 Q3

Calculate the single premium payable for a temporary reversionary annuity of £12,000 per annum payable monthly in arrear to a female life currently aged 55 exact on the death of a male life currently aged 50 exact. No payment is made after 20 years from the date of purchase.

Basis:

Rate of interest 4% per annum

Mortality of male life PMA92C20

Mortality of female life PFA92C20

Expenses Nil

Answer:

[2,388]

2. CT5 September 2010 Q6

Calculate:

(a) $\bar{A}_{\overline{30:40}}$

(b) $\bar{a}_{30:40:\overline{20}|}$

Basis:

$\mu = 0.01$ throughout for the life aged 30 now

$\mu = 0.02$ throughout for the life aged 40 now

$\delta = 4\%$ per annum

Answer:

[(a) .10476, (b) 10.763]

3. CT5 September 2010 Q13

On 1 January 2009, a life insurance company issued 10,000 joint life whole life assurance policies to couples. Each couple comprised one male life aged 60 exact and one female life aged 55 exact when the policy commenced. Under each policy, a sum assured of £100,000 is payable immediately on the death of the second of the lives to die.

Premiums under each policy are payable annually in advance while at least one of the lives is alive.

The life insurance company uses the following basis for calculating premiums and net premium reserves:

Mortality PMA92C20 for the male

PFA92C20 for the female

Interest 4% per annum

Expenses Nil

(i) Calculate the annual premium payable under each policy.

During the calendar year 2009, there was one claim for death benefit, in respect of a policy where both the male and the female life died during the year. In addition, there were 20 males and 10 females who died during the year.

(ii) Calculate the mortality profit or loss for the group of 10,000 policies for the calendar year 2009.

Answer:

[(i) 1,420.83, (ii) −68,169.01]

4. CT5 April 2011 Q9

A male life aged 52 exact and a female life aged 50 exact take out a whole life assurance policy. The policy pays a sum assured of £100,000 immediately on first death. Premiums are payable for a period of five years, monthly in advance.

Calculate the monthly premium payable.

Basis:

Mortality PMA92C20 (male life), PFA92C20 (female life)

Rate of interest 4% per annum

Expenses Nil

Answer:

[627.85]

5. CT5 September 2011 Q5

(a) Write down the random variable form of $\bar{A}^1_{x:y}$.

(b) Calculate $\bar{A}^1_{x:y}$ on the following assumptions:

μ_x= 0.02 for all x

μ_y = 0.03 for all y

δ = 4% per annum

Answer:

[(a) $\bar{Z} = \begin{cases} v_i^{T_x} & \text{if} \quad T_x \le T_y \\ 0 & \text{if} \quad T_x > T_y \end{cases}$, (b) 0.22222]

6. CT5 September 2011 Q7

A special joint life last survivor annuity of £10,000 per annum is payable continuously in respect of a male and female life each aged 60 exact. Payments commence on the first death and continue for 5 years after the second death.

Calculate the expected present value of this annuity.

Basis:

Mortality PMA92C20 (male life), PFA92C20 (female life)

Rate of interest 4% per annum

Expenses Nil

Answer:

[54,934]

7. CT5 April 2012 Q4

A joint life assurance contract provides a death benefit of £100,000 payable immediately on the second death of two lives, a male life currently aged 60 exact and a female life currently aged 55 exact.

Calculate the expected present value of the contract.

Basis:

Mortality PMA92C20 (male life), PFA92C20 (female life)

Rate of interest 4% per annum

Expenses Nil

Answer:

[27104]

8. CT5 September 2012 Q11

A special joint life annuity of £500 per week is payable in arrear in respect of a male life aged 65 exact and a female life aged 62 exact. The annuity has the following features:

• The annuity is guaranteed in any event for the first 5 years at the level of £500 per week.

• At the end of the guarantee period if both lives are still surviving the annuity continues at the same level until one life dies at which time it reduces to two thirds of the initial level and continues at this reduced level until the second life dies.

• At the end of the guarantee period if only one life has survived the annuity reduces to two-thirds of the initial level and continues at this reduced level until the second life dies.

• At the end of the guarantee period if both lives have previously died then the annuity ceases.

Calculate the expected present value of this annuity.

Basis:

Mortality PMA92C20 (male life), PFA92C20 (female life)

Rate of interest 4% per annum

Expenses Nil

Answer:

[395,258]

9. CT5 September 2012 Q12

A life insurance company issues a special endowment assurance policy for a 25 year term to two lives x and y. Under this policy, a sum assured of £100,000 is paid immediately on the second death within the 25-year term. At the end of 25 years a sum of £50,000 is paid to each survivor.

