



Subject: PRLI2

Chapter: Unit 3 & 4

Category: Assignment 2 solutions

Answer 1:

A unit-linked policy has the following profit vector:

Policy Year	Net Cashflow
1	-37
2	-15
3	-8
4	38
5	49

- i) The provisions required at the end of year 2 and year 1 are:

$${}_2V = \frac{8}{1.065} = 7.512$$

$${}_1V = \frac{1}{1.065} \left\{ 15 + (1 - 0.025) \frac{8}{1.065} \right\} = 20.961$$

- ii) Before zeroisation, the net present value (based on a risk discount rate of 10%) is

$$\begin{aligned} \text{NPV} &= \frac{-37}{1.1} + \frac{-15 \times 0.975}{1.1^2} + \frac{-8 \times 0.975^2}{1.1^3} + \frac{38 \times 0.975^3}{1.1^4} + \frac{49 \times 0.975^4}{1.1^5} \\ &= 0.114 \end{aligned}$$

After zeroisation the profit in year 1 will become:

$$\begin{aligned} \text{Profit in Year 1} &= -37 + \frac{-15 \times 0.975}{1.065} + \frac{-8 \times 0.975^2}{1.065^2} \\ &= -57.434 \end{aligned}$$

NPV after zeroisation =

$$\begin{aligned} &\frac{-57.434}{1.1} + \frac{38 \times 0.975^3}{1.1^4} + \frac{49 \times 0.975^4}{1.1^5} \\ &= -0.67 \end{aligned}$$

NPV is reduced after zeroisation as the funds are set aside earlier to accumulate at an interest rate that is lower risk discount rate

Answer 2:

- i) The profit vector is the expected profit at the end of each policy year per policy in force at the beginning of that policy year.

The vector of expected profit per policy issued is called the profit signature, For example if $(\text{PRO})_t$ is the profit vector, ${}_t-1P_x * (\text{PRO})_t$ is the profit signature, where ${}_t-1P_x$ is the probability of the policy issued to a life aged x is inforce at the beginning of year t .

ii)

Age (x)	Qx^d	Qx^s	$(aq)_x^d$	$(aq)_x^s$	(ap)	$_{t-1}(ap)$
50	0.001971	10%	0.001971	0.09980	0.898226	1
51	0.002732	10%	0.002732	0.09973	0.897541	0.898226
52	0.003152	10%	0.003152	0.09968	0.897163	0.806195
53	0.003539	0%	0.003539	0.00000	0.996461	0.723288

Year	Profit Vector	Probability In-Force	Discount Factor	Expected Profit	Premium Signature
	From Ques	$_{t-1}(ap)$	using RDR	$a*b*c$	$P*_{t-1}(ap)*Disc$
1	7,211.25	1	0.930233	6,708.14	15,000.000
2	(1,262.18)	0.8982261	0.865333	(981.05)	12,533.387
3	(1,063.56)	0.80619493	0.804961	(690.20)	10,464.401
4	(6,082.67)	0.72328842	0.748801	(3,294.37)	8,733.280
Total				1,742.519	46,731.069

Profit Margin = $1,742.519 / 46,731.069 = 3.73\%$

iii)

Option B

iv)

Year	Zeroised Profit Vector	Probability in Force	Discount Factor	PV
1	1,553.31	1	0.93023	1,444.94
2	0	0.898226	0.86533	-
3	0	0.806195	0.80496	-
4	0	0.723288	0.74880	-
Total PV				1,444.94

Profit Margin = $1,444.94 / 46,731.069 = 3.09\%$

Answer 3:

Decrement Table

Year	Lx	qx	wx	No. of deaths	No. of surrenders	No. of maturity	No. survivors
1	1	0.003358	10%	0.003358	0.0996642		0.8969778
2	0.896978	0.004903	5%	0.00439788	0.044629		0.847951
3	0.847951	0.00565	5%	0.00479092	0.042158		0.801002
4	0.801002	0.006352	5%	0.00508796		0.79591404	0

Unit Fund

Year	Premium	Allocated premium	Bid offer Spread	Fund before deduction	Pol Admin charge	Fund after deduction	Growth rate	FmC	Fund after growth and FMC
1	5,500.00	5,362.50	5,094.38	5,094.38	60.00	5,034.38	4%	52.36	5,183.40
2	5,500.00	5,362.50	5,094.38	10,277.78	60.00	10,217.78	3.50%	105.75	10,469.65
3	5,500.00	5,362.50	5,094.38	15,564.03	60.00	15,504.03	3.50%	160.47	15,886.20
4	5,500.00	5,362.50	5,094.38	20,980.58	60.00	20,920.58	3.50%	216.53	21,436.27

