INSTITUTE OF ACTUARIAL & QUANTITATIVE STUDIES

Subject: Pricing & Reserving for Life Insurance Products - 2

Chapter: Unit 1 & 2

Category: Assignment solutions

1.

i) Starting from LHS

$(Ia)_{\overline{n}|} = v + 2v^2 + 3v^3 + \text{---------------} + nv^n$ (A) [1]

Multiplying by (1+i) both sides, we get

$(1+i)\,(Ia)_{\overline{n}|} = 1 + 2v + 3v^2 + \text{---------------} + nv^{n-1}$ (B) [0.5]

By subtraction (B – A), we get:

$i\,(Ia)_{\overline{n}|} = 1 + v + v^2 + v^3 + \text{---------------} + v^{n-1} - nv^n$

$= \ddot{a}_{\overline{n}|} - nv^n$ [0.5]

So:

$(Ia)_{\overline{n}|} = (\ddot{a}_{\overline{n}|} - nv^n) / i$

[2]

ii) Value of X:

The present value of contributions: [0.5]

$= 2{,}00{,}000 * \ddot{a}_{\overline{10}|} + X\,(Ia)_{\overline{9}|}$

i= 10%, v = 0.9091, d = i/(1+i) = 9.091%

$\ddot{a}_{\overline{10}|} = (1 - v^{\wedge 10})/d$

$= 6.7590$ [0.5]

$(Ia)_{\overline{9}|} = (\ddot{a}_{\overline{9}|} - 9v^9) / i$

Or, $\ddot{a}_{\overline{9}|} = (1 - v^{\wedge 9})/d$

$= 6.3349$

$(Ia)_{\overline{9}|} = (6.3349 - 9 * .4241) / 0.1$

$= 25.1804$ [1]

Thus present value of contribution is

= 2,00,000*6.759026 + X * 25.18046

= 13,51,805.20 + X * 25.18046 [0.5]

The present value of higher education expense:

= 30,00,000 * (1.06)^10 * v^{10} @10%

= 20,71,350 [0.5]

Substituting and equating the values:

20,71,350 = 13,51,805.20 + X * 25.18046

X = 28,575 approx [1]

[4]

[6 Marks]

2.

$=100000 \int_0^{20} v^t \, {}_tP_{40}^{h} \, (\mu_{40+t} + \sigma_{40+t})$ dt

$=100000 \int_0^{20} e^{-\ln(1.05)t} \, {}_tP_{40}^{hh} 0.006$dt

${}_tP_{40}^{hh} = {}_tP_{40}^{hh}$ = exp $(-\int_{40}^{40+t} (\mu_s + \sigma_s)$ ds

$e^{-0.006t}$

Therefore, value =

$=100000$ X $.006 \int_0^{20} e^{-\ln(1.05)t} \, e^{-0.006t}$dt

$=600 \int_0^{20} e^{-\ln(1.05)t} \, e^{-0.006t}$dt

$=600 \int_0^{20} e^{--5479}$tdt

$=600 \times (-e^{--5479t}/.5479)_0^{20}$

$=600$ (-6.10097+18.25151)

$=7290.3$

CHAPTER NAME

PRACTICE/NOTES/ASSIGNMENT

3.

i) Assuming claim is payable only at the end

Premium = P $\ddot{a}^{(12)}_{40:\overline{20|}}$

=P $\left[\ddot{a}_{40:\overline{20|}} - \frac{11}{24}\left(1 - \frac{D_{60}}{D_{40}}\right)\right]$

=P $\left[13.927 - \frac{11}{24}\left(1 - \frac{882.85}{2052.96}\right)\right]$

=13.66576724 P

Claim Cost

75,00,000 $A'_{40:\overline{20|}}$ + 50,00,000 $A'_{[40]:\overline{20|}}$

75,00,000 $\left[A'_{40:20} - \frac{D_{60}}{D_{40}}\right]$ + 50,00,000 $\left[A'_{[40]:\overline{20|}} - \frac{D_{60}}{D_{[40]}}\right]$

75,00,000 $\left[0.46433 - \frac{882.85}{2052.96}\right]$ + 50,00,000 $\left[0.46423 - \frac{882.85}{2052.54}\right]$

=2,57,193.968202- 1,70,521.965824

=4,27,714.965824

Alternate solution (if claim is paid immediately)

=4,27,714.965824* 1.04^0.5

=4,36,185.3914

Claim Investigation expense

=0.03 x 427714.965824

=12,831. 448975

Alternate solution (if claim is paid immediately)

