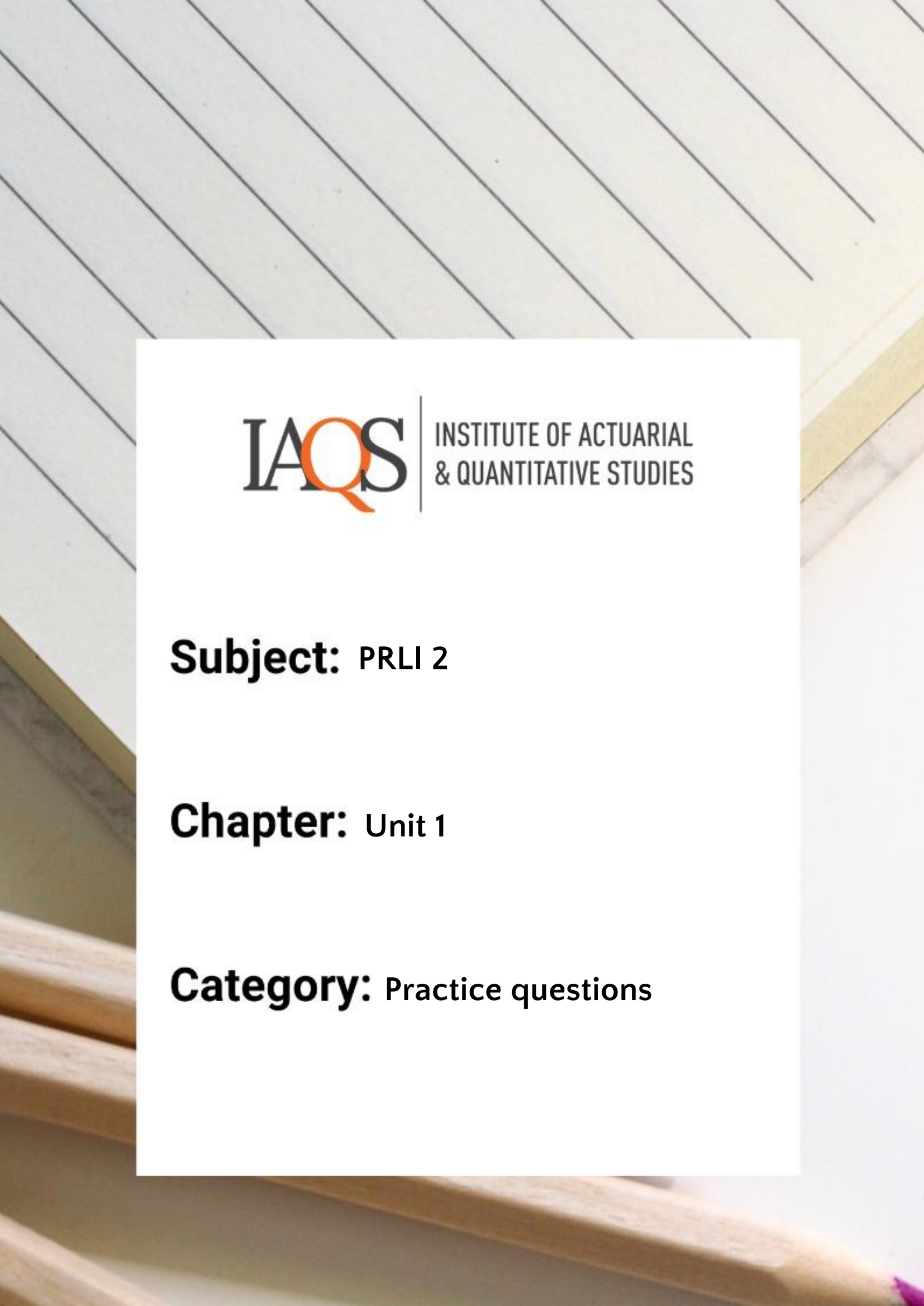


INSTITUTE OF ACTUARIAL & QUANTITATIVE STUDIES

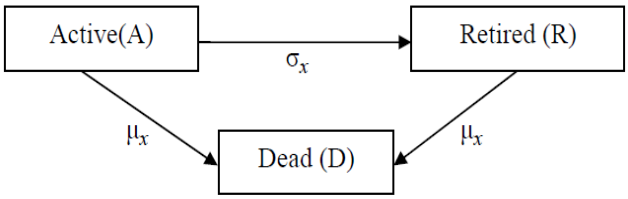
Subject: PRLI 2

Chapter: Unit 1

Category: Practice questions

1. CT5 April 2010 Q9

A life insurance company models the experience of its pension scheme contracts using the following three-state model:



(i) Derive the dependent probability of a life currently Active and aged x retiring in the year of age x to $(x + 1)$ in terms of the transition intensities. [2]

(ii) Derive a formula for the independent probability of a life currently Active and aged x retiring in the year of age x to $(x + 1)$ using the dependent probabilities. [4]

[Total 6]

2. CT5 April 2010 Q10

The decrement table extract below is based on the historical experience of a very large multinational company's workforce.

