Lecture 3



Class: MSc

Subject: Pricing & Reserving for Life Insurance Products - 2

Subject Code:

Chapter: Unit 3 Chapter 1

Chapter Name: Profit Testing



Today's Agenda

1. Introduction

- 1. Unit-Linked Contracts
- 2. Unit Funds & Non-Unit Funds
- 3. Accumulating with profit contracts
- 4. Unitised with profit contracts

2. Profit Testing

- 1. Evaluating expected cash flows
- 2. Profit tests for annual premium contracts
- 3. Pricing using profit tests



1 Introduction

In this section, we describe policies for which the benefit takes the form of an accumulating fund of premiums, *ie* unit-linked (UL) and accumulating with-profits (AWP) contracts.

Under UL contracts the insurance company has no discretion about the policy benefits that are paid but, under AWP, significant components of the benefit amounts are discretionary, following a similar rationale to conventional with-profits contracts.



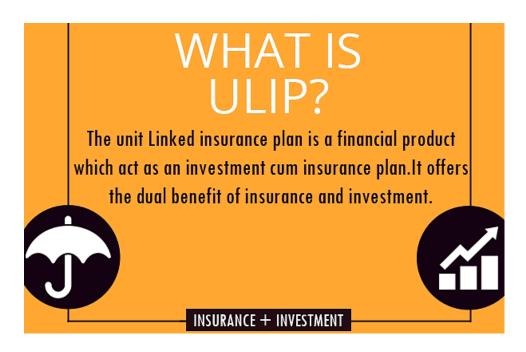


Unit-linked assurances (typically whole life or endowment) have benefits which are directly linked to the value of the underlying investments. Each policyholder receives the value of the units allocated to the policy. There is no pooling of investments or allocation of the pooled surplus.

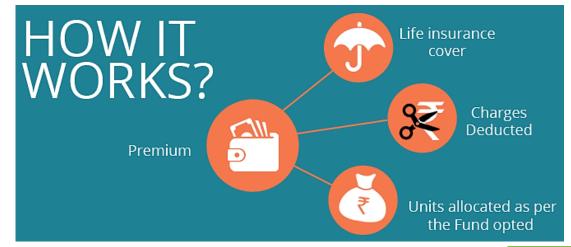
As each premium is paid, a specified proportion (the 'allocation percentage') is invested in an investment fund chosen by the policyholder.

The investment fund is divided into units which are priced continuously.

When each investment allocation is made, the number of units purchased by the policyholder is recorded. The value at the date of death or maturity of the cumulative number of units purchased is the sum assured under the policy.











Suppose a policyholder's annual premium is Rs. 100,000 for 20 years, with a sum assured of Rs. 10 Lakhs. After deduction of applicable charges of Rs. 3000 the amount is invested in the fund chosen by the policyholder. Assuming the fund's NAV is Rs. 10 on the day he invested, so he is eligible to get 9700 units [Rs 97,000/NAV of Rs. 10].



https://www.comparepolicy.com/blogs/unit-linked-insurance-plan-ideal-investment-cum-insurance-plan/



In order to price and value unit-linked contracts, details of allocation percentages (usually specified in the policy) and an assumption about the future growth in the price of the units purchased are needed. The calculations involve projecting the expected profit flows on a year-on-year basis, and discounting these to obtain a measure of the expected profitability of the contract.

So, with unit-linked contracts, the policyholder's basic entitlement is expressed in terms of units, which represent a portion of a fund or funds run by the life insurer. The value of these units moves in line with the performance of the fund.



Important terminology that we will use when discussing unit-linked contracts includes:

- unit account, or unit fund this is the total value of the units in respect of the policy at any time.
- **bid and offer price** the *bid price* (which is the cash-in value of each unit) and the *offer price* (which is the price that has to paid to purchase a unit in the fund). The difference between the two (with the offer price being greater than the bid price) is called the *bid-offer spread*, and this difference is money that the insurance company makes from each unit purchased and which helps cover its costs and enable it to make profits
- **charges** the company deducts money, either by cancelling units or by reducing the unit growth credited to the unit account on a periodic basis, for instance monthly. The company normally reserves the right to vary charges in the light of experience.