=0.03 x 43,6185.3914

=13,085.56174

Commission Expense

$0.3P\ P\ \ddot{a}^{(12)}_{40:\overline{1|}} + 0.05\ P\ \ddot{a}^{(12)}_{40:\overline{20|}}$

$=0.3\ P\left[\ddot{a}_{40:1} - \frac{11}{24}\left(1 - \frac{D_{41}}{D_{40}}\right)\right] + 0.05P \times 13.66576724$

$=0.3P\left[\frac{1}{1.04}\ x\ (1 - 0.000937) - \frac{11}{24}\ x\ \left(1 - \frac{1972.15}{2052.96}\right)\right] + 0.683288362\ P$

$=0.3P \times 0.942596 + 0.683288362 = 0.966067\ P$

Management Expense

$=0.1P \times \ddot{a}^{(12)}_{40:\overline{20|}} \times 1.04^{0.5}$

$=0.1P \times 13.66576724 \times 1.04^{0.5}$

$=1.39364\ P$

Underwriting Expense

= 500

Calculation of Monthly Premium

13.66576724 P = 4,27,714.965824 +12,831.448975 +0.966067P +1.39364P +500

11.30607 P = 4,41,046.4148

P = 39009.7

Alternate solution (if claim is paid immediately)

13.66576724 P = 4,36,185.3914 +13,085.56174 +0.966067P +1.39364P +500

11.30607 P = 4,49,770.95

P = 39,781.37

Monthly premium =39,009.7/12 = 3,250.8

Alternate solution (if claim is paid immediately)

Monthly premium = 39,781.37/12 = 3,315.1

ii) Calculation of Reserve

I. Policies in which only accidental claim has been made

(12)

$=75{,}00{,}000\ A'_{45:\overline{15}|}$ - 39,009.7 $\ddot{a}_{45:\overline{15}|}$

$=75{,}00{,}000\left[A_{45:15} - \frac{D_{60}}{D_{45}}\right]$

$-39{,}009.7\left[\ddot{a}_{45:\overline{15}|} - \frac{11}{24}\left(1 - \frac{D_{60}}{D_{45}}\right)\right]$

$=75{,}00{,}000\left[0.56206 - \frac{882.85}{1677.97}\right]$ - $39{,}009.7\left[11.386 - \frac{11}{24}\left(1 - \frac{882.85}{1677.97}\right)\right]$

=75,00,000 x 0.035918293 – 39,009.7x 11.14485172

=2,69,387.1975- 4,34,750.6352

=-1,65,363.4377

Alternate solution (if claim is paid immediately)

$75{,}00{,}000\ A'_{45:\overline{15}|}$ - 39,781.37 $\ddot{a}_{45:\overline{15}|}$

$=75{,}00{,}000\left[A_{45:15} - \frac{D_{60}}{D_{45}}\right]$

$-39{,}781.37\left[\ddot{a}_{45:\overline{15}|} - \frac{11}{24}\left(1 - \frac{D_{60}}{D_{45}}\right)\right]$

$=75{,}00{,}000\left[0.56206 - \frac{882.85}{1677.97}\right]$ - $39{,}781.37\left[11.386 - \frac{11}{24}\left(1 - \frac{882.85}{1677.97}\right)\right]$

=75,00,000 x 0.035918293 – 39,781.37 x 11.14485172

=2,69,387.1975- 4,43,357.4699

=-1,73,970.2724

II. **Policies in which only illness claim**

(12)

$=50,00,000\ A'_{45:15}\ - 39,009.7\ \ddot{a}_{45:15}$

=50,00,000 x 0.035918293 – 4,34,750.6352

= -2,55,159.1702

Alternate solution (if claim is paid immediately)