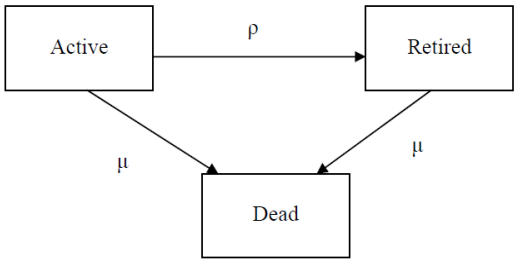
Age (x)	*Number of employees* $(al)_x$	*Deaths* $(ad)_x^d$	*Withdrawals* $(ad)_x^w$
40	10,000	25	120
41	9,855	27	144
42	9,684		

Recent changes in working conditions have resulted in an estimate that the annual independent rate of withdrawal is now 75% of that previously used.

Calculate a revised table assuming no changes to the independent death rates, stating your results to one decimal place. [7]

3. CT5 April 2011 Q5

A pension scheme uses the following model to calculate probabilities, where the transition intensities are μ = 0.05 and ρ = 0.08.



Calculate:
(a) the dependent probability of retirement
(b) the independent probability of death from active service
using the Kolmogorov equations. [5]

4. CT5 April 2012 Q10

An insurance company writes policies that provides benefits of £1,000 in the event of becoming disabled due to accident and £10,000 on death.
(a) Construct a multiple state transition model for these policies.
(b) Give a formula for the expected present value of the benefits.
[6]

5. CT5 September 2013 Q20

The following is an extract of a decrement table assumed for a funeral plan, showing deaths (*d*) and withdrawals (*w*):

Age x	$(al)_x$	$(ad)_x^d$	$(ad)_x^w$
85	10,000	1,400	2,300
86	6,300	1,000	1,100
87	4,200		

It has been established that the independent rates of decrement of withdrawal are now only 50% of those assumed in the table above for the ages of 85 and 86. The underlying independent mortality rates are unchanged.

Construct a revised decrement table to reflect this change. [7]

6. CT5 April 2014 Q8

A double decrement table is to be constructed from two single decrement tables. The modes of decrement are α and β. The basis for each of the single decrement tables is shown below:

Basis:

In the table for single decrement α: $l_{x+t}^{\alpha} = l_x^{\alpha} - t^3 d_x^{\alpha}$ for $0 \le t \le 1$

In the table for single decrement β: $l_{x+t}^{\beta} = l_x^{\beta} - t^5 d_x^{\beta}$ for $0 \le t \le 1$

The *l* function represents the number of lives and the *d* function the number of decrements in the appropriate table.

(i) Show that

$${}_t p_x^{\beta} \mu_{x+t}^{\beta} = 5t^4 q_x^{\beta} \text{ for } 0 \le t \le 1$$

(ii) Hence or otherwise show that

$$(aq)_x^{\beta} = q_x^{\beta}\left(1 - \frac{5}{8} q_x^{\alpha}\right)$$

7. CT5 September 2015 Q6

The employees of a manufacturing company are subject to two modes of decrement, mortality and withdrawal from employment.

The independent forces of mortality and withdrawal for employees aged 50 and 51 are given in the following table:

Age	μ_x^d	μ_x^w
50	0.0010	0.15
51	0.0015	0.10

Calculate, showing all your workings, the probability that a new employee aged 50 exact will die as an employee at age 51 last birthday. State any assumptions that you make. [5]

8. CT5 September 2015 Q7
A critical illness scheme provides a benefit of 100,000 on death or earlier diagnosis of a critical illness.

(i) Draw and label the appropriate transition diagram. [3]

(ii) Set out an expression for the expected present value of this benefit. [3]
[Total 6]

9. CT5 April 2015 Q3

Suppose α and β are the only two independent modes of decrement and $\mu_x^{\beta} = \frac{1}{4}\mu_x^{\alpha}$

Express $(aq)_x$ in terms of μ_x^{β}.