Calculate the annual premium paid continuously under this policy assuming this is paid throughout the term or until the second death if earlier.

Basis:

Mortality Life x: $\mu x = 0.02$ for all x

Life y: $\mu y = 0.03$ for all y

Force of interest 5% per annum

Expenses Nil

Answer:

[1905.23]

10. CT5 April 2013 Q9

A male life currently aged 65 exact purchases a special joint life annuity of £10,000 per annum payable monthly in advance together with additional benefits detailed below.

On the death of the male life, the annuity reduces to £5,000 per annum payable monthly in advance to a female life until her death, assuming she survives him. The female life is currently aged 62 exact.

The policy additionally provides benefits of:

• An annuity certain (extra to the above and not dependent on the survival status of each life) of £10,000 per annum payable monthly in advance and paid only for ten years, and

• £10,000 payable immediately on the death of each life.

Calculate the expected present value of the total benefits.

Basis:

Mortality Male life PMA92C20

Female life PFA92C20

Interest 4% per annum

Expenses Nil

Answer:

[241387]

11. CT5 April 2013 Q11

Two lives are both aged 45 exact.

Calculate:

(i) The probability of both lives surviving to age 65 exact.

(ii) The present value of an annuity of £1,000 per annum increasing by 3% each year payable annually in advance so long as both lives survive.

(iii) The present value of a 20-year term assurance with a benefit of £100,000 payable immediately on the second death.

Basis:

Mortality μ_x = 0.05 for all x for both lives

Interest 4% per annum

Answer:

[(i) 0.13534, (ii) 9628, (iii) 25872]

12. CT5 April 2014 Q7

A Joint Life Annuity is issued to a male aged 65 exact and a female aged 62 exact. The annuity is payable quarterly in arrears for the first payment commencing 3 months after issue.

The annuity has the following conditions:

• £10,000 per annum whilst both lives survive.

• If the male life predeceases the female life the annuity reduces to £7,500 per annum payable for the remainder of her lifetime.

• If the female life predeceases the male life the annuity reduces to £6,000 per annum payable for the remainder of his lifetime.

Calculate the expected present value of the annuity.

Basis: Mortality PMA92C20 (male) and PFA92C20 (female)

Interest 4% per annum

Expenses Ignore

Answer:

[£151974]

13. CT5 April 2015 Q10

(i) Calculate : $\bar{A}^{1}_{40:50}$.

Basis:

Mortality μx = 0.04 throughout life for the life aged 40

μx = 0.06 throughout life for the life aged 50

Rate of interest 5% per annum

Two lives aged 40 and 50 exact purchase a policy with the benefit in part (i) above and a sum assured of 75,000. The benefit is funded by a premium payable continuously for a 30-year period or until the first death if earlier. The premium is paid at a level annual rate for the first 20 years, then reduces by 25% to be paid at the lower level annual rate for the remainder of the period.

(ii) Calculate the initial level annual premium using the basis in part (i) above.

Answer:

[(i) 0.26884, (ii) 3065.7]

14. CT5 April 2015 Q11

A special joint life annuity is issued to a male life now aged 65 exact and a female life now aged 62 exact.

The annuity is payable monthly in arrear and is subject to the following conditions:

• The amount of the annuity while both lives survive is 100,000 per annum.

• If the male life dies first leaving the female life surviving the annuity reduces to 50,000 per annum payable until she dies.

• If the female life dies first leaving the male life surviving the annuity reduces to 75,000 per annum payable until he dies.

• In addition, if either life is alive at the 10th and 20th anniversaries of the policy a cash lump sum of 20,000 is paid at each date.

Calculate the expected present value of the annuity.

Basis: Mortality PMA92C20 and PFA92C20

Interest 4% per annum

Expenses Nil

Answer:

[1479537]

15. CT5 September 2015 Q11

An assurance policy provides a benefit of 10,000 payable immediately on the death of the last survivor of a male life aged 55 exact and a female life aged 50 exact.

(i) Calculate, showing all your workings, the expected present value for this policy.

(ii) Derive an expression for the variance of the value of this policy.

Basis:

Mortality PFA92C20

Interest 4% per annum

Answer:

[(i) 2,235, (ii) $10,000^2 \left\{ {}^2\bar{A}_{\overline{55:50}} - \left(\bar{A}_{\overline{55:50}}\right)^2 \right\}$]

16. CT5 April 2016 Q10

A life insurance policy for a male life aged 55 exact provides the following benefits:

- 50,000 payable immediately on his death, if this occurs before the age of 65 exact.
- On survival to age 65 exact, a refund of 25% of total premiums paid without interest.
- On death of the male at any time, a pension of 5,000 per annum is payable monthly in advance to his widow (who is 5 years younger than him) for the remainder of her life, should she survive him. (This benefit is available throughout the lifetime of the male.)