$=50,00,000\ A'_{45:15}\ - 39,781.37\ \ddot{a}_{45:15}$

=50,00,000 x 0.035918293 – 4,43,357.4699

= -2,63,766.0049

III. **Policies in which no claim has been made**

=1,25,00,000 x 0.035918293 – 4,34,750.6352

= 14,228.0273

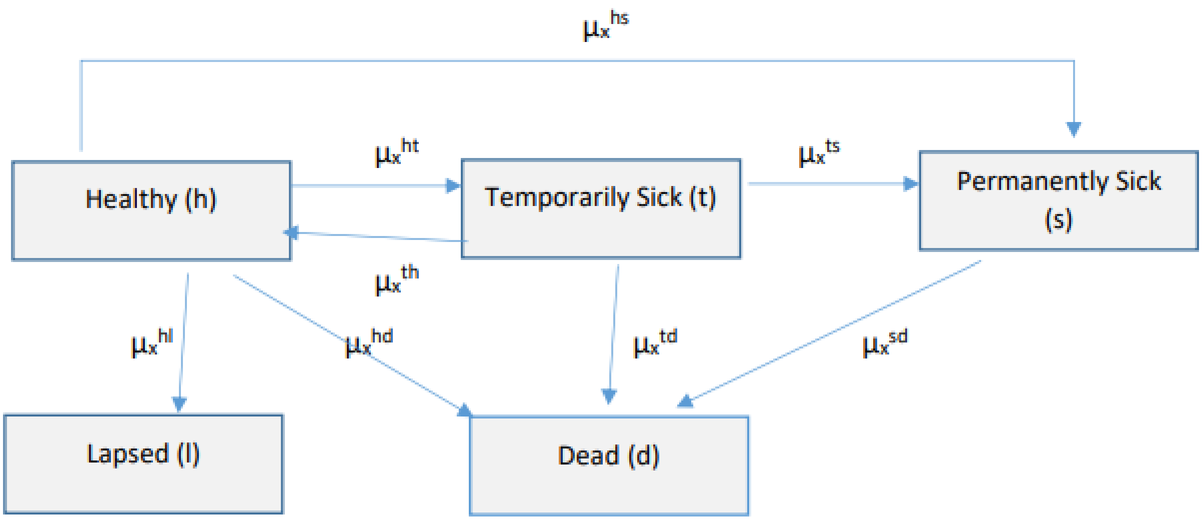
Alternate solution (if claim is paid immediately)

=1,25,00,000 x 0.035918293 – 4,43,357.4699

= 5,621.1926

4. i) Three key states to include will be healthy, sick and dead. However, the sick states would be either of the two: a temporarily (recoverable) sick state and a permanent (nonrecoverable) sick state as different benefit levels are paid in each. It would also be sensible to include lapse, as these policies are paid for by annual premiums so they can be lapsed if policyholders stop paying their premiums early.

We also need to include the transitions where there are changes in payments.



ii) EPV(premiums) = EPV(benefits) [1]

EPV of temporary sickness benefit:

$10{,}000\int_{r=0}^{15} v^r \, {}_rp_{50}^{ht} \, dr$ [1]

EPV of permanent sickness benefit:

$20{,}000\int_{r=0}^{15} v^r \, {}_rp_{50}^{hs} \, dr$ [0.5]

EPV of death benefit: On death at time r, a benefit of INR 1,00,000 would be payable, regardless of which state is then occupied

$100{,}000\int_{r=0}^{15} v^r \left({}_rp_{50}^{hh} \mu_{50+r}^{hd} + {}_rp_{50}^{ht} \mu_{50+r}^{td} + {}_rp_{50}^{hs} \mu_{50+r}^{sd}\right) dr$ [1.5]

EPV of premium:

Premiums are paid by healthy lives. So the expected present value is:

$P\int_{r=0}^{15} v^r \, {}_rp_{50}^{hh} \, dr$ [1]

So by setting EPV of benefits = EPV of premium, we obtain:

$P = \left\{10{,}000\int_{r=0}^{15} v^r \, {}_rp_{50}^{ht} \, dr + 20{,}000\int_{r=0}^{15} v^r \, {}_rp_{50}^{hs} \, dr + 100{,}000\int_{r=0}^{15} v^r \left({}_rp_{50}^{hh} \mu_{50+r}^{hd} + {}_rp_{50}^{ht} \mu_{50+r}^{td} + {}_rp_{50}^{hs} \mu_{50+r}^{sd}\right) dr\right\} / \left\{\int_{r=0}^{15} v^r \, {}_rp_{50}^{hh} \, dr\right\}$

[2]

[7]

[12 Marks]

5.

i) Gross premium for the compound reversionary policy:

EPV premiums = $P\ddot{a}_{[40]25} = 15.887P$

EPV expenses = 1200 + 500 * $\bar{A}_{[40]}$ + 2% *P * ($\ddot{a}_{[40]25}$ -1)

=1200 + 500 * (1.04)$^{0.5}$ * 0.23041 + 0.02*P*(15.887-1)

= 1317.487 + 0.29774P

EPV benefits = $2{,}00{,}000\left(v^{0.5}q_{[40]} + v^{1.5}(1.04)\,{}_{1|}q_{[40]} + v^{2.5}(1.04)^2\,{}_{2|}q_{[40]} + \ldots\right)$

