



Subject: PRLIP - 2

Chapter:

Category: Assignment
2 solutions

1. Sensitivity analysis may be carried out at an individual policy level and at a portfolio level. · At an individual policy level, sensitivity analysis allows the company to understand the impact of mis-estimating parameter values in the model. · It can help show what the reductions would be in profits emerging, return on capital or other metrics targeted. · This may help assess what margins may be included in the parameter values for the risk that are not borne out in reality. · At a portfolio level, sensitivity analysis can be used to assess the impact of shifts in mix of business. · Some parts of the portfolio may be more profitable than others and this analysis will highlight the possible impact on overall profitability of the product. · Sensitivity analysis on the volume of business can also be used to assess the overall profits emerging. · This may be useful to validate the viability of any development expenditure associated that may be associated with the pricing exercise. · This can also help the company to understand the possible risk to its capital position if volumes are more than expected.

2.

Annual premium 100,000.00

Risk discount rate 8.0%

■ Allocation % (1st yr) 90.0%

Allocation % (2nd yr +) 101.50%

Interest on Unit investments 6.0%

Interest on non-unit reserves 4.0%

Man charge 0.50%

B/O spread 5.0%

■ Minimum death benefit 5,00,000

	INR	%prm	Total
Initial expense /commission	2500	20.0%	12500
Renewal expense/commission	500	4.5%	5000
Multiple decrement table:			

X	q_x^d	q_x^s		
40	0.000788	0.10		
41	0.000962	0.05		
42	0.001104	0.05		
43	0.001208	0.05		
x	$(aq)_x^d$	$(aq)_x^s$	(ap)	$t-1(ap)$
40	0.000749	0.09996	0.899291	1.000000
41	0.000938	0.04998	0.949086	0.899291
42	0.001076	0.04997	0.948951	0.853504
43	0.001178	0.04997	0.948852	0.809934

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i)

Answer : a)

Unit fund (per policy at start of year)

	yr 1	yr 2	yr 3	yr 4
value of units at start of year	0	90,177	196,809	309,274
alloc	90,000	101,500	101,500	101,500
B/O	4,500	5,075	5,075	5,075
interest	5,130	11,196	17,594	24,342
management charge	453	989	1,554	2,150
value of units at year end	90,177	196,809	309,274	427,891

ii)

Answer : a)

Cash flows (per policy at start of year)

	yr 1	yr 2	yr 3	yr 4
unallocated premium	10,000	-1,500.00	-1,500	-1500.00
B/O spread	4,500	5,075.00	5,075	5075.00
Expenses and commission	12,500	5,000.00	5,000	5000.00
interest	80	-57.00	-57.00	-57.00
man charge	453	989	1,554	2150.20
extra death benefit	307	284	205	85
end of year cashflow	2226	-777	-133	583

ACTUARIAL
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iii)

probability in force	1	0.899291	0.853504	0.809934
discount factor	0.925925	0.857338	0.793832	0.735029
expected p.v. of profit	1718.98			
Premium signature	100000	83267.69	73174.25	64295.17
Expected p.v. of premiums	320,737.09			
Profit Margin	0.54%			

iv)

Answer: a)

To calculate the expected provisions at the end of each year we have (utilising the end of year cashflow figures and decrement tables in (i) above):

$${}_2V = 133 / 1.04 = 127.88$$

$$\text{Cashflow is year 2} = -777 - (ap)_{41} \cdot {}_2V$$

$$\Rightarrow \text{Cashflow is year 2} = -777 - (ap)_{41} \cdot 127.88 = -898.37$$

$$\Rightarrow {}_1V = 898.37 / 1.04$$

$$\Rightarrow {}_1V = 863.82$$

These need to be adjusted as the question asks for the values in respect of the beginning of the year. Thus we have:

$$\text{Year 2 } 127.88(ap)_{41} = 121.37$$

$$\text{Year 1 } 863.82(ap)_{40} = 776.83$$

v)

Answer: a)

Based on the expected provisions calculated in (a) above, the cash flow for years 2 and will be zeroised whilst year 1 will become:

$$2226 - 776.83 = 1,449.17$$

Hence the table below can now be completed for the revised profit margin revised end of year cash flow:

	1,449.17	0	0	583
probability in force	1	0.899291	0.853504	0.809934
discount factor	0.925925	0.85733	0.793832	0.735029
expected p.v. of profit	1688.90			

3.

