



Subject: Pricing & Reserving
for Life Insurance
Products - 2

Chapter: Unit 1 & 2

Category: Assignment
Questions

1. i) Prove that

$$(Ia)_{\overline{n}|i} = (\ddot{a}_{\overline{n}|i} - nv^n) / i$$

Where i = effective rate of interest p.a. and n = term (2)

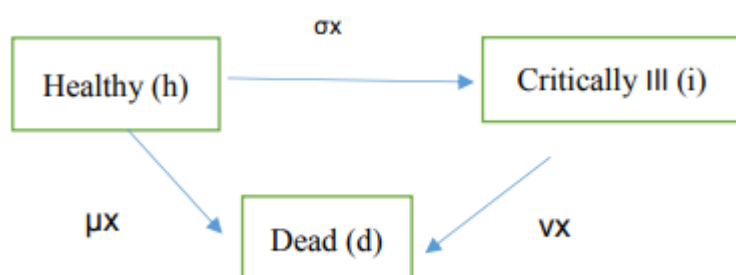
ii) Mr Gary is working as an executive in a multinational company. He is planning to start saving for his child's higher education who is currently 8 years old. The expenses for higher education will be incurred after 10 years from now.

The current cost of higher education is INR 30,00,000 which increases with an inflation of 6% p.a.

Mr Gary plans to start the contribution by INR 2,00,000 p.a. immediately. From second year onwards, at the start of each year, he wishes to increase the previous year's contribution by INR X till the 10th year. Assume that the interest earned on the savings is 10% p.a. effective, calculate how much should be the increase per year (i.e. X) so as to meet the fund requirement for higher education of the child at the end of tenth year. (4) [6]

2. A life insurance company sells a term assurance and critical illness policy with a 20-year term to a life aged 40 exact. The policy provides a benefit of Rs 100,000 payable immediately on death or earlier diagnosis of critical illness. No further benefit is paid in the event of death within the term after a prior critical illness claim has been paid.

The company prices the policy using the following multiple state model:



Calculate the expected present value of the benefits under the policy.

Basis: $i = 5\%$ per annum

$\mu x = 0.004$ at all ages

$v x = 0.005$ at all ages

$\sigma x = 0.002$ at all ages

[7]

3. A life insurance company is planning to launch the following product with the specifications as follows:

The term of the policy is 20 years and the premium is payable monthly in advance for the duration of the policy.

The sum assured is Rs 50,00,000. It will be paid out in the following respects:

- o Section 1: Accidental Disability benefit – 100% of the sum assured will be paid out.
- o Section 2: Critical Illness – On occurrence of any of the listed Critical Illnesses 150% of the sum assured will be paid out.
- o Only one claim from each section is allowed so maximum two claims can be made in the policy.

The following are the details on expenses:

- o Commission of 35% of each monthly premium is payable for entire first policy year
- o 5% of the monthly premium is payable as commission from the second policy year onwards
- o Underwriting expense is Rs 500 paid out at the inception of the policy
- o Management expense is 10% of monthly premium amount payable at the end of each month
- o A claim investigation expense as 3% of the claimed amount

It is assumed that Accidental Disability claims and Critical Illness claims are independent

These policies will be sold to people aged 40 exact.

The following can be assumed as the basis for pricing and reserving:

- o Interest rate: 4% per annum
- o Rates for Accidental Disability claims: AM92 Select
- o Rates for Critical Illness claims: AM92 Ultimate

i) You are required to derive the monthly premium payable for this policy for the policyholder. (9)

ii) Calculate prospective reserve at the end of the 5th year. (4) [13]

4. A life insurance company sells health insurance policies to healthy individuals. The policy would pay an income of INR 20,000 per year during the period of permanent sickness (i.e. from which it is not possible to recover) and the income is halved to INR 10,000 per year in case of temporary sickness (from which it is possible to recover), with all the sickness benefits ceasing at age 65 years. Also in case of death before 65 years, a benefit of INR 1,00,000 is payable.

Level Annual premiums of INR “P” p.a. are payable continuously to age 65 or to earlier death, except that premium is waived during any period of sickness.

i) Draw and label a transition diagram suitable for modeling the pricing process. (5)

ii) Basis the transition rates in your diagram and probabilities in the below form:

tp_x^{ij} = Probability that a person aged x in state i is in state j at time $x+t$

Construct a formula, using integrals, to calculate the level premium “P” for this contract (ignoring expenses), for a life aged exactly 50 at entry. (7) [12]

5. A life insurance company plans to review the premium rates for with-profit whole life policies.

The company pays compound reversionary bonuses (4% of accumulated sum assured) on the currently sold whole life policies.

Bonuses are added at the end of the policy year.

The sum assured is payable immediately on the death of the life assured and premiums are payable annually in advance ceasing with the policyholder’s death or on reaching age 65 if earlier.

The current product was priced using the following basis:

Mortality AM92 Select

Interest 4% per annum

Initial expenses 1200 and

Renewal expense 2% of second and subsequent premiums

Claim expense 500 at termination of contract

i) Calculate the gross premium for a sum assured of Rs. 200,000 and a life aged 40 exact at outset, using the equivalence principle. (4)

ii) The company plans to change the method of bonus declaration to simple reversionary for the new with-profit product being proposed. However, it still wants to charge the

same premium from the policyholders. Calculate the revised bonus rates as a % of sum assured. (4)

iii) After 10 years, bonuses totaling Rs. 100,000 have been declared for the compound reversionary bonus contract. Calculate the net premium reserve for that policy at that time, using AM92 ultimate mortality and interest of 4% per annum. (4) [12]

6. A life insurance company issues a single premium insurance policy for a term of 20 years to lives aged 45 years exact, with the following benefits:

- A benefit of Rs 100,000 payable at end of year of death during the term of the policy.
- A benefit of Rs 100,000 payable at end of year on diagnosis of cancer during the term of the policy.
- If the life insured survives the period of 20 years and has also not been diagnosed with cancer during the term, then the premium paid is returned to the policyholder at the end of 20th year.
- No death benefit is payable in the event an earlier benefit has been paid on diagnosis of cancer.