$= \frac{200000}{1.04^{0.5}}\left(q_{[40]} + (1.04)\,{}_{1|}q_{[40]} + (1.04)^2\,{}_{2|}q_{[40]} + \ldots\right)$

$= \frac{200000}{1.04^{0.5}} = 1{,}96{,}116.14$

Using the equivalence principle,

15.887P = 1,96,116.14 + 1317.487 + 0.29774P

P = 12,664 [4]

ii) Revised bonus rates for simple reversionary policy:
Since company plans to charge the same premium rate, simple reversionary bonus rate can be calculated the equating the EPV of benefits of the two policy. Let's consider x% as simple bonus rate:

EPV of benefits = $2.00{,}000 \times x\%\left(I\bar{A}\right)_{[40]} + 2{,}00{,}000 \times (1 - x\%)\bar{A}_{[40]}$

= $\left(1.04^{0.5}\right)\left[2.00{,}000 \times x\%(IA)_{[40]} + 2{,}00{,}000 \times (1 - x\%)A_{[40]}\right]$

EPV of benefits to be equated with the EPV of benefits as per compound reversionary bonus

Therefore, $\left(1.04^{0.5}\right)\left[2.00{,}000 \times x\%(IA)_{[40]} + 2{,}00{,}000 \times (1 - x\%)A_{[40]}\right] = 1{,}96{,}116.14$

Solving equation, X% = 9% is the simple reversionary rate. [4]

iii) Premium as per net premium basis:

$$P\ddot{a}_{40:25} = 2,00,000\bar{A}_{40}$$

P = 2,960.54

Net premium reserve at time 10,

$$_{10}V = 3,00,000\bar{A}_{50} - P\ddot{a}_{50:15}$$

$=3,00,000 * 1.04^{0.5} * 0.32907 - 2,960.54 * 11.253$

=67,361.10

[4]

[12 Marks]

6.

Let force of decrement due to death be represented as μ_x , force of decrement due to diagnosis of cancer as σ_x.

Value of the benefits on death and Cancer is

$$V= 100,000\int_0^{20} v^t \, {}_tp_{45}^{hh} (\mu_{45+t} + \sigma_{45+t})\, dt$$

$$100,000\int_0^{20} e^{-\ln(1.05)t} \, {}_tp_{45+t}^{hh} 0.008dt$$

$${}_tp^{hh} = \exp(-\int_{45}^{45+t}((\mu_s+\sigma_s)ds)$$

$$= \exp(-\int_{45}^{45+t}(0.008ds)$$

$$= e^{-0.008t}$$

The question refers to decrements due to death and does not explicitly state whether deaths after cancer are included in this or not. Hence marks would be given to both kinds of interpretation i.e 0.006 taken as death excluding cancer and 0.006 including death after cancer which means death excluding after cancer is 0.006 – 0.002*0.01 = 0.00598

Therefore, $V = 800 * \int_0^{20} e^{-\ln(1.05)t} . e^{-0.008t} dt$

$800 * \int_0^{20} e^{-\ln(1.05)t} . e^{-0.008t} dt$

$= 800 \int_0^{20} e^{-0.05679t} dt$

$=800 * [-e^{-.05679t} / .05679]_0^{20}$

= 800*(−5.65531+17.60873) = 9562.8

Value of the Return of Premium

$=P * e^{\int_{45}^{65}(0.006+0.002)dx} * v^{20}$

$=P * e^{-0.008*(65-45)} * v^{20}$

$=P * e^{-0.16} * 0.37689$

Premium = Value of Benefits on Death and Cancer + Value of Return of Premium + Commission + Expenses + Profit Margin

So the Premium = P = 9562.8 + 0.321164 * P + 0.02*P + 0.02*P + 0.1 * P

Premium = 17,747.15

[8 Marks]

7.

