



INSTITUTE OF ACTUARIAL
& QUANTITATIVE STUDIES

Subject: Probability and
Statistics 1

Chapter: Unit 1 & 2

Category: Assignment
Questions

1.

- i) The k th moment.
- ii) The k th moment about α .
- iii) The k th central moment.
- iv) The coefficient of skewness.

2.

A newly established life insurance company is analyzing the experience of policy withdrawal by the policyholders of a portfolio of 10,000 policies, on basis of the channel through which the policy was sold. The table below shows the split of the number of policies sold by the respective channels and the probability of a policyholder withdrawing the policy in a year

Channel	Agency	Bank	Online
Probability (p)	0.05	0.08	0.14
Number of policies (n)	2000	3500	4500

It can be assumed that the withdrawal by any individual policyholder during any year is independent of withdrawal by other policyholders of same or different channel.

- i) Calculate the probability that a randomly selected policyholder will withdraw in a particular year.
- ii) Calculate the probability that a randomly selected policyholder will withdraw in a particular year given that the policy was not sold through online channel.
- iii) Calculate the probability that a randomly selected policy was sold by Bank given that the policyholder withdrew last year.

3.

The following are the marks scored by 24 students in a theory paper conducted for 60 marks:

22, 31, 26, 22, 19, 21, 33, 27, 53, 27, 34, 27, 46, 30, 21, 30, 17, 60, 36, 32, 26, 33, 27, 33

i) Display the above data in a stem and leaf diagram.

ii) Calculate the median and mode.

iii) Calculate the interquartile range.

4.

Obtain the recursive relation for the binomial distribution (n, p) of the form

$$P(X = x) = g(x, n, p) P(X = x-1); x = 1, 2, \dots, n; 0 < p < 1$$

where $g(x, n, p)$ is a general function of x, n and p

5.

Suppose that 30% of passengers in a queue at a taxi stand at Town A wish to go to Town B which is 150km far away.

i) Calculate the probability that a four- seater taxi owner (excluding driver) will need to ask 15 passengers to get 4 passengers for the trip to Town B.

ii) Find the average number of persons to be asked in order to fulfil the taxi needs.

6.

A continuous random variable X has probability density function

$$f(x) = \frac{3x^2}{\theta^3}; 0 < x < \theta \text{ and } 0 \text{ otherwise.}$$

- i) Calculate the mode and median of X .
- ii) Calculate the probability that X is less than the ratio of the mode to the median of X .

7.

The table below shows the maximum monthly temperatures (in ° Celsius) recorded in a year for two cities, A and B.

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
City A	21	26	26	29	31	33	35	32	30	26	21	20
City B	18	25	25	32	37	68	42	32	29	25	20	16

- i) Draw boxplot diagrams for the maximum monthly temperatures of these cities.
- ii) Use the boxplot diagrams to compare and contrast the two data sets.

8.

The random variable X has a lognormal distribution with the same mean and variance as that of the χ_9^2 distribution.

Calculate $P(X > 9)$.

9.

In a bakery, the time taken to prepare an exotic cake is normally distributed with mean 2 hours and standard deviation 15 minutes.

Calculate the probability that the time taken for two randomly selected exotic cakes differs by no more than 25 minutes.

10.

Assume that a company has 99 employees and they are drawing the same salary.

- i) Calculate the arithmetic mean and the standard deviation of salaries of these employees.

Later, one more employee has joined the company and his salary is Rs 1000 more than those of the existing employees.

- ii) Calculate the arithmetic mean and the standard deviation of salaries of the 100 employees and comment.

11.

The results of an experts committee on an investigation of car accidents, only due to tyre burst or collision with road divider, are summarized below.

$P(\text{Accidents due to tyre burst}) = 0.6$

$P(\text{Accidents due to collision with road divider}) = 0.4$

It is known from past records that the death casualty in car accidents due to tyre burst and on collision with road divider are 30% and 50% respectively.