- i) The vector of balancing items in the projected revenue accounts for each policy year is called the profit vector. Profit vector gives the expected profit at the end of each policy year per policy in force at the beginning of that policy year.

(1)

- ii) The objective specified for expected level of profit is termed as "profit criterion".
E.g. NPV = 40% of Initial Sales Commission
Profit Margin = 3% of the EPV of Premium Income

(2)

- iii) Risk discount rate = risk free rate + margin for risk
= 10% + 3%
= 13%

$$\text{EPV profit}_{(@13\%)} = -250v + 150v^2 + 200v^3 + 225v^4, \text{ where } v = \frac{1}{1.13}$$

$$= 172.84$$

Let the first year premium be P.

$$\begin{aligned} \text{EPV premiums}_{(@13\%, \text{AM92 Ultimate})} &= P (1 + 1.04 {}_1p_{42}v + 1.04^2 {}_2p_{42}v^2 + 1.04^3 {}_3p_{42}v^3) \\ &= P (1 + 0.919338 + 0.845094 + 0.776754) \\ &= 3.541186 P \end{aligned}$$

$$\text{Profit margin} = (\text{EPV profit} / \text{EPV premiums})$$

$$5\% = \frac{172.84}{3.541186 P}$$

$$P = 976.17$$

The company must charge a premium of Rs 976.17 in the first year of the contract.

- 4.
- i) *Experience basis* is used to calculate the expected future profits of a contract for comparison with a stated profit criterion. It represents the best estimate of expected future experience.

Pricing basis is used while setting premium. Assuming that the risk discount rate reflects fully the uncertainties in the assumptions, the pricing basis represents the insurer's realistic expected outlook.

Valuation (or reserving) basis is used to calculate the reserves to be held by an Insurer. It represents prudent assumptions (pessimistic as compared to best estimate) of expected future experience. (3)

- ii) While the student actuary is correct that reserves are held such that there is an acceptably low probability of insolvency occurring in the future, it is incorrect to assume that everyone will die on the day after the valuation date. This is because:

- A company whose reserving basis is extremely pessimistic will be holding extremely high reserves leading to large capital requirement. This will require higher profits resulting in higher premiums/charges for customer.
- The reserving basis chosen has to satisfy any local legislation and professional guidance which exists to protect the interests of the policyholders. (2)

- iii) We need the profit vector of the policy in order to calculate the non unit reserves. It can be derived by dividing the given profit signature with the probability of surviving to the start of that year.

$$= (-20, -35.20/{}_1P_{[70]}, 50/{}_2P_{[70]}, -28.50/{}_3P_{[70]}, -20/{}_4P_{[70]}, 90.21/{}_5P_{[70]})$$

$$= (-20, -35.79, 52.12, -30.65, -22.27, 104.40)$$

Let reserves required at the end of policy year n be denoted as ${}_nV$.

No reserve will be required at the end of the fifth year. ${}_5V = 0$.

$${}_4V = \frac{22.27}{1.06} = 21.01$$

After setting up ${}_4V$, the revised profit in year five will be 0.

$${}_3V = \frac{(30.65 + 21.01 {}_1p_{73})}{1.06} = 48.06$$

After setting up ${}_3V$, the revised profit in year four will be 0.

The cashflow of year 3 is sufficient to set up this reserve. Hence ${}_2V = 0$.

Allowing for this, the revised profit in year three will be

$$\begin{aligned} &= 52.12 - 48.06 {}_1p_{72} \\ &= 5.54 \end{aligned}$$

$${}_1V = \frac{35.79}{1.06} = 33.76$$

After setting up ${}_1V$, the revised profit in year two will be 0.