PV of Net CF = PV of Premium – PV of benefits – PV of Expenses – PV of Commission

Let maximum initial commission be I

Now

PV of Premium = 30,000 $*\ddot{a}_{[45]:\overline{20}|}$
= 30,000 * 11.888
= 356,640

PV of Benefits = 1,000,000 * $A_{[45]:\overline{20}|}$

= 1,000,000 * 0.32711
= 327,110

PV of Expenses = 5,000 + 2.5% * 30,000 * ($\ddot{a}_{[45]:\overline{20}|}$ - 1)
+ 500 * $p_{[45]}$ * 1.06^(-1) * (1 + 1.019231*v*$_1p_{[46]}$ + 1.019231^2 * v^2*$_2p_{[46]}$....
+ 1.019231^18 * v^18*$_{18}p_{[46]}$)
= 5,000 + 2.5% * 30,000 * ($\ddot{a}_{[45]:\overline{20}|}$ - 1)
+ 500 * $p_{[45]}$ * 1.06^(-1) * $\ddot{a}_{[46]:\overline{19}|}$ (@4%)
= 5,000 + 2.5% * 30,000 * (11.888 - 1)
+ 500 * (9783.3371/9798.0837) * 1.06^(-1) * 13.316
= 19,438

PV of Renewal commission= 2% * 30,000 * ($\ddot{a}_{[45]:\overline{20}|}$ - 1)
= 2% * 30,000 * (11.888 - 1)
= 6,533

PV of Net CF = 356,640 – 327,110 – 19,438 – 6,533 – I
= 3,559 – I

Margin = PV of Net CF / Annual Premium
= (3,559 –I) / Annual Premium
= (3,559 – I)/ 30,000 = 10%

I = Rs. 559 **[7 Marks]**

8.

i) The assumption of Independence of Decrements is used to derive single decrement table from a multiple decrement table.

Mathematically:

$(a\mu)_x^j = \mu_x^j$ for all j and all x

When we look at transition intensities (forces of decrement), we are looking at infinitesimally small time interval in which there is only time for one decrement. Thus it is reasonable to assume that the independent and dependent forces of decrement are equal. **(3)**

ii) In case of term assurance, it is likely that people who lapse their policies have a lower than average mortality **(1)**

iii)

a) $1000\int_0^5 e^{-\delta t}({}_tp_{45}^{PI}\mu_{45+t} + {}_tp_{45}^{PP}\nu_{45+t})\,dt$ **(1)**

b) $250\int_0^5 e^{-\delta t}\ {}_tp_{45}^{IP}\nu_{45+t}dt$ **(1)**

c) $150\int_0^5 e^{-\delta t}\ {}_tp_{45}^{II}dt$ **(1)**

d) $250\int_0^5 e^{-\delta t}\ {}_tp_{45}^{PP}\ \vartheta_{45+t}\,dt$ **(1)**

iv) Lets assume that decrements are independent. Hence the given independent forces of decrement can be assumed to apply when both decrements occur together in the same population.

$$(aq)_{45}^C = \frac{0.05}{0.18}(1 - e^{-0.18}) = 0.045758$$
$$(aq)_{45}^S = \frac{0.13}{0.18}(1 - e^{-0.18}) = 0.118972$$
$$(ap)_{45} = 1 - 0.045758 - 0.118972 = 0.835270$$

$$(aq)_{46}^C = \frac{0.06}{0.16}(1 - e^{-0.16}) = 0.055446$$
$$(aq)_{46}^S = \frac{0.1}{0.16}(1 - e^{-0.16}) = 0.092410$$
$$(ap)_{46} = 1 - 0.055446 - 0.092410 = 0.852144$$

The probability of being in state Inforce at the beginning of age 47 =

0.835270 × 0.852144 = 0.711770 **(4)**

[12 Marks]

9.

i) Under with profit policies the contract between policyholder and the company is such that, whatever is the surplus arising on the underlying fund, would be shared between policyholders and shareholders in a predetermined manner.
Now, as the surplus arises over future years the same is shared with policyholders in form of bonuses and hence the need of bonuses under with profit policies.

[1]

ii) Investment Surplus
Expense Surplus
Mortality Surplus
Lapse / Surrender Surplus
Reinsurance Surplus [2]

iii) Terminal bonus could be a strategy to build lower guarantees over the lifetime of with profit policies.
For example if the proportion of RB and TB is skewed towards TB then it would means lower guarantees to Policyholders and hence the companies would have greater freedom in terms of choosing the investment assets.
By doing this they can take risk of investing in real assets like equities and real estate which can lead to better final returns to policyholders, which would be distributed through higher TB. [2]

[5 Marks]

10.

i) Let Monthly Premium be P

EPV of Premiums = 12 * P * $\ddot{a}^{(12)}_{50:\overline{10}|}$ at 6%

Now: $\ddot{a}^{(12)}_{50:\overline{10}|}$ = $\ddot{a}_{50:\overline{10}|}$ - (11/24)*(1 – ${}_{10}p_{50}$ * v^{10})