10. CT5 September 2017 Q7
A population is subject to two modes of decrement, ··and ··as defined below:

$\mu_x^{\alpha} = 1/(110 - x)$ for $0 \le x < 110$; and

$\mu_x^{\beta} = 0.03$ for $0 \le x < 110$

You are given that $\int_0^1 te^{-0.03t}dt = 0.490112$

Determine the value of $(aq)_{40}^{\beta}$.
[Total 7]

11. CT5 April 2018 Q9
A life insurance company issues 20-year critical illness term assurance policies.

The benefits, payable during the policy term, are a lump sum of 50,000 payable immediately on diagnosis of a critical illness, or a benefit of 100,000 immediately on death if earlier.

Premiums are payable monthly in advance during the term of the policy, ceasing on any claim.
(i) Draw a transition state model for this policy labelling your diagram. [3]

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A policy is issued to a life aged 45 exact, with an annual premium rate of 750.
(ii) Calculate the value of the policy to the company. [6]

Basis:
Mortality mx = 0.004 for all x
Critical Illness sx = 0.002 for all x
Force of Interest ∂• = 4% throughout
[Total 9]

12. CM1A April 2019 Q6

A life insurance company issues a 20-year term assurance with additional permanent disability benefit. The benefits provided are:

- on death (whether the life was previously healthy or permanently disabled) a lump sum payment of £150,000 payable immediately
- on permanent disability a lump sum of £75,000 payable immediately.

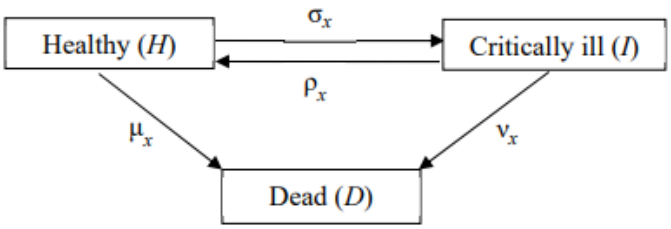
(i) Draw a transition state model for this policy, labelling your diagram. [2]

(ii) Calculate the total expected present value of the benefits. [8]

Basis:
Force of mortality from healthy 0.03 for all ages
Force of mortality from permanent disability 0.08 for all ages
Force of permanent disability 0.001 for all ages
Force of interest 5% per annum
[Total 10]

13. CM1A September 2020 Q7

A life insurance company sells a policy with a 20-year term that provides a benefit of $100,000 payable immediately on death or on earlier diagnosis of a critical illness. No further benefit is paid in the event of death after a critical illness claim has been paid. Premiums of P are paid annually in advance throughout the term or until a claim, if earlier. The company prices the policy using the following multiple state model using the forces of transition σ_x, ρ_x, μ_x and ν_x for a life aged x.



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Basis:
μ_x = 0.025 for all ages
σ_x = 0.015 for all ages
Interest rate is 3% per annum effective

(i) Determine, with a reason, the values for ρ_x and v_x that should be used in the pricing model for this policy. [1]

The present value of the benefits on this policy is given by the following formula:

$$a \times \int_{t=c}^{d} b \times e^{zt}\, dt.$$

(ii) State the values of a, b, c, d and z. [3]

(iii) Calculate the present value of the benefits for this policy based on your answers to part (ii). [2]

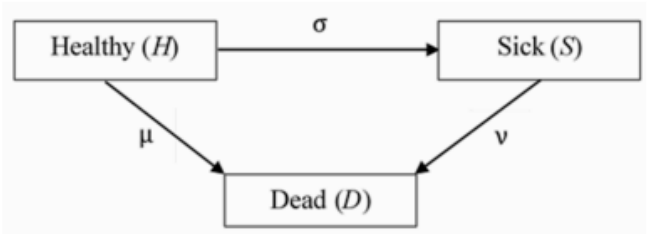
The present value of the annual premiums for this policy is given by the following formula:

$$P \sum_{t=f}^{g} e^{th}.$$

(iv) State the values of f, g and h. [2]

(v) Calculate the annual premium for this policy based on your answers to parts (iii) and (iv). [1]
[Total 9]

14. CM1A April 2021 Q2
A life insurance company uses the following three-state model, with constant forces of transition, to price its stand-alone critical illness policies.



Under these policies, a lump sum benefit is payable when a life becomes critically ill during the policy term. No other benefits are payable.

PRACTICE QUESTIONS 1

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A 30-year policy with sum assured $150,000 is issued to a healthy life aged 35 exact.

The expected present value of the benefit at outset is given by the following formula:

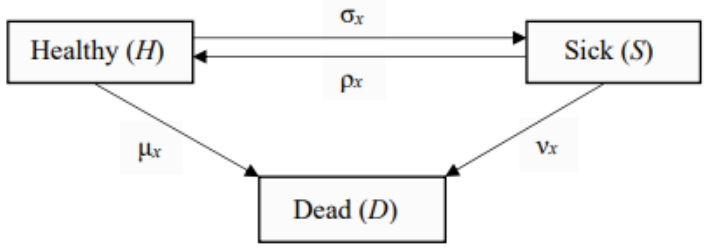
$$m \times \int_a^b n \times e^{zt}\ dt.$$

(i) State the numerical values of a, b, m, n and z. [3]
(ii) Calculate the expected present value of the benefit for this policy based on your answer to part (i).