- The revised profit in year one will be

$$\begin{aligned} &= -20 - 33.76 {}_1p_{70} \\ &= -53.20 \end{aligned}$$

- The revised profit vector is
 $(-53.20, 0, 5.54, 0, 0, 104.40)$

5.

E OF ACTUARIAL
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i)

Unit fund represents the value of unit of the policyholder at any time.

It does not necessarily represent the amount that the policyholder is entitled to at any point. For example, on surrender, policyholder will get the surrender value which is the proportion of unit fund.

On death, maturity or surrender, the units held will be used to pay the benefit.

Non-unit fund will arise from the following sources:

- a. premium less cost of allocation i.e. the difference between the premium paid and the amount invested in the unit fund
- b. expenses incurred by the insurer
- c. interest earned on non-unit fund
- d. FMC taken from unit fund
- e. Cost of outgoes if outgo amount is greater than the value of units
- f. Profit on surrender

ii)

The life company sets up non unit reserves if the overall cash flow in any month other than month one is negative without reserves. The policy should be self-financing after month one.

Reserves are set up early so that money can be released as and when required to eliminate the negative cash flow. Setting up reserves will result in delay of profits but gives the comfort to the company that capital is sufficient to support the policy throughout the term.

6.

(i)

Year	Age	Profit vector	qx	Px	p(x-1)	Profit signature	Present value
1	30	-250	0.00059	0.99941	1	(250.00)	(235.85)
2	31	-400	0.000602	0.999398	0.99941	(399.76)	(355.79)
3	32	-600	0.000617	0.999383	0.998808	(599.29)	(503.17)
4	33	1500	0.000636	0.999364	0.998192	1,497.29	1,185.99

NPV @ 6% = 91.18

[3]

(ii) $2V = 600 / 1.03 = 582.52$ $1V = (400 + 582.52 * 0.99941) / 1.03 = 953.57$ Year 1 revised cashflow = $-250 - 953.57 * 0.99941 = -1203$ NPV = $-1203 / 1.06 + 1185.99 = 51.08$

[3]

(iii) The NPV after zeronising negative cashflows to achieve a single financing phase is smaller. This is because the negative cashflows have been accelerated hence being discounted less. [1]

[7 Marks]

7.

(i) Decrement table

Age	Ind prob death	Ind prob surr	Dep prob surr	Dep prob mort	Prob surv at end	Prob surv at start
61	0.006433	0.06000	0.05981	0.00624	0.93395	1.00000
62	0.009696	0.06000	0.05971	0.00941	0.93089	0.93395
63	0.011344	0.06000	0.05966	0.01100	0.92934	0.86940
64	0.012716	0.06	0.05962	0.01233	0.92805	0.80797

Unit fund

Year	1	2	3	4
Premium	30,000	30,000	30,000	30,000
Fund at start	-	11,790	38,723	69,525
Allocated premium	12,000	27,000	30,000	33,000
Less: Bid offer spread	600	1,350	1,500	1,650
Policy fee	-	-	-	-
Fund before interest	11,400	37,440	67,223	100,875
Plus: Interest	570	1,872	3,361.14	5,043.76
Fund after interest	11,970	39,312	70,584	105,919
Less: FMC	180	590	1,059	1,589
Fund at end	11,790	38,723	69,525	104,330

Non-unit fund

Unallocated prem	18,000	3,000	-	(3,000)
Add: Bid offer spread	600	1,350	1,500	1,650
Policy fee	-	-	-	-
Less: Commission	6,000	600	600	600.00
Less: Expenses	5000	2000	2000	2000
Fund before interest	7,600	1,750	(1,100)	(3,950)
Add: Interest	266.00	61.25	(38.50)	(138.25)
Less: Maturity benefit				4,841.16
Add: FMC	180	590	1,059	1,589
Less: Additional death benefit	550.43	576.32	335.34	-
Non-unit cash flow	7,495.12	1,824.62	(415.08)	(7,340.63)