Calculate the single premium payable assuming the following:

- Interest rate - 5% per annum
- Force of decrement due to death - 0.006 at all ages
- Force of decrement due to diagnosis - 0.002 at all ages
- Force of decrement due to death after diagnosis of cancer - 0.010 at all ages
- Initial commission of 2% paid at the start of the policy
- Initial expense of 2% of premium at start of policy
- Profit Margin of 10% of premium. [8]

7. A pricing Actuary is designing a 20 year Endowment assurance product with regular premiums for Male life aged 45 years exact. The sum assured on offer is Rs 1,000,000 payable at the end of year of death or end of policy term, whichever happens first. An annual premium of Rs 30,000 is paid annually in advance.

Calculate the maximum initial commission that could be paid to the agent if the target profit margin for the product is 10%, where the profit margin is defined as expected present value of net future cash-flows divided by one annual premium.

Mortality: AM92 Select

Rate of interest: 6% per annum

Expenses: Initial: Rs 5,000

Renewal (starting from start of year 2): 2.5% of premium Rs 500 fixed (increasing at 1.9231% compounded per annum from third year)

Renewal commission: 2% of premium starting 2nd policy year

Tax: NIL [7]

8. i) State and explain the assumption required in order to derive a single decrement table from a multiple decrement table. (3)
- ii) Give an example where the above assumption stated in i) might not hold true in practice. (1)
- iii) An insurance policy can take three states: Inforce (I), Paidup (P) and Claim (C). The policy is Inforce when issued but can subsequently be made Paidup by stopping the payment of premiums. The Paidup policy can also move back to Inforce state once it starts paying premiums. Claim can be made from both Inforce as well as Paidup states and it is an absorbing exit state. Let the force of transition be denoted from $I \rightarrow P$ as σx , from $P \rightarrow I$ as ρx , from $I \rightarrow C$ as μx and from $P \rightarrow C$ as νx . For a policyholder aged 45 years with five years remaining to maturity, write down an expression in integral form to calculate expected present value of:
- a) A Claim of Rs 1000, if the policy is currently in the Paidup state. (1)
- b) A Claim of Rs 250 from the Paidup state, if the policy is currently in the Inforce state. (1)
- c) Premiums of Rs 150 payable continuously if the policy is currently in Inforce state. Assume that no unpaid premiums of the Paidup state are payable while moving to Inforce state. (1)
- d) Same as b) but assume policy is currently in Paidup state. (1)
- iv) Assume in the above sub question iii) that instead of Paidup state, the policy can be Surrendered (S) which is an absorbing exit state. The independent force of decrement from Claim and Surrender is 0.05 and 0.13 for age 45 and 0.06 and 0.10 for age 46 respectively.
- Calculate the probability of being in state Inforce at the beginning of age 47. (4) [12]

9. i) Why would a company distribute bonuses in a with profit product? (1)
- ii) List the sources of surplus in a life fund, which could impact the bonuses for with profit policies. (2)
- iii) Describe how terminal bonus could be a useful tool in enhancing the overall returns for policyholders. (2)
- [5]

10. A Life insurance company, specializing in with profit products, sells limited pay with profit Endowment policies to male lives aged 50 years exact. The sum assured plus declared reversionary bonuses are payable on survival to the end of the term or on death if earlier.

Other details:

Premium payable term: 10 years

Policy term: 15 years

Sum Assured: INR 1,000,000

Initial Commission: 20 % of sum of monthly premiums of first year, payable at outset

Initial expenses: INR 2,500 + 35% of sum of monthly premiums of first year, incurred at start of policy

Renewal commission: 2% of renewal premium payable from second year

Renewal expenses: INR 400 at start of every year from second year to 15th year + 1.5% of renewal premium from second year

Mortality: AM92 Ultimate

Future reversionary bonus: 1.92308% of the sum assured, compounded and vesting at the end of each policy year

Interest rate: 6% per annum

Premiums are payable monthly in advance and benefits are paid at the end of year of death.

i) Calculate the monthly premium. (6)

ii) At the end of 10th year for a surviving policy the company has declared bonus at a constant rate of 2% over last 10 years.

Calculate the Gross Premium Reserve at the end of 10th year basis following reserving assumptions.

Mortality: AM92 Ultimate

Interest rate: 5% per annum

Future bonuses: 4% of total of base sum assured and accrued bonuses

Expenses: same as pricing. (4)
[10]

11. A life insurance company sells a 20-year with-profits endowment assurance policy to a male life aged 30. The basic sum assured under this policy is INR 400,000. The company expects to pay a simple regular bonus at the rate of 40 per 1000 sum assured which would vest at the end of each policy year.

Calculate the net premium prospective reserve for this policy at time 7 assuming –

- Premiums are payable annually in advance throughout the term of the policy
- Bonuses declared till date have been in line with expectation
- Death Benefit is payable at the end of policy year of death
- Mortality is AM92 Ultimate and
- Interest is 6% pa.

[6]