Basis:
μ = 0.01
σ = 0.02
v = 0.04
Interest: 3% p.a. effective [3]
[Total 6]

15. CM1A September 2021 Q3

A life insurance company uses the following multiple state model to price its sickness policies.



Describe, in words, what each of the following integral expressions represents. You may assume the time periods are measured in years.

(a) $2,000 \times \int_0^{20} e^{-\delta t} \times {}_t p_{40}^{HS}\, dt$

(b) $1,000 \times \int_0^{20} e^{-\delta t} \times {}_t p_{40}^{\overline{HH}}\, dt$

(c) $20,000 \times \int_0^{20} e^{-\delta t} \times {}_t p_{40}^{HS} \times v_{40+t}\, dt$

[6]

16. CM1A September 2021 Q10

The table below is an extract from a multiple decrement table that is currently used to model the deaths and withdrawals of employees working for a large company in the hospitality industry. No decrements occur other than by death or withdrawal.

Age (x)	*Number of employees* $(al)_x$	*Number of deaths* $(ad)_x^d$	*Number of withdrawals* $(ad)_x^w$
47	50,000	390	1,500

Recent experience has resulted in an estimate that, at all ages:

- the annual independent force of mortality for employees is now 60% of that implied by the q_x rates in the ELT15 (Females) table.
- the annual independent probability of withdrawal for employees is now 250% of that used to construct the above table.

(i) Calculate, showing all working, the revised independent forces of mortality and withdrawal, each to six significant figures, for age 47. You should state any assumptions that you make. [7]

(ii) Construct the revised multiple decrement table, showing your results to two decimal places. [5]

(iii) Identify any concerns with the use of this revised multiple decrement table to model the future deaths and withdrawals of employees of the company. [3]

[Total 15]

17. CM1A April 2022 Q5

At a particular insurance company, actuarial students' study for a maximum of 3 years.

Students are subject to the following decrements:

- Mortality
- Not progressing with their studies but staying with the company (withdrawal)
- Leaving the company to join another employer (transfer).

The forces of mortality, withdrawal and transfer are assumed to be independent and to be constant over individual years of study.

In addition, at the end of each year of study, a proportion of students will complete their studies and will be deemed to have qualified.

The following forces of decrement will apply for each year of study:

Year of study	Force of mortality	Force of withdrawal	Force of transfer
1	0	0.2	0.25
2	0.025	0.1	0.35
3	0.030	0	0.45

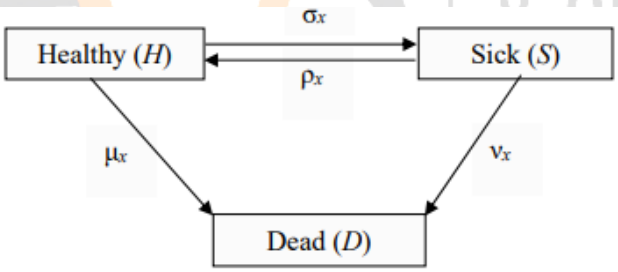
The proportion of students who qualify each year is as follows:

End of year of study	Proportion of students who qualify (%)
1	3
2	12
3	100

Calculate, showing all working, the probability that a student who starts the programme will qualify while still being at the company. [7]

18. CM1A April 2022 Q6

An insurance company issues a 30-year combined death and sickness policy to a healthy life aged 25 exact. The company uses the following multiple state model in respect of the policy.



The expected present value of the benefits provided by the policy are set out in expressions (a), (b) and (c) below:

(a) $50,000\int_0^{30} e^{-\delta t}\left({}_tp_{25}^{HH}\times\mu_{25+t}+{}_tp_{25}^{HS}\times\nu_{25+t}\right)dt$

(b) $20,000\int_0^{29.5} e^{-\delta t}\times{}_tp_{25}^{HH}\times\sigma_{25+t}\int_0^{29.5-t}{}_{0.5+s}p_{25+t}^{\overline{SS}}\times\nu_{25.5+t+s}\,ds\,dt$

(c) $5,000\int_0^{30} e^{-\delta t}\,{}_tp_{25}^{HS}dt$

Describe, in words, the benefits provided by the policy. You may assume the time periods are measured in years.

PRACTICE QUESTIONS 1

PRLI 2