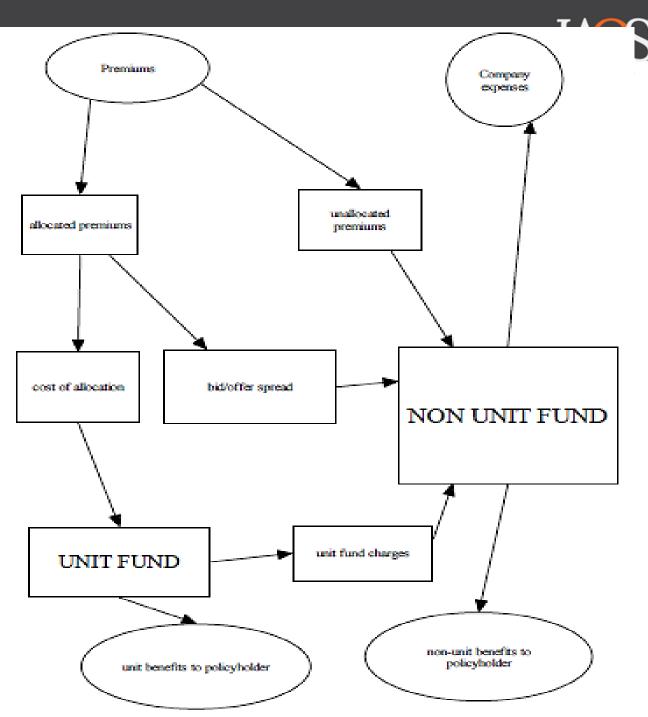
1.2 Unit Funds & Non-Unit Funds

The most important thing to bear in mind with unit-linked contracts is that we have two 'worlds' to keep track of: the unit world, and a cash (or non-unit) world. The policyholder pays premiums to acquire units, and the eventual benefit is normally denominated in these units, so we will need to keep track of the number of units bought by a policyholder, how they are growing, and what charges we are deducting from them.

However, the policyholder pays the life insurance company in real money. So we need to keep track of the cash not used to buy units, because that cash is a source of profit to the life insurance company. Conversely, if the policyholder dies there might be a cash denominated sum insured, and so we need to keep track of the cash outgo on claims. Another very significant cash outgo to consider is the company's expenses. These include expenses incurred in underwriting and maintaining the policy, as well as commission payments to whoever sold it.

Unit Funds & Non-Unit Funds

The inter-relationship between the policyholder, the company, units and cash can be encapsulated in the following diagram.



1.3 Accumulating with-profits contracts

Under an accumulating with-profits (AWP) contract, the basic benefit takes the form of an accumulating fund of premiums. If the accumulating fund at time t is denoted by F_t , the simplest form of an AWP contract follows the following recursive formula:

$$F_t = (F_{t-1} + P)(1+b_t)$$

This example assumes that annual premiums of P are payable at the start of each year. b_t is the annual bonus interest declared for year t.

The bonus will reflect both the returns achieved on the underlying assets over the period plus any additional profits made on the contract in this time. As it is discretionary, it does not exactly reflect these amounts, and in practice the insurer tends to smooth out the variations in returns and profits achieved from year to year to produce a bonus interest rate that is more stable over time than the underlying asset returns, for example.

A key feature of the regular bonus interest is that it cannot be negative, whereas for certain asset types (eg equity portfolios) actual returns can be negative.



1.3 Accumulating with-profits contracts

Sometimes, as was often the case in the UK in the past, part of the bonus interest would be guaranteed. One way of including a guaranteed bonus interest rate of g per annum is shown in the following recursive formula:

$$F_t = (F_{t-1} + P)(1+g)(1+b_t)$$

An alternative approach is just to guarantee that the value for bt in any given year cannot be lower than g.

It is unusual for any guaranteed rate to be applied to AWP in modern conditions (other than the degenerate case where g = 0).

As with conventional with-profits, the regular bonuses under AWP can be reduced so as to retain profit for subsequent deferred payment as a terminal bonus. The contractual benefit under an AWP policy (payable on death or maturity as appropriate) could then be defined as:

$$B_t = F_t + T_t$$

where T_t is the amount of terminal bonus payable on a claim at time t. The purpose and rationale for paying terminal bonus is the same under AWP as it is for conventional with-profits.

Apart from the terminal bonus component, these simple AWP contracts operate in a very similar way to a deposit account administered by a bank.



1.4 Unitised (accumulating) with-profit contracts

Many companies that sell AWP administer the contract in unitised form (called *unitised with-profits* (UWP)). The policyholder is allocated units, and the fund value at any time for any policy is equal to the number of units held multiplied by the current price (or value) of each unit at that time.

In this