The policy is funded by premiums payable annually in advance for five years, or until the death of the male life, if earlier.

Basis: Male mortality PMA92C20

Female mortality PFA92C20

Rate of interest 4 % per annum

Expenses Nil

Calculate, showing all your workings, the premium for this policy.

Answer:

[4138]

17. CT5 April 2016 Q11

On 1 January 2012, a life insurance company issued joint life whole life assurance policies. Each policy was issued to a male life aged 65 exact and a female life aged 60 exact. A sum assured of 75,000 is payable immediately on the death of the second of the lives to die.

Premiums of 1,395.11 are payable annually in advance for each policy while at least one of the lives is alive.

At the beginning of 2014, there were 5997 policies in force. For all of these policies, both lives were still alive. During 2014, the following experience was observed:

- for 2 policies, both lives died
- for 12 policies, only the male life died
- for 8 policies, only the female life died

Calculate, showing all your workings, the mortality profit or loss for the group of policies for the calendar year 2014.

Basis:

Mortality PMA92C20 for the male

PFA92C20 for the female

Rate of interest 4% per annum

Expenses Ignore

Answer:

[207774.03]

18. CT5 September 2016 Q11

A life insurance company issues a joint annuity policy to a male aged 60 exact and a female aged 62 exact. Under the policy:

- an annuity of 50,000 per annum is guaranteed to be payable for a period of 10 years and thereafter for the lifetime of the male.

• on the death of the male, an annuity of 20,000 per annum is payable to the female, if she is still alive. This annuity commences on the monthly payment date next following, or coincident with, the date of his death or from the 10th policy anniversary, if later. It is payable for the lifetime of the female.

• all annuities are payable monthly in arrears.

Determine the expected present value of the policy.

Basis:

Mortality PMA92C20 for the male

PFA92C20 for the female

Rate of interest 4% per annum

Expenses Ignore

Answer:

[804230]

19. CT5 April 2017 Q9

An assurance policy provides a benefit of 1 payable immediately on the death of the last survivor of a male life aged 55 exact and a female life aged 50 exact.

Determine:

(i) the expected present value of this policy.

(ii) the variance of the present value of this policy.

Basis:

Force of mortality Male life – a constant force of 0.03

Female life – a constant force of 0.02

Force of interest 4%

Answer:

[(i) 0.20635, (ii) 0.04553]

20. CT5 April 2017 Q10

A special joint-life deferred annuity policy provides the following benefits:

• 20,000 payable immediately on each death at any age

• a pension payable monthly in advance after 10 years at a rate of 10,000 per annum if both lives are alive and 5,000 per annum if only one life is alive

Premiums are payable monthly in advance until the first death for a maximum of 10 years.

Show that the monthly premium payable for a male life aged 55 exact and a second female life aged 50 exact is approximately 1,114.

Basis: First life mortality PMA92C20

Second life mortality PFA92C20

Rate of interest 4% per annum

Expenses Ignore

21. CT5 September 2018 Q2

A reversionary continuous annuity begins on the death of life x, if a second life y is then alive. Payment continues during the lifetime of y.

(a) State, using random variables, the present value of this annuity.

(b) Give an expression for the expected present value of this annuity in terms of assurance functions

Answer:

[(a) $\bar{Z} = \begin{cases} \bar{a}_{\overline{T_y}|} - \bar{a}_{\overline{T_x}|} & \text{if } T_y > T_x \\ 0 & \text{otherwise} \end{cases}$, (b) $E(\bar{Z}) = \bar{a}_y - \bar{a}_{xy} = \frac{1-\bar{A}_y}{\delta} - \frac{1-\bar{A}_{xy}}{\delta} = \frac{\bar{A}_{xy}-\bar{A}_y}{\delta}$]

22. CM1A April 2019 Q4

A life insurance company provides the following benefits:

- an annuity, on survival to age 65, of £15,000 per annum payable monthly in advance
- a spouse's annuity of £8,000 per annum payable monthly in advance on the death of the policyholder, provided that the policyholder survives to age 65.

No benefit is payable if the policyholder dies before age 65.

Calculate the single premium in respect of a female policyholder currently aged exactly 50 who has a male spouse currently aged exactly 53.

