INSTITUTE OF ACTUARIAL & QUANTITATIVE STUDIES

Subject: PRLI2

Chapter: Unit 1 & 2

Category: Assignment 1 solutions

Answer 1:

Let P be the monthly premium for the contract. Then

$$12P\,\ddot{a}^{(12)}_{[35:\overline{25}|} = 12P\left[\ddot{a}_{[35]:\overline{25}|} - \frac{11}{24}\left(1 - v^{25}\,25p_{[35]}\right)\right]$$ [1]

$$= 12P\left[16.029 - \frac{11}{24}\left(1 - 0.37512 \times \frac{9287.2164}{9892.9151}\right)\right]$$ [1]

$= 188.7848P$ [0.5]

EPV of death benefits:

$$260000\,\bar{A}^{1}_{[35]:\overline{25}|} - 10000\,(I\bar{A})^{1}_{[35]:\overline{25}|} = 10000 \times (1.04)^{0.5} \times \left[26A^{1}_{[35]:\overline{25}|} - (IA)^{1}_{[35]:\overline{25}|}\right]$$ [1]

$$A^{1}_{[35]:\overline{25}|} = A_{[35]:\overline{25}|} - v^{25}\,{}_{25}p_{[35]} = 0.38350 - 0.35220 = 0.03135$$ [0.5]

$$(IA)^{1}_{[35]:\overline{25}|} = (IA)_{[35]} - v^{25}\,{}_{25}p_{[35]}[25A_{60} + (IA)_{60}]$$ [1]

$$= 7.47005 - 0.35220 * [25 \times 0.45640 - 8.36234 = 0.50722$$

$$10198.04(26 \times 0.03135 - 0.50722) = 3139.76$$ [1]

EPV of annuity:

$$v^{25}\,{}_{25}p_{[35]}[47000\ddot{a}_{60} + 3000\,(I\ddot{a})_{60}]$$
$$= 0.35220 * [47000 \times 14.134 + 3000 \times 150.053 = 392456.13$$

EPV of expenses:

$$235 + 60\left(\ddot{a}^{@0\%}_{[35:\overline{25}|} - 1\right) = 235 + 60\left(e_{[35]} - \frac{l_{60}}{l_{[35]}}(1 + e_{60})\right)$$

$$= 235 + 60\left(43.909 - \frac{9287.2164}{9892.9151} \times 20.67\right) = 1648.95$$

Initial and renewal commission expense:

$$0.030 \times 12P + 0.05 \times 12P\left(\ddot{a}^{(12)}_{[35:\overline{25}|} - \frac{1}{12}\right) = 3.6P + 0.6P \times (15.732 - 0.08333)$$
$$= 12.98924P$$

188.7848 P = 3139.76 + 392456.13 +12.98924P +1648.95
175.79558P = 397244.8
P = 2259.7

Answer 2:

i)

	Outcome	Cash flow
(1)	HHH	25000,0,0
(2)	HHS	25000,0, -35000
(3)	HHD	25000,0,-50000
(4)	HSH	25000,-35000,0
(5)	HSS	25000,-35000,-35000
(6)	HSD	25000,-35000,-50000
(7)	HD	25000,-50000

ii) Completing the set of transition probabilities:

$$p^{HH}_{50+t} + p^{HD}_{50+t} + p^{HS}_{50+t} = 1$$

$$\therefore p^{HH}_{50+t} = 0.65$$

$$p^{SS}_{50+t} + p^{SH}_{50+t} + p^{SD}_{50+t} = 1$$

$$\therefore p^{SS}_{50+t} = 0.25$$

For each outcome

	Outcome	Cash flow	Probability
(1)	HHH	25000,0,0	0.4225
(2)	HHS	25000,0, -35000	0.1300
(3)	HHD	25000,0,-50000	0.0975
(4)	HSH	25000,-35000,0	0.1100
(5)	HSS	25000,-35000,-35000	0.0500
(6)	HSD	25000,-35000,-50000	0.0400
(7)	HD	25000,-50000	0.1500

iii) The associated outcomes may be considered as sum of two random variables as mentioned below:

(a) Random Variable X – Discounted value at time 0 in respect of claim payable at time 1

(b) Random Variable Y – Discounted value at time 0 in respect of claim payable at time 2

At time 1		Random Variable X
Status	Cash Flow	Discounted Value at Time 0
Health	0	0
Sick	35000	32407.40741
Died	50000	46296.2963

At time 2		Random Variable Y
Status	Cash Flow	Discounted Value at Time 0
Health	0	0
Sick	35000	30006.85871
Died	50000	42866.94102