Non Unit Fund

Year	Premium less cost of Allocation	Admin charge	Expenses	Commission	Death Benefit cost	Surrender Benefit cost	Maturity Benefit cost	FMC	Non-Unit Investment Income	Profit
1	405.62	60.00	265	275.00	4.69	-89.7	0	52.36	-1.86	61.13
2	405.62	60.00	75	137.50	13.32	-44.8	0	105.75	6.33	396.68
3	405.62	60.00	0	137.50	23.00	-44.7	0	160.47	8.20	518.49
4	405.62	60.00	0	137.50	34.68	-	59.62	216.53	8.20	458.56

i)

a)

Premium less cost of Allocation	Admin charge	Expenses	Commission	Death Benefit cost	Surrender Benefit cost	Maturity Benefit cost	FMC	Non Unit Investment Income	Profit
405.62	60	265	275	1395.85	0	0	52.36	-1.86	-1,419.73

(3)

b)

Premium less cost of Allocation	Admin charge	Expenses	Commission	Death Benefit cost	Surrender Benefit cost	Maturity Benefit cost	FMC	Non Unit Investment Income	Profit
405.62	60	265	275	0	-900	0	52.36	-1.86	876.12

(2)

c)

Premium less cost of Allocation	Admin charge	Expenses	Commission	Death Benefit cost	Surrender Benefit cost	Maturity Benefit cost	FMC	Non Unit Investment Income	Profit
405.62	60	265	275	0	0	0	52.36	-1.86	-23.88

(2)

ii)

a)

Year	Premium less cost of Allocation	Admin charge	Expenses	Commission	Death Benefit cost	Surrender Benefit cost	Maturity Benefit cost	FMC	Non Unit Investment Income	Profit
2	405.62	60	75	137.5	0	-	0	105.75	6.33	365.20

(1.5)

b)

Year	Premium less cost of Allocation	Admin charge	Expenses	Commission	Death Benefit cost	Surrender Benefit cost	Maturity Benefit cost	FMC	Non Unit Investment Income	Profit
2	405.62	60	75	137.5	0	0	0	105.75	6.33	365.20

(1.5)

iii)

Profit vector per policy	Profit signature	NPV@5%
		1,062.94
61.13	61.14	1054.95
396.68	355.82	751.88
518.49	439.65	349.82
458.56	367.31	-

(8)

119 Model

Answer 4:

Annual premium 100,000.00
 Risk discount rate 8.0%
 Allocation % (1st yr) 90.0%
 Allocation % (2nd yr +) 101.50%
 Interest on Unit investments 6.0%
 Interest on non-unit reserves 4.0%
 Man charge 0.50%
 B/O spread 5.0%
 Minimum death benefit 5,00,000

	INR	%prm	Total
Initial expense /commission	2500	20.0%	12500
Renewal expense/commission	500	4.5%	5000

Multiple decrement table:

X	q^d_x	q^s_x		
40	0.000788	0.10		
41	0.000962	0.05		
42	0.001104	0.05		
43	0.001208	0.05		
x	$(aq)^d_x$	$(aq)^s_x$	(ap)	$t-1(ap)$
40	0.000749	0.09996	0.899291	1.000000
41	0.000938	0.04998	0.949086	0.899291
42	0.001076	0.04997	0.948951	0.853504
43	0.001178	0.04997	0.948852	0.809934

i)

Answer : a)

Unit fund (per policy at start of year)

	yr 1	yr 2	yr 3	yr 4
value of units at start of year	0	90,177	196809	309274
alloc	90,000	101,500	101,500	101,500
B/O	4,500	5,075	5075	5,075
interest	5,130	11,196	17,594	24,342
management charge	453	989	1,554	2,150
value of units at year end	90,177	196,809	309,274	427,891

ii)

Answer : a)