- i) Calculate $P[\text{Tyre burst/a death casualty due to car accident}]$
- ii) Comment on the most probable cause of death, between the two, in car accidents.

12.

A secretary is given 100 computer passwords and only one which is correct opens a file. Since the secretary has no information on the correct password, she tries to open using one of the passwords. She randomly chooses one and discards it if incorrect until she finds the correct one.

i) Calculate the probability that she obtains the correct password in the third attempt.

A security system has been set up so that if three incorrect passwords are tried before the correct one, the computer file is locked and access to it is denied.

ii) Calculate the probability that the secretary will gain access to the file.

The secretary selects a password tries it and if it does not work, puts it back with the other passwords before randomly selecting a new password.

iii) Calculate the probability that the correct password is found on the tenth attempt.

13.

The number of hospitalisations due to COVID19 have varied experience over months and data corresponding to a group illustrates that one should expect:

Two hospitalisations in October

Three hospitalisations in November

One hospitalization in December

Determine the probability that there will be less than five hospitalisations in the period October to December given the hospitalization in each month are independent and are assumed to follow Poisson distribution.

14.

A health insurance company has recently launched a new one year health insurance product which pays a fixed sum assured on the incidence of Heart, Cancer and Liver related ailments in the next one year.

Even after a claim under one ailment, coverage continues for the other ailments. A fixed sum assured would be paid on the incidence of the pre-defined ailment and no further claims can then arise for that particular ailment.

It can be assumed that these three risks are independent.

Sum Assured for Heart, Cancer and Liver related ailments are INR 20 lakhs, INR 25 lakhs and INR 15 lakhs respectively.

The company estimates the probabilities of claim arising in the next year to be 0.01 for Heart related ailments, 0.02 for Cancer and 0.005 for Liver related ailments.

- i) Determine, for a single policy, using suitable Bernoulli variables, the mean and standard deviation of the total claim amount to be paid over the next year.
- ii) You are informed that a claim has been reported under a policy. Given that there is a claim under the policy, calculate the expected pay-out on this claim.
- iii) Why does mean claim pay-out in part (i) differ from the expected claim pay-out in part (ii)?

15.

The following is the frequency distribution of 125 claims made from a certain portfolio.

Number of Claims	0	1	2	3	4	5	6
Frequency	10	?	35	?	15	7	8

The mean number of claims of this distribution is 2.504. However, some frequencies are missing.

- i) Calculate the missing frequencies.
- ii) Compute median and comment on symmetry of the distribution.

16.

At a hospital patients were categorised as 'critical', 'serious' and 'stable on arrival'.

It is seen from past records of the hospital that 20% patients were critical, 35% were serious and rest were stable on arrival. The hospital claims that the probabilities of deaths for critical, serious and stable patients respectively are 0.50, 0.20 and 0.05.

Given that a patient survived, what is the probability that patient was critical upon arrival.

17.

An insurance company has branches in two locations. A sample data on claims collected from these locations are summarised below.

Claims information	Location 1	Location 2
Number of claims	28	15
Claim amount: Mean	130.2	140.7
Claim amount: SD	55.1	66.2

Calculate, up to one decimal place, the mean and standard deviation of claim amounts for the combined data.

18.

Assume that a company has 99 employees and they are drawing the same salary.

i) Calculate the arithmetic mean and the standard deviation of salaries of these employees.

Later, one more employee has joined the company and his salary is Rs 1000 more than those of the existing employees.

ii) Calculate the arithmetic mean and the standard deviation of salaries of the 100 employees and comment.

19.

It is known that the value of two observations 3 and 5 have frequencies f_1 and f_2 respectively. Show that the variance of these observations is $\frac{4f_1f_2}{(f_1+f_2)^2}$.

20.

In an arrangement, a group of nine persons sit in a row at random and A, B and C belong to this group. Calculate the probability that

- i) A, B and C sit together in a particular order.
- ii) A, B and C sit together in any order.
- iii) A and C occupy the end seats.
- iv) B always occupies the middle seat in the row.

21.

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