INSTITUTE OF ACTUARIAL & QUANTITATIVE STUDIES

Subject: Probability & Statistics - 1

Chapter: Unit 3

Category: Practice Questions

1. CT3 April 2005 Q3

Claim sizes in a certain insurance situation are modelled by a distribution with moment generating function $M(t)$ given by

$M(t)=(1-10t)^{-2}$

Show that $E(X^2) = 600$ and find the value of $E(X^3)$

Ans:
$E(X^3) = 24000$

2. CT3 September 2008 Q3

(i) Let Y be the sum of two independent random variables X_1 and X_2, that is,

$$Y = X_1 + X_2.$$

Show that the moment generating function (mgf) of Y is the product of the mgfs of X_1 and X_2.

(ii) Let X_1 and X_2 be independent gamma random variables with parameters (α_1, λ) and (α_2, λ), respectively.

Use mgfs to show that $Y=X_1+X_2$ is also a gamma random variable and specify its parameters.

Ans:
(i) $E(e^{tX_1})\ E(e^{tX_2})= M_{X1}(t)*M_{X2}(t)$
(ii) $M_{Xi}(t) =(1-1/\lambda)^{-\alpha i}$
(iii) Therefore, $M_Y(t) =(1-1/\lambda)^{-(\alpha 1+\alpha 2)}$
So that Y is a gamma r.v. with parameters $(\alpha 1+\alpha 2, \lambda)$

3. CT3 April 2009 Q11

The number of claims, X, which arise in a year on each policy of a particular class is to be modelled as a Poisson random variable with mean λ.

Let $\underline{X} = (X_1, X_2, \ldots, X_n)$

be a random sample from the distribution of X, and let

$$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$$

(i) (a) Use moment generating functions to show that $\sum_{i=1}^{n} X_i$ has a Poisson distribution with mean $n\lambda$.

(b) State, with a brief reason, whether or not the variable $2X_1 + 5$ has a Poisson distribution.

Ans:

(i) (a) –

(b) No

One reason is that $E[2X_1 + 5] = 2\lambda + 5$, which is not equal to $V[2X_1 + 5] = 4\lambda$

[Note: another obvious reason is that $2X_1 + 5$ can only takes values 5, 7, 9, … , not 0, 1, 2, 3,…]

4. CT3 September 2009 Q3

Let X be a random variable with moment generating function $MX(t)$ and cumulant generating function $CX(t)$, and let $Y = aX + b$, where a and b are constants. Let Y have moment generating function $MY(t)$ and cumulant generating function $CY(t)$.

(i) Show that $C_Y(t)=bt+C_X(at)$

(ii) Find the coefficient of skewness of Y in the case that $M_X(t)=(1-t)^{-2}$ and $Y = 3X + 2$ (you may use the fact that $C_Y'''(0)=E[(Y-\mu_Y)^3]$)

Ans:

(i) –

(ii) 1.414

5. CT3 April 2014 Q4

Let X be a random variable with probability density function:

$$f(x) = \begin{cases} \frac{1}{2}e^{x} & ; \quad x \le 0 \\ \frac{1}{2}e^{-x} & ; \quad x > 0 \end{cases}$$

(i) Show that the moment generating function of X is given by:

$M_X(t)=(1-t^2)^{-1}$ for $|t|<1$

(ii) Hence find the mean and the variance of X using the moment generating function in part (i).

Ans:

(i) –

(ii) $E(X)=0$, $V(X)=2$

6. CS1A September 2020 Q4

A random variable Y has probability density function

$$f(y) = ae^{-5y}, \quad y > b,$$

where a, b are positive constants.

The moment generating function of Y is denoted by $M_Y(t)$

(i) Write down the bounds of the integration required to calculate $M_Y(t)$.

(ii) Identify which one of the following options gives the correct expression for $M_Y(t)$

A1 $a\dfrac{e^{-(1-5t)b}}{1-5t}$

A2 $\dfrac{a}{b}\dfrac{e^{-(1-5t)b}}{1-5t}$

A3 $\dfrac{a}{b}\dfrac{e^{-(5-t)b}}{5-t}$

A4 $a\dfrac{e^{-(5-t)b}}{5-t}$

(iii) Write down the condition on t for MY(t) to be finite.

(iv) Determine an expression giving the constant a in terms of b, using your answer for $M_Y(t)$ from part (ii).

Ans:

(i) Integrate from b to plus infinity

(ii) **A4**

(iii) $t < 5$

(iv) Evaluating the function at $t = 0$ gives 1

We obtain a = 5e^(5b)

7. CS1A September 2021 Q7

Let X_i, i=1,2,…,n be independent random variables, each following an exponential distribution with parameter b. We consider the random variable $Y=\sum_{i=1}^{n} X_i$

(i) Justify why $M_Y(t)$, the moment generating function (MGF) of variable Y, is given by

$$M_Y(t)=(1-t/b)^{-n}$$

Let Z be a random variable such that the MGF of Z is $M_z(t)=\sqrt{M_Y(t)}$

(ii) Determine the value of b for which Z follows a chi-square distribution, specifying the degrees of freedom of the chi-square distribution.

Ans:

(i) –

(ii) b=0.5 with 'n' degrees of freedom

8. CS1A April 2023 Q3

An Actuary determines that the claim size for a certain class of accident is a random variable, X, with moment-generating function:

$M_X(t)=\frac{1}{(1-2500t)^4}$, where t < 1/2500

Determine, using Mx(t), the standard deviation of the claim size for this class of accident.

Ans:

SD = 5,000