Cash flows (per policy at start of year)

	yr 1	yr 2	yr 3	yr 4
unallocated premium	10,000	-1,500.00	-1,500	-1500.00
B/O spread	4,500	5,075.00	5,075	5075.00
Expenses and commission	12,500	5,000.00	5,000	5000.00
interest	80	-57.00	-57.00	-57.00
man charge	453	989	1,554	2150.20
extra death benefit	307	284	205	85
end of year cashflow	2226	-777	-133	583

iii)

probability in force	1	0.899291	0.853504	0.809934
discount factor	0.925925	0.857338	0.793832	0.735029
expected p.v. of profit	1718.98			
Premium signature	100000	83267.69	73174.25	64295.17
Expected p.v. of premiums	320,737.09			
Profit Margin	0.54%			

iv)

Answer: a)

To calculate the expected provisions at the end of each year we have (utilising the end of year cashflow figures and decrement tables in (i) above):

$${}_2V = 133/1.04 = 127.88$$

$$\text{Cashflow is year 2} = -777 - (ap)_{41} \cdot {}_2V$$

$$\Rightarrow \text{Cashflow is year 2} = -777 - (ap)_{41} \cdot 127.88 = -898.37$$

$$\Rightarrow {}_1V = 898.37/1.04$$

$$\Rightarrow {}_1V = 863.82$$

These need to be adjusted as the question asks for the values in respect of the beginning of the year. Thus we have:

$$\text{Year 2 } 127.88(ap)_{41} = 121.37$$

$$\text{Year 1 } 863.82(ap)_{40} = 776.83$$

v)

Answer: a)

Based on the expected provisions calculated in (a) above, the cash flow for years 2 and will be zeroised whilst year 1 will become:

$$2226 - 776.83 = 1,449.17$$

Hence the table below can now be completed for the revised profit margin revised end of year cash flow:

	1,449.17	0	0	583
probability in force	1	0.899291	0.853504	0.809934
discount factor	0.925925	0.85733	0.793832	0.735029
expected p.v. of profit :	1688.90			

Answer 5

- i) The vector of balancing items in the projected revenue accounts for each policy year is called the profit vector. Profit vector gives the expected profit at the end of each policy year per policy in force at the beginning of that policy year. (1)
- ii) The objective specified for expected level of profit is termed as "profit criterion".
E.g. NPV = 40% of Initial Sales Commission
Profit Margin = 3% of the EPV of Premium Income (2)
- iii) Risk discount rate = risk free rate + margin for risk
= 10% + 3%
= 13%

$$\text{EPV profit}_{(@13\%)} = -250v + 150v^2 + 200v^3 + 225v^4, \text{ where } v = \frac{1}{1.13}$$

$$= 172.84$$

Let the first year premium be P.

$$\begin{aligned}\text{EPV premiums}_{(@13\%, \text{AM92 Ultimate})} &= P (1 + 1.04 {}_1p_{42}v + 1.04^2 {}_2p_{42}v^2 + 1.04^3 {}_3p_{42}v^3) \\ &= P (1 + 0.919338 + 0.845094 + 0.776754) \\ &= 3.541186 P\end{aligned}$$

Profit margin = (EPV profit / EPV premiums)

$$5\% = \frac{172.84}{3.541186 P}$$

$$P = 976.17$$

The company must charge a premium of Rs 976.17 in the first year of the contract.

Answer 6

- i) *Experience basis* is used to calculate the expected future profits of a contract for comparison with a stated profit criterion. It represents the best estimate of expected future experience.

Pricing basis is used while setting premium. Assuming that the risk discount rate reflects fully the uncertainties in the assumptions, the pricing basis represents the insurer's realistic expected outlook.

Valuation (or reserving) basis is used to calculate the reserves to be held by an Insurer. It represents prudent assumptions (pessimistic as compared to best estimate) of expected future experience. (3)

- ii) While the student actuary is correct that reserves are held such that there is an acceptably low probability of insolvency occurring in the future, it is incorrect to assume that everyone will die on the day after the valuation date. This is because:

- A company whose reserving basis is extremely pessimistic will be holding extremely high reserves leading to large capital requirement. This will require higher profits resulting in higher premiums/charges for customer.
- The reserving basis chosen has to satisfy any local legislation and professional guidance which exists to protect the interests of the policyholders. (2)

- iii) We need the profit vector of the policy in order to calculate the non unit reserves. It can be derived by dividing the given profit signature with the probability of surviving to the start of that year.

$$\begin{aligned}&= \left(-20, -35.20/{}_1p_{[70]}, 50/{}_2p_{[70]}, -28.50/{}_3p_{[70]}, -20/{}_4p_{[70]}, 90.21/{}_5p_{[70]} \right) \\ &= (-20, -35.79, 52.12, -30.65, -22.27, 104.40)\end{aligned}$$

Let reserves required at the end of policy year n be denoted as ${}_nV$.