Probability for occurrence

At time 1	Random Variable X	
Status	Discounted Value at Time 0	Probability for cash flow
Health	0	0.65
Sick	32407.40741	0.20
Died	46296.2963	0.15

At time 2	Random Variable Y	
Status	Discounted Value at Time 0	Probability for cash flow
Health	0	0.6825
Sick	30006.85871	0.18
Died	42866.94102	0.1375

<u>Mean Calculation</u>

At time 1	Random Variable X		
Status	Discounted Value at Time 0	Probability for cash flow	Probability * Discounted value
Health	0	0.65	0
Sick	32407.40741	0.20	6481.48148
Died	46296.2963	0.15	6944.44445

<u>Mean of Random variable X = 13425.92593</u>

At time 2	Random Variable Y		
Status	Discounted Value at Time 0	Probability for cash flow	Probability * Discounted value
Health	0	0.6825	0
Sick	30006.85871	0.18	5401.23457
Died	42866.94102	0.1375	5894.20439

<u>Mean of Random variable Y = 11295.43896</u>

Mean of NPV = Premium – E [Random variable X] – E[Random variable Y]
= 25000 – 13425.59259 - 11295.43896
= 278.63511

<u>Variance Calculation:</u>

The random variables can be considered as independent,

At time 1	Random Variable X			
Status	Discounted Value at Time 0	X^2	Probability for cash flow	Probability * X^2
Health	0	0	0.65	0
Sick	32407.40741	1020240055	0.20	210048011
Died	46296.2963	2143347051	0.15	321502057

Variance of Random variable X = E[(X^2)] - { E[X]}^2
= 531550068 – 13425.92593^2
= 351294581

At time 2	Random Variable Y			
Status	Discounted Value at Time 0	Y^2	Probability for cash flow	Probability * Y^2
Health	0	0	0.6825	0
Sick	30006.85871	900411569	0.18	162074082
Died	42866.94102	1837574632	0.1375	252666512

Variance of Random variable Y = E[(Y^2)] - { E[Y]}^2
= 414740594 – 11295.43896^2
= 287153652

Variance of NPV = Variance [Random variable X] + Variance [Random variable Y]
= 638448233

Answer 3:

Option C

Workings for reference
EPV of no claim bonus is as below = 15,000 * e –∫4060(0.03+0.04) dx * e –∫40600.05 dx
= 15,000 * e (–0.07*20) * e (–0.05*20)
= 15,000 * 0.2466 * 0.3679
= 1,360.86

Answer 4:

i) Future loss random variable (FLRV)

FLRV at time zero is:

$$\begin{cases} (3{,}00{,}800 - 15000K_{50})v^{T_{50}} + 500 + 50a_{\overline{K_{50}}|} - P\,\ddot{a}_{\overline{K_{50+1}}|} & K_{50} < 15 \\ 500 + 50a_{14} - P\,\ddot{a}_{\overline{15}|} & K_{50} \geq 15 \end{cases}$$

Just before the payment of 10th premium, the life is aged 59 years, and the balance of term is 6 years. So, the FLRV is now:

$$\begin{cases} (1{,}65{,}800 - 15000\,K_{59})v^{T_{59}} + 50\ddot{a}_{\overline{K_{59}+1}|} - P\,\ddot{a}_{\overline{K_{59}+1}|} & K_{59} < 6 \\ 50\,\ddot{a}_{\overline{6}|} - P\ddot{a}_{\overline{6}|} & K_{59} \geq 6 \end{cases}$$

ii) The premium equation using the principle of equivalence is:

$$P\, \ddot{a}_{50:\overline{15}|} = 315800\, \bar{A}_{50^1:\overline{15}|} - 15000\, (\bar{I}\bar{A})_{50^1:\overline{15}|} + 500 + 50\, (\ddot{a}_{50:\overline{15}|} - 1)$$

$\ddot{a}_{50:\overline{15}|} = 11.253$ (From the tables

$$\bar{A}_{50^1:\overline{15}|} \approx (1+i)^{0.5}\, A_{50^1:\overline{15}|} = 1.04^{0.5}[A_{50:\overline{15}|} - \frac{D_{65}}{D_{50}}]$$

$$\frac{D65}{D50} = \frac{689.23}{1366.61} = 0.50434$$

$$= 1.04^{0.5}[0.56719 - 0.50434] = 0.06410$$

The increasing term assurance factor is

$$(\bar{I}\bar{A})_{50^1:\overline{15}|} \approx (1+i)^{0.5}(IA)_{50^1:\overline{15}|} = 1.04^{0.5}[(IA)_{50} - \frac{D_{65}}{D_{50}}\, ((IA)_{65} + 15A_{65})]$$

$$= 1.04^{.5} \times [8.55929 - 0.50434 \times (7.89442 - 15 \times 0.52786)] = 0.59615$$