Profit margin

Year	1	2	3	4	
Profit	7,495.12	1,824.62	(415.08)	(7,340.63)	
Prob surv at start	1	0.93395	0.86940353	0.80797	
Discount factor profit	0.934579439	0.873438728	0.816297877	0.762895212	
PV profit	7,004.79	1,488.43	(294.58)	(4,524.73)	3,673.91
Discount factor premium	1.00	0.93	0.87	0.82	
PV premium	30,000.00	26,185.60	22,781.12	19,786.29	98,753.01
Profit margin	3.72%				

(ii) Profit margin could be higher or lower depending on the definition of the surrender benefit. If a sizeable surrender penalty is imposed such that the unit fund mostly covers the surrender benefit, profit margins could improve.

(iii) To cover high initial expenses.

8.

Let P be the annual premium payable

$$P \ddot{a}_{[61]:4} = 500000 A_{[61]:4} + (100 + .025P) (\ddot{a}_{[61]:4} - 1) + 800$$

$$P * 3.730 = 500000 * .85654 + (100 + .025 P * (3.730 - 1) + 800$$

Solving for P

$$P (0.975 * 3.730 + 0.025) = 500000 * .85654 + 100 * 2.730 + 500$$

$$P = 429343 / 3.662$$

$$P = 117250$$

Reserve required on the policy per unit sum assured

$${}_1V_{61:4} = 1 - \frac{\ddot{a}_{62:3}}{\ddot{a}_{61:4}} = 1 - \frac{2.857}{3.722} = .23240$$

$${}_2V_{61:4} = 1 - \frac{\ddot{a}_{63:2}}{\ddot{a}_{61:4}} = 1 - \frac{1.951}{3.722} = .47582$$

$${}_3V_{61:4} = 1 - \frac{\ddot{a}_{64:1}}{\ddot{a}_{61:4}} = 1 - \frac{1}{3.722} = .73133$$

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Multiple Decrement Table

Multiple Decrement Table			
X	$q_{[x]}^d = (aq)_{[x]}^d$	$q_{[x]}^s$	$(aq)_{[x]}^s = q_{[x]}^s (1 - (aq)_{[x]}^d)$
61	0.006433	0.05	0.04968
62	0.009696	0.05	0.04952
63	0.011344	0.05	0.04943
64	0.012716		
T	$(ap)_{[61]+t-1 }$	${}_{t-1}(ap)_{[61]}$	
1	0.943887	1.00000	
2	0.940784	0.94389	
3	0.939226	0.88799	
4	0.987284	0.83403	

Year	Premium	Expense	Openeing Reserve	Interest	Death Claim	Surrender Claim	Maturity Claim	Closing Reserve	Profit
1	117250.7681	800	0	5822.538404	3,216.50	5,825.02		109,679.67	3,552.12
2	117250.7681	3,031.27	116200	11520.97494	4,848.00	11,612.52		223,821.92	1,658.04
3	117250.7681	3,031.27	237910	17606.47494	5,672.00	17,387.12		343,442.08	3,234.78
4	117250.7681	3,031.27	365665	23994.22494	6,358.00		493,642.00	-	3,878.72
Year	Profit Signature	Discount Factor	NPV of Profit Signature						
1	3,552.12	0.9259259	3,289.00						
2	1,565.00	0.8573388	1,341.74						
3	2,872.45	0.7938322	2,280.25						
4	3,234.97	0.7350299	2,377.80						
NPV of Profit Signature			9,288.79						
Year	Premium	v_{t-1}	Discount Factor	NPV Premium					
1	117250.7681	1.00000	1	117,250.77					
2	117250.7681	0.94389	0.925926	102,473.91					
3	117250.7681	0.88799	0.857339	89,263.98					
4	117250.7681	0.83403	0.793832	77,629.38					
		NPV of Premium		386,618.04					
Profit Margin =		2.40% NPV profit signature/ NPV premium							

9.

i) Calculating the dependent probabilities:

x	$(aq)_x^d$	$(aq)_x^s$	$(ap)_x$	$_{x-42}(ap)_x$
42	0.000922	0.099908	0.899170	1.000000
43	0.001150	0.099885	0.898965	0.899170
44	0.001327	0.000000	0.998673	0.808323