No reserve will be required at the end of the fifth year. ${}_5V = 0$.

$${}_4V = \frac{22.27}{1.06} = 21.01$$

After setting up ${}_4V$, the revised profit in year five will be 0.

$${}_3V = \frac{(30.65 + 21.01 {}_1p_{73})}{1.06} = 48.06$$

After setting up ${}_3V$, the revised profit in year four will be 0.

The cashflow of year 3 is sufficient to set up this reserve. Hence ${}_2V = 0$.

Allowing for this, the revised profit in year three will be

$$= 52.12 - 48.06 {}_1p_{72}$$

$$= 5.54$$

$${}_1V = \frac{35.79}{1.06} = 33.76$$

After setting up ${}_1V$, the revised profit in year two will be 0.

The revised profit in year one will be

$$= -20 - 33.76 {}_1p_{70}$$

$$= -53.20$$

The revised profit vector is

$$(-53.20, 0, 5.54, 0, 0, 104.40)$$

Answer 7

i)

Unit fund represents the value of unit of the policyholder at any time.

It does not necessarily represent the amount that the policyholder is entitled to at any point. For example, on surrender, policyholder will get the surrender value which is the proportion of unit fund.

On death, maturity or surrender, the units held will be used to pay the benefit.

Non-unit fund will arise from the following sources:

- premium less cost of allocation i.e. the difference between the premium paid and the amount invested in the unit fund
- expenses incurred by the insurer
- interest earned on non-unit fund
- FMC taken from unit fund
- Cost of outgoes if outgo amount is greater than the value of units
- Profit on surrender

ii)

The life company sets up non unit reserves if the overall cash flow in any month other than month one is negative without reserves. The policy should be self-financing after month one.

Reserves are set up early so that money can be released as and when required to eliminate the negative cash flow. Setting up reserves will result in delay of profits but gives the comfort to the company that capital is sufficient to support the policy throughout the term.

Answer 8:

(i)

Year	Age	Profit vector	qx	Px	p(x-1)	Profit signature	Present value
1	30	-250	0.00059	0.99941	1	(250.00)	(235.85)
2	31	-400	0.000602	0.999398	0.99941	(399.76)	(355.79)
3	32	-600	0.000617	0.999383	0.998808	(599.29)	(503.17)
4	33	1500	0.000636	0.999364	0.998192	1,497.29	1,185.99

NPV @ 6% = 91.18

[3]

(ii) $2V = 600 / 1.03 = 582.52$ $1V = (400 + 582.52 * 0.99941) / 1.03 = 953.57$ Year 1 revised cashflow = $-250 - 953.57 * 0.99941 = -1203$ NPV = $-1203 / 1.06 + 1185.99 = 51.08$

[3]

(iii) The NPV after zerosing negative cashflows to achieve a single financing phase is smaller. This is because the negative cashflows have been accelerated hence being discounted less. [1]

[7 Marks]

Answer 9

(i) Decrement table

Age	Ind prob death	Ind prob surr	Dep prob surr	Dep prob mort	Prob surv at end	Prob surv at start
61	0.006433	0.06000	0.05981	0.00624	0.93395	1.00000
62	0.009696	0.06000	0.05971	0.00941	0.93089	0.93395
63	0.011344	0.06000	0.05966	0.01100	0.92934	0.86940
64	0.012716	0.06	0.05962	0.01233	0.92805	0.80797

Unit fund

Year	1	2	3	4
Premium	30,000	30,000	30,000	30,000
Fund at start	-	11,790	38,723	69,525
Allocated premium	12,000	27,000	30,000	33,000
Less: Bid offer spread	600	1,350	1,500	1,650
Policy fee	-	-	-	-
Fund before interest	11,400	37,440	67,223	100,875
Plus: Interest	570	1,872	3,361.14	5,043.76
Fund after interest	11,970	39,312	70,584	105,919
Less: FMC	180	590	1,059	1,589
Fund at end	11,790	38,723	69,525	104,330