Hence Premium P is:

$$11.253\, P = 315800\ \times 0.06410 - 15000 \times 0.59615 + 500 + 50 \times (11.253 - 1)$$
$$= Rs.\, 1094.19$$

iii)

$${}_9V = 180800\, \bar{A}_{59^1:\overline{6}|} - 15000(\bar{I}\bar{A})_{59^1:\overline{6}|} - (1094.19 - 50)\ddot{a}_{59:\overline{6}|}$$

$$\bar{A}_{59^1:\overline{6}|} \approx 1.04^{0.5} A_{59^1:\overline{6}|} = 1.04^{0.5}[A_{59:\overline{6}|} - \frac{D_{65}}{D_{59}}]$$

$$\frac{D_{65}}{D_{59}} = \frac{689.23}{924.60} = 0.745307$$
$$= 1.04^{0.5}[0.79446 - 0.745307] = 0.05013$$

And increasing term assurance factor is:
$$(\bar{I}\bar{A})_{59^1:\overline{6}|} \approx (1.04)^{0.5}(IA)_{59^1:\overline{6}|} = 1.04^{0.5}[(IA)_{59} - \frac{D_{65}}{D_{59}}((IA)_{65} + 6A_{65})]$$

$$1.04^{0.5}[8.42588 - 0.745307 \times (7.89442 - 6 * 0.52786)] = 0.18520$$

Annuity function $\ddot{a}_{59:\overline{6}|}$= 5.344

The reserve is:
$$9V = 180800 \times 0.05013 - 15000 \times 0.18520 - 1044.19 \times 5.344 = 704.66$$

Assumption: Reserving basis are the same as that used to calculate the premium

Answer 5:

The cash flows expected to occur in the 30th month are:

- Premium income of INR 5000 provided policyholder is alive at the beginning of the month
- Renewal expense outgo, provided policyholder is alive at the beginning of the month.
- Death benefit outgo based on reduced sum assured of the contract, provided the policyholder dies in the 30th month.
- Renewal commission outgo based on the premium of the contract, provided the policyholder is alive at the beginning of the 30th month.

Reduced sum assured at the start of the 30th month is = 5,000,000 – 20,000 $\ddot{s}_{\overline{30}|}^{@1\%}$
=4297345

Expense outgo amount for 30th month = INR 50 * ($1.03^{28/12}$) = INR 53.57
The probability that the policyholder survives to the beginning of the 24th month is:

$\frac{l_{57}}{l_{55}}$, where all mortality rates considered allow for the 3-year set back in AM92 table

i.e. for l_{57}, we seek value of l_{54} from AM92 table (and similarly for all other ages)
Assuming deaths are uniformly distributed over the year of age, we can calculate the value of l_x at the non - integer age using linear interpolation between the values at adjacent integer ages. So, the required probability is equal to:

$$\frac{(\frac{6}{12}l_{57} + \frac{6}{12}l_{58})}{l_{55}} = \frac{(\frac{6}{12} * 9595.9715 + \frac{6}{12} * 9557.8179)}{9660.5021} = 0.991345$$

The probability that the policyholder dies in the 30th month is:

$$\frac{1}{12} * \frac{d_{57}}{l_{55}} = \frac{38.1536}{12 * 9660.5021} = 0.000329$$

Expected premium income = 5000 x 0.991345 = 4956.7
Expected expense outgo = 53.57 x 0.991345 = 53.1
E(xpected death outgo = 4297345 x 0.000329 = 1414.3
Expected renewal commission outgo = 5% x 4956.7 = 247.8

So, net cash flow = income – outgo = premium – expenses – death – commission
= 3,241.4

Answer 6:

i) $\mu_d = \textit{force of death} = 0.01$
$\mu_s = \textit{force of sickness} = 0.04$
$\mu_{lap} = \textit{force of lapse} = 0.05$

Probability of surviving the duration of 1 year without exit:

$(ap) = e^{-(0.1)}$ = 0.90483742

Dependent rates can be calculated as:

$$(aq)^d = \frac{\mu^d}{\mu^d + \mu^s + \mu^{lap}}(1 - e^{-(\mu^d + \mu^s + \mu^{lap})}) = \frac{0.01}{0.1}(1 - e^{-(0.1)})$$
$$= 0.0095163$$
$$(aq)^s = \frac{\mu^s}{\mu^d + \mu^s + \mu^{lap}}(1 - e^{-(\mu^d + \mu^s + \mu^{lap})}) = \frac{0.04}{0.1}(1 - e^{-(0.1)}) = 0.038065$$
$$(aq)^{lap} = \frac{\mu^{lap}}{\mu^d + \mu^s + \mu^{lap}}(1 - e^{-(\mu^d + \mu^s + \mu^{lap})}) = \frac{0.05}{0.1}(1 - e^{-(0.1)}) = 0.047581$$

ii) PV Outgo due to payment of Survival Benefit

$$= 300{,}000\ x\ 0.90483742^5\ x\ (1.05)^{-5} = 142{,}570$$

PV Outgo due to death
= 800,000 x $((aq)^d\ v^1 + (ap)\ (aq)^d\ v^2 + (ap)^2\ (aq)^d v^3 + (ap)^3\ (aq)^d v^4 + (ap)^4\ (aq)^d v^5)$

$=$

= 800000 x 0.0344016 = 27,521

PV Outgo due to sickness
= 500000 x $((aq)^s\ v^1 + (ap)\ (aq)^s\ v^2 + (ap)^2\ (aq)^s v^3 + (ap)^3\ (aq)^s v^4 + (ap)^4\ (aq)^s v^5)$
= 500000 x 0.137606 = 68,803

PV Outgo due to lapsation
= 60% x 100,000 x $((aq)^{lap}\ v^1 + 2\ x\ (ap)\ (aq)^{lap}\ v^2 + 3\ x\ (ap)^2\ (aq)^{lap} v^3$ -
$4\ x\ (ap)^3\ (aq)^{lap} v^4 + 5\ x\ (ap)^4\ (aq)^{lap} v^5)$

=)

= 60000 x 0.465322 = 27,919

PV of Premium
= 100,000 x (1 + (ap) x $v^1 + (ap)^2\ x\ v^2 + (ap)^3\ x\ v^3 + (ap)^4\ x\ v^4$
= 100,000 x 3.795783
= 379,578
Net Present Value = 379,578 – 68,803 – 27,521 – 27,919– 142,570 = 112,765

Answer 7:

Let P be the monthly premium

Present value of premiums = present value of decreasing benefits + present value of annuity benefit + present value of initial expenses + present value of regular expenses + present value of claim expenses

Present value of premium = $12Pa^{(1\ddot{2})}_{[30]:30} =$

$$a^{(12)}_{[30]:30} \approx \ddot{a}_{[30]:30} - \frac{11}{24}\left(1 - \frac{D_{60}}{D_{[30]}}\right)$$
$$\frac{D_{60}}{D_{[30]}} = \frac{882.85}{3059.68} = 0.288543$$
$$a^{(12)}_{[30]:30} = 17.759 - \frac{11}{24} * (1 - 0.288543) = 17.433$$

Present value of premiums = 209.195 P

Present value of decreasing benefits = $515000\bar{A}^{1}_{[30]:30} - 15000(I\bar{A})^{1}_{[30]:30}$

$$\bar{A}^{1}_{[30]:30} \approx 1.04^{0.5} * \left(A_{[30]:30} - \frac{D_{60}}{D_{[30]}}\right)$$
$$= 1.04^{0.5} * (0.31697 - .288543) = 0.02899$$
$$(I\bar{A})^{1}_{[30]:30} \approx 1.04^{0.5}\left((IA)_{[30]} - \frac{D_{60}}{D_{[30]}}((IA)_{60} + 30A_{60})\right)$$
$$= 1.04^{0.5}\big(6.91644 - 0.288543(8.36234 + 30 * 0.4564)\big) = 0.56376$$

Present value of decreasing benefits = $515000\ x\ 0.02899 - 15000\ x\ 0.56376$ = 6473.45

Present value of annuity benefit = $20000 * \frac{D_{60}}{D_{[30]}} a^{(12)}_{60}$

$$a^{(12)}_{60} = \ddot{a}^{(12)}_{60} - \frac{1}{12} \approx \ddot{a}_{60} - \frac{11}{24} - \frac{1}{12} = 15.632 - \frac{13}{24} = 15.09033$$

