INSTITUTE OF ACTUARIAL & QUANTITATIVE STUDIES

Subject: Probability & Statistics - 1

Chapter: Unit 3

Category: Practice Questions

1. CT3 September 2018 Q2

A random variable, X, has the probability generating function Gx(t) where

$$G_X(t) = 0.4096 + 0.4096t + 0.1528t^2 + 0.0256t^3 + 0.0016t^4$$

(i) Determine the probability $P(X = 3)$ using $GX(t)$. [1]

You are now given that X follows a binomial distribution.

(ii) Determine the parameter values of the distribution of X. [3]
[Total 4]

2. CT3 September 2017 Q7

The annual number of claims an insurance company incurs, N, is believed to follow a Poisson distribution with mean λ. The value of each claim Xi, $i = 1, 2, \ldots$ follows a known distribution with mean μ and variance σ^2. The value of each claim is independent of the value of any other claim and of the number of claims. Let
$S = X1 + X2 + \ldots + XN$ denote the total claims in any given year.

(i) Write down an expression for the moment generating function of S in terms of the moment generating function of Xi. [1]

(ii) Derive formulae for the mean and variance of S using your answer to part (i). [5]
[Total 6]

3. CT3 April 2014 Q4

Let X be a random variable with probability density function:

$$f(x) = \begin{cases} \frac{1}{2}e^{x} & ; \quad x \le 0 \\ \frac{1}{2}e^{-x} & ; \quad x > 0 \end{cases}$$

(i) Show that the moment generating function of X is given by:

$$M_X(t) = (1 - t^2)^{-1},$$

for $|t| < 1$.

(ii) Hence find the mean and the variance of X using the moment generating function in part (i). [3]
[Total 6]

4. CT3 September 2014 Q3

Let N be a random variable describing the number of withdrawals from a bank branch each day. It is assumed that N is Poisson distributed with mean μ. Let X_i, the random variable describing the amount of each withdrawal, be exponentially distributed with mean $1/\lambda$. All X_i are independent and identically distributed. Let S denote the total amount withdrawn from that branch in a day i.e.

$$S = \sum_{i=1}^{N} X_i$$

with $S = 0$ if $N = 0$.

(i) Derive the moment generating function of S. [4]

(ii) Calculate the mean and variance of S if $\mu = 100$ and $\lambda = 0.025$. [3]
[Total 7]

5. CT3 April 2009 Q11

The number of claims, X, which arise in a year on each policy of a particular class is to be modelled as a Poisson random variable with mean λ.

Let $\underline{X} = (X_1, X_2, \ldots, X_n)$

be a random sample from the distribution of X, and let

$$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$$

(i) (a) Use moment generating functions to show that

$$\sum_{i=1}^{n} X_i$$

has a Poisson distribution with mean $n\lambda$.

(b) State, with a brief reason, whether or not the variable $2X1 + 5$ has a Poisson distribution.

(c) State, with a brief reason, whether or not $\underline{X}$ has a Poisson distribution in the case that $n = 2$.

(d) State the approximate distribution of $\underline{X}$ in the case that n is large.
[8]

6. CT3 September 2009 Q3

Let X be a random variable with moment generating function $MX(t)$ and cumulant generating function $CX(t)$, and let $Y = aX + b$, where a and b are constants. Let Y have moment generating function $MY(t)$ and cumulant generating function $CY(t)$.

(i) Show that

$$C_Y(t) = bt + C_X(at).$$

(ii) Find the coefficient of skewness of Y in the case that

$M_X(t) = (1 - t)^{-2}$ and $Y = 3X + 2$ (you may use the fact that $C_Y'''(0) = E[(Y - \mu_Y)^3]$).

[5]
[Total 7]

7. CT3 September 2008 Q3

(i) Let Y be the sum of two independent random variables $X1$ and $X2$, that is, $Y = X1 + X2$.
Show that the moment generating function (mgf) of Y is the product of the mgfs of $X1$ and $X2$.

(ii) Let X_1 and X_2 be independent gamma random variables with parameters (α_1, λ) and (α_2, λ), respectively .

Use mgfs to show that $Y = X_1 + X_2$ is also a gamma random variable and specify its parameters. [2]
[Total 4]

8. CT3 April 2006 Q9

The total claim amount on a portfolio, S, is modelled as having a compound distribution $S = X1 + X2 + + XN$ where N is the number of claims and has a Poisson distribution with mean , Xi is the amount of the ith claim, and the Xi s are independent and identically distributed and independent of N. Let $MX(t)$ denote the moment generating function of Xi.

(i) Show, using a conditional expectation argument, that the cumulant generating function of S, $CS(t)$, is given by

$$C_S(t) = \lambda\{M_X(t) - 1\}.$$

Note: You may quote the moment generating function of a Poisson random variable from the book of Formulae and Tables. [4]

(ii) Calculate the variance of S in the case where = 20 and X has mean 20 and variance 10. [2] [Total 6]

9. CT3 April 2005 Q3

Claim sizes in a certain insurance situation are modelled by a distribution with moment generating function $M(t)$ given by

$$M(t) = (1 - 10t)^{-2}.$$

Show that $E[X^2] = 600$ and find the value of $E[X^3]$.

10. CS1A September 2020 Q4

A random variable Y has probability density function

$$f(y) = ae^{-5y}, \quad y > b,$$

where a, b are positive constants.

The moment generating function of Y is denoted by MY(t).

(i) Write down the bounds of the integration required to calculate MY(t). [1]

(ii) Identify which one of the following options gives the correct expression for MY(t).

A1 $\quad a\dfrac{e^{-(1-5t)b}}{1-5t}$

A2 $\quad \dfrac{a}{b}\dfrac{e^{-(1-5t)b}}{1-5t}$

A3 $\quad \dfrac{a}{b}\dfrac{e^{-(5-t)b}}{5-t}$

A4 $\quad a\dfrac{e^{-(5-t)b}}{5-t}$

(iii) Write down the condition on t for MY(t) to be finite. [1]

(iv) Determine an expression giving the constant a in terms of b, using your answer for MY(t) from part (ii). [3] [Total 7]

11. CS1A September 2021 Q7

Let X_i, $i = 1, 2, \ldots, n$ be independent random variables, each following an exponential distribution with parameter b. We consider the random variable $Y = \sum_{i=1}^{n} X_i$.

(i) Justify why $M_Y(t)$, the moment generating function (MGF) of variable Y, is given by

$$M_Y(t) = \left(1 - {}^{t}/_{b}\right)^{-n}$$ [2]

Let Z be a random variable such that the MGF of Z is $M_z(t) = \sqrt{M_Y(t)}$.

(ii) Determine the value of b for which Z follows a chi-square distribution, specifying the degrees of freedom of the chi-square distribution. [3]

[Total 5]

12. CS1A April 2023 Q3

An Actuary determines that the claim size for a certain class of accident is a random variable, X, with moment-generating function:

$$M_X(t) = \frac{1}{(1-2500t)^4}, \quad \text{where} \quad t < \frac{1}{2500}$$

Determine, using Mx(t), the standard deviation of the claim size for this class of accident. [4]