Non-unit fund

Unallocated prem	18,000	3,000	-	(3,000)
Add: Bid offer spread	600	1,350	1,500	1,650
Policy fee	-	-	-	-
Less: Commission	6,000	600	600	600.00
Less: Expenses	5000	2000	2000	2000
Fund before interest	7,600	1,750	(1,100)	(3,950)
Add: Interest	266.00	61.25	(38.50)	(138.25)
Less: Maturity benefit				4,841.16
Add: FMC	180	590	1,059	1,589
Less: Additional death benefit	550.43	576.32	335.34	-
Non-unit cash flow	7,495.12	1,824.62	(415.08)	(7,340.63)

Profit margin

Year	1	2	3	4	
Profit	7,495.12	1,824.62	(415.08)	(7,340.63)	
Prob surv at start	1	0.93395	0.86940353	0.80797	
Discount factor profit	0.934579439	0.873438728	0.816297877	0.762895212	
PV profit	7,004.79	1,488.43	(294.58)	(4,524.73)	3,673.91
Discount factor premium	1.00	0.93	0.87	0.82	
PV premium	30,000.00	26,185.60	22,781.12	19,786.29	98,753.01
Profit margin	3.72%				

(ii) Profit margin could be higher or lower depending on the definition of the surrender benefit. If a sizeable surrender penalty is imposed such that the unit fund mostly covers the surrender benefit, profit margins could improve.

(iii) To cover high initial expenses.

Answer 10

Let P be the annual premium payable

$$P \ddot{a}_{[61]:4} = 500000 A_{[61]:4} + (100 + 0.025P) (\ddot{a}_{[61]:4} - 1) + 800$$

$$P * 3.730 = 500000 * .85654 + (100 + 0.025 P * (3.730 - 1)) + 800$$

Solving for P

$$P (0.975 * 3.730 + 0.025) = 500000 * .85654 + 100 * 2.730 + 500$$

$$P = 429343 / 3.662$$

$$P = 117250$$

Reserve required on the policy per unit sum assured

$${}_1V_{61:4} = 1 - \frac{a_{62:3}}{a_{61:4}} = 1 - \frac{2.857}{3.722} = .23240$$

$${}_2V_{61:4} = 1 - \frac{a_{63:2}}{a_{61:4}} = 1 - \frac{1.951}{3.722} = .47582$$

$${}_3V_{61:4} = 1 - \frac{a_{64:1}}{a_{61:4}} = 1 - \frac{1}{3.722} = .073133$$

Multiple Decrement Table

Multiple Decrement Table			
X	$q_{[x]}^d = (aq)_{[x]}^d$	$q_{[x]}^s$	$(aq)_{[x]}^s = q_{[x]}^s (1 - (aq)_{[x]}^d)$
61	0.006433	0.05	0.04968
62	0.009696	0.05	0.04952
63	0.011344	0.05	0.04943
64	0.012716		
T	$(ap)_{[61]+t-1}$	${}_{t-1}(ap)_{[61]}$	
1	0.943887	1.00000	
2	0.940784	0.94389	
3	0.939226	0.88799	
4	0.987284	0.83403	

Year	Premium	Expense	Openeing Reserve	Interest	Death Claim	Surrender Claim	Maturity Claim	Closing Reserve	Profit
1	117250.7681	800	0	5822.538404	3,216.50	5,825.02		109,679.67	3,552.12
2	117250.7681	3,031.27	116200	11520.97494	4,848.00	11,612.52		223,821.92	1,658.04
3	117250.7681	3,031.27	237910	17606.47494	5,672.00	17,387.12		343,442.08	3,234.78
4	117250.7681	3,031.27	365665	23994.22494	6,358.00		493,642.00	-	3,878.72
Year	Profit Signature	Discount Factor	NPV of Profit Signature						
1	3,552.12	0.9259259	3,289.00						
2	1,565.00	0.8573388	1,341.74						
3	2,872.45	0.7938322	2,280.25						
4	3,234.97	0.7350299	2,377.80						
NPV of Profit Signature			9,288.79						
Year	Premium	${}_{t-1}V_{[61]}$	Discount Factor	NPV Premium					
1	117250.7681	1.00000	1	117,250.77					
2	117250.7681	0.94389	0.925926	102,473.91					
3	117250.7681	0.88799	0.857339	89,263.98					
4	117250.7681	0.83403	0.793832	77,629.38					
		NPV of Premium		386,618.04					
Profit Margin =		2.40%		NPV profit signature/NPV premium					