Now $\ddot{a}_{50:\overline{10}|}$ = 7.694 solving it we get

So: $\ddot{a}^{(12)}_{50:\overline{10}|}$ = 7.694 - (11/24)*(1 – 1.06^10 * (9287.2164/9712.0728))
= 7.694 – (11/24)* (1- 0.533968)
= 7.4804

Hence:
EPV of Premiums = 12 * P * 7.4804
= 89.7648 * P

EPV of benefits = 10,00,000 * (q_{50} * v + 1.019231 * v^2 * ${}_{1|}q_{50}$ + ...
+ 1.019231^14 * v^{15} * ${}_{14|}q_{50}$
+ 1.019231^15 * v^{15} * ${}_{15}p_{50}$)
= 10,00,000 * (1/1.019231) * (q_{50} * v * 1.019231 + 1.019231^2 * v^2 * ${}_{1|}q_{50}$ +
..... + 1.019231^15 * v^{15} * ${}_{14|}q_{50}$ + 1.019231^15 * v^{15} * ${}_{15}p_{50}$)
+ 10,00,000 * (1/1.019231) * v^{15} * ${}_{15}p_{50}$ * (1.019231^16 - 1.019231^15)
(All at 6%)

Converting to 4% (combination of 1/019231 and 6 % leads to 4%)
EPV of benefits = 10, 00,000 * (1/1.019231) * ($A_{50:\overline{15}|}$ at 4%)
+ 10,00,000 * v^{15} (at 6%) * ${}_{15}p_{50}$ * (1.019231^15-1.019231^14)

Now using $A_{50:\overline{15}|}$ = 0.56719 we get

EPV of benefits = 5, 66,004.1

(Above step involves converting from 6% series to 4% by taking a common factor of 1.019231 outside the bracket).

EPV of Expense and Commission =

$(20 + 35)/100 * 12*P + (2 + 1.5)/100 * {}_1p_{50} * 1.06\text{^}{-1} * 12P * \ddot{a}^{(12)}_{51:\overline{9}|}$

$+ 2,500 + 400 * {}_1p_{50} * 1.06\text{^}{-1} * \ddot{a}_{51:\overline{14}|}$ at 6%

= 9.3333 P + 6,115.06

Now equating EPV of Premiums = EPV of benefits + EPV of Expense and commission

89.7648*P = 5, 66,004.1 + 9.3333*P + 6,115.06

80.4315*P = 5, 72,119.16

We get P = 7,113.12 INR [6]

ii) Sum Assured at the end of 10^{th} year + Accrued bonuses

X = 10,00,000 * 1.02 ^ 10

X = 12, 18,994

PV of future benefits at the end of 10^{th} Year =

$= X/1.05 * q_{60} + X/1.05^2 * 1.04 * {}_{1|}q_{60} + X/1.05^3 * 1.04^2 * {}_{2|}q_{60}$

$+ X/1.05^4 * 1.04^3 * {}_{3|}q_{60} + X/1.05^5 * 1.04^4 * {}_{4|}q_{60}$

$+ X/1.05^5 * 1.04^5 * {}_{5}p_{60}$

= 0.952948 * X

= 11, 60,774

PV of expenses = 400 * $\ddot{a}_{60:\overline{5|}}$ at 5%

$= 400 * (1 + 1.05^{-1} * l_{61}/ l_{60} + 1.05^{-2} * l_{62}/ l_{60} + 1.05^{-3} * l_{63}/ l_{60} + 1.05^{-4} * l_{64}/ l_{60})$

= 1,787.39

Reserves required = PV of Benefits + PV of Expenses

= 11, 62,561 INR [4]

[10 Marks]

11. To calculate the net premium reserves we would need to calculate the Net Premium. Let the net premium be P.

Thus, $P * \ddot{a}_{30:\overline{20|}} = 400000 * A_{30:\overline{20|}}$

P = 400000 * 0.315817 / 12.08705 = 10451.42

We need to calculate the net premium prospective reserve at time 7. To do so we would need the bonus that has vested in the policy till date. As the bonuses are assumed to be in line with expectations till time 7, 7 years of simple bonus at the rate of 40 per thousand SA should be added.

So the total bonus that has been added till date equals 7 * 4% * 400000 = 112000 (16000 is added for each year gone by)

So the guaranteed SA now is = 400000 + 112000 = 512000

Thus the prospective net premium reserves at time 7 is

$512000 * A_{37:\overline{13|}} - 10451.42 * \ddot{a}_{37:\overline{13|}}$

= 512000 * 0.471638 - 10451.42 * 9.333835

= 143926.89 **[6 Marks]**