Calculating the unit fund:

Year	Unit Fund at Start	Allocated Premium	Bid Offer Spread	Interest	Annual Management charge	Unit Fund at End
1	0.00	3000.00	75.00	263.25	39.85	3148.40
2	3148.40	8000.00	200.00	985.36	149.17	11784.59
3	11784.59	9250.00	231.25	1872.30	283.45	22392.19

Calculating the non unit cashflows:

Year	Unallocated Premium	Bid Offer Spread	Expenses	Commission	Interest	Annual Management charge	Death Cost	Maturity Cost
1	7000.00	75.00	2000.00	1000.00	285.25	39.85	15.54	0.00
2	2000.00	200.00	750.00	300.00	80.50	149.17	9.45	0.00
3	750.00	231.25	750.00	300.00	-4.81	283.45	0.00	3354.37

Year	End of year cashflow	Probability Inforce	Discount Factor	Expected PV of profit	Premium signature	Expected PV of premiums	Profit Margin
1	4384.56	1.000000	0.884956	3083.46	10000.00	24287.61	12.70%
2	1370.22	0.899170	0.783147		8991.70		
3	-3144.48	0.808323	0.693050		8083.23		

ii) Reserves do not reduce profits; they only defer the release of profits.

So, if the interest earned on reserves is equal to the risk discount rate than there is no effect of deferring the profit as the opportunity cost on money held as reserves is zero.

Therefore, the profit margin will be same as calculated earlier.

[3]

iii) Since the mortality rates and surrender rate is the same in the valuation basis and pricing basis, we will be able to use the dependent probabilities derived in i).

To calculate the expected provisions at the end of each year we have:

$${}_2V = 4532.25 / 1.07 = 4235.75$$

$${}_1V \times 1.07 - (ap)_{43} \times {}_2V = -(-760.10) \rightarrow {}_1V = 4269.06$$

These need to be adjusted as the question asks for the values in respect of the beginning of the year.

Thus we have:

$$\text{Year 2} \quad 4235.75 \times (ap)_{43} = 3807.79$$

$$\text{Year 1} \quad 4269.06 \times (ap)_{42} = 3838.61$$

10.

i) The unit fund will be projected as under:

Policy year	Premium allocated	Cost of allocation	Plus fund b/fd	Fund before charge	Fund management charge	Policy admin. Fees	Fund c/fd
1	17500.00	15750.00	15750.00	16852.50	84.26	500.00	16268.24
2	27500.00	24750.00	41018.24	43889.52	219.45	500.00	43170.07
3	27500.00	24750.00	67920.07	72674.47	363.37	500.00	71811.10
4	27500.00	24750.00	96561.10	103320.38	516.60	500.00	102303.78

The expected profit will be calculated by projecting the non unit cashflows as under:

Policy year	Profit on allocation	Expenses	Non unit interest	Fund management charge	Policy admin. fees	Non unit death cost	Profit in each year
1	9250.00	3500.00	402.50	84.26	500.00	730.98	6005.78
2	250.00	1100.00	-59.50	219.45	500.00	327.45	-517.50
3	250.00	1100.00	-59.50	363.37	500.00	0.00	-46.13
4	250.00	1100.00	-59.50	516.60	500.00	0.00	107.10

ii) The non-unit reserves required are:

Start of year 3: $46.13/1.07 = 43.11$

End of year 2: $43.11 \times 0.985 + 517.50 = 559.96$

Start of year 2: $559.96/1.07 = 523.33$

End of year 1: $523.33 \times 0.985 = 515.48$

The calculations of revised profit are as under:

Policy year	Non unit reserve b/fd	Interest on reserves	Increase in reserves	Profit ignoring reserves	Profit allowing for reserves
1	0.00	0.00	515.48	6005.78	5490.30
2	523.33	36.63	-480.87	-517.50	0.00
3	43.11	3.02	-43.11	-46.13	0.00
4	0.00	0.00	0.00	107.10	107.10