Present value of annuity benefit = 87,084

Present value of initial expense = 850

Present value of regular expense = $50\ddot{a}_{[30]:\ddot{30}@0\%}$

$$\ddot{a}_{[30]:30@0\%} = \ddot{a}_{[30]@0\%} - (v@0\%)^{30}\frac{l_{60}}{l_{[30]}}\ \ddot{a}_{60@0\%}$$

Now v@0% =1

And $\ddot{a}_{[30]@0\%} = 1 + e_{[30]}$

$$\ddot{a}_{[30]:30@0\%} = 1 + e_{[30]} - \frac{l_{60}}{l_{[30]}}(1 + e_{60})$$

$$= 1 + 48.764 - \frac{9287.2164}{9923.7497}(1 + 20.67) = 29.484$$

So, present value of regular expenses = 50 x 29.484 = 1474.2

Present value of claim expenses = 0.75% x present value of annuity benefit = 653.12

So, from equation of value we get:

209.195 P = 6473.45 + 87084 + 850 + 1474.2 + 653.12

P = 461.46

Answer 8:

i) **Option B**

Benefit upon completion of 8 policy anniversaries = 100000 x $(1+3\%)^8$ = 126,677

Age of policyholder = 38

q_{38} = 90 % x 0.000813 = 0.000732

Reserve = 126,677 x (0.000732/1.045 + (1-0.000732)/1.045^2)

=116006

ii) The reserves will increase.

- Currently the effective discount rate is 4.5%. Under the revised regulations, the benefits will increase at 3% p.a. and be discounted at 6.5%, resulting in an effective discount rate of ~3.5%.

As effective discount rate is lower, the reserves will increase.

Answer 9:

Correct answer – A

The probability of a life aged x , who is currently sick, staying in the sick state for at least t years is given by:

$${}_t p_{40}^{\overline{SS}} = \exp\left(-\int_0^t (\rho_{x+s} + \nu_{x+s})\, ds\right).$$

Since the transition intensities are assumed to be constant, the expression simplifies to:

$${}_t p_{40}^{\overline{SS}} = e^{-t(\rho+\nu)}.$$

The expected present value of the sickness benefit is then:

$$2{,}000\int_0^{20} e^{-\delta t}\ {}_t p_{40}^{\overline{SS}}\, dt = 2{,}000\int_0^{20} e^{-(\delta+\rho+\nu)t}\, dt$$

$$= \left[-\frac{2{,}000}{\delta+\rho+\nu} e^{-(\delta+\rho+\nu)t}\right]_0^{20}$$

$$= \frac{2{,}000}{\ln 1.04 + 0.05}\left[1 - e^{-20(\ln 1.04 + 0.05)}\right]$$

$$= £18{,}652.72$$

Answer 10:

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i) Starting from LHS

$(Ia)_{\overline{n}|} = v + 2v^2+3v^3+\text{---------------}+ nv^n$ (A) [1]

Multiplying by (1+i) both sides, we get

$(1+i)\ (Ia)_{\overline{n}|} = 1+ 2v + 3v^2+\text{---------------}+ nv^{n-1}$ (B) [0.5]

By subtraction (B – A), we get:

$i\ (Ia)_{\overline{n}|} = 1 + v + v^2+ v^3+\text{---------------}+ v^{n-1} - nv^n$

$= ä_{\overline{n}|} - nv^n$ [0.5]

So:

$(Ia)_{\overline{n}|} = (ä_{\overline{n}|} - nv^n) / i$

[2]

ii) Value of X:

The present value of contributions: [0.5]

$=2,00,000 * ä_{\overline{10}|} + X\ (Ia)_{\overline{9}|}$

i= 10%, v = 0.9091, d = i/(1+i) = 9.091%

$ä_{\overline{10}|} = (1-v^{\wedge 10})/d$

$= 6.7590$ [0.5]

$(Ia)_{\overline{9}|} = (ä_{\overline{9}|} - 9v^9) / i$

Or, $ä_{\overline{9}|} = (1-v^{\wedge 9})/d$

$= 6.3349$

$(Ia)_{\overline{9}|} = (6.3349 - 9 * .4241) / 0.1$

$= 25.1804$ [1]

Thus present value of contribution is

= 2,00,000*6.759026 + X * 25.18046

= 13,51,805.20 + X * 25.18046 [0.5]

The present value of higher education expense:

$= 30,00,000 * (1.06)^{\wedge}10 * v^{10}$ @10%

= 20,71,350 [0.5]

Substituting and equating the values:

20,71,350 = 13,51,805.20 + X * 25.18046

X = 28,575 approx [1]

[4]

[6 Marks]