Basis:

Mortality Female PFA92C20

Male PMA92C20

Interest rate 4% per annum

Expenses Ignore

Answer:

[122,396.14]

23. CM1A September 2019 Q3

(i) Explain what is meant by the expression ${}_{5|17}q^{1}_{40:40}$

Two lives, each aged exactly 40, are independent with respect to mortality and are each subject to a constant force of mortality of 0.01 per annum.

(ii) Calculate the value of the expression in part (i).

Answer:

[(ii) 0.13040]

24. CM1A September 2019 Q9

A man aged 60 exact purchases a whole life level annuity of £20,000 per annum payable monthly in arrears with payment guaranteed for the first five years.

In addition, a reversionary annuity of £10,000 per annum is payable to the man's wife, who is two years younger. This reversionary annuity commences on the monthly payment date following the man's death or on completion of the five-year guaranteed period, if later. The annuity is payable monthly in arrears until the wife's death.

Calculate the single premium payable using the following basis:

Interest: 4% per annum

Mortality: PMA92C20 for the policyholder

PFA92C20 for the spouse

Expenses: Initial expenses of £250 plus £10 on each annuity payment date

Answer:

[£ 333657.80]

25. CM1A September 2020 Q4

An insurance company issues 20-year joint life term assurances. The sum assured of $150,000 is payable at the end of the year of the first death, if it occurs within the policy term. The premium is payable monthly in advance throughout the term of the policy or until the first death, if earlier.

Calculate the monthly premium for a policy issued to a male life aged 55 exact and a female life aged 53 exact.

Basis

Mortality of male life: PMA92C20

Mortality of female life: PFA92C20

Interest rate: 4% per annum

Expenses: Ignore

Assume that lives are independent with respect to mortality.

Answer:

[121.96]

26. CM1A April 2021 Q1

Calculate ${}_{3|5}q_{45:\overline{45}}^{1}$ assuming AM92 mortality for both individuals and that the individuals are independent with regards to mortality.

Answer:

[0.012451366]

27. CM1A September 2021 Q2

Calculate, showing all working:

(a) $\mu_{55:60}$

(b) ${}_5p_{55:60}$

(c) ${}_2q^{1}_{60:60}$

Assume lives are independent with regards to mortality and that both lives are subject to the PFA92C20 mortality table.

Answer:

[(a) 0.002894, (b) 0.97843082, (c) 0.004483816]

28. CM1A September 2021 Q7

A life insurance company issues a reversionary annuity policy to a male and female, both aged exactly 65.

The annuity of $30,000 p.a., payable monthly in arrears, commences on the first death, and payments cease on the death of the second life, or on the 15th anniversary of the policy inception if earlier.

Calculate, showing all working, the single premium for the policy.

Basis:

Mortality: PMA92C20 for the male life

PFA92C20 for the female life

The lives are independent with respect to mortality.

Interest: 4% p.a.

Expenses: Initial: $250 incurred at the outset

Renewal: 3% of each annuity payment

Answer:

[46,189]

29. CM1A September 2022 Q4

(i) Describe, in words, the meaning of the following:

$\bar{A}_{45\,50:\overline{20}|}^{\,2}$

Two lives, x and y, are independent with respect to mortality. Life x is subject to a constant force of mortality of 0.025, and life y is subject to a constant force of mortality of 0.02.

(ii) Calculate the probability that life y dies more than 3 years after life x. When performing your calculation, you should set out an appropriate integral function and show all your working.

Answer:

[(ii) 0.52320]

30. CM1A April 2023 Q8

A life insurance company issues a retirement policy to a male policyholder aged 60 exact. The policy provides the following benefits:

- An immediate whole life level annuity of £25,000 per annum payable monthly in advance.
- A reversionary annuity of £15,000 per annum payable monthly in advance to the policyholder's surviving female spouse, who is currently aged 62 exact.
- A lump sum of £200,000 payable immediately to the policyholder's family if the policyholder dies before age 65.

Calculate the expected present value of the policy benefits at outset. You should show all working.

Basis:

Mortality Male PMA92C20

Female PFA92C20

Interest 4% per annum effective

Answer:

[416,027]

31. CM1A September 2023 Q3

(i) Explain in words what is meant by ${}_{5|}q^{1}_{x:y}$

(ii) Calculate, showing all working, the value of the expression in part (i).

Basis:

x is subject to a constant force of mortality of 0.025.

y is subject to a constant force of mortality of 0.01.

You should assume x and y are independent with respect to mortality.

Answer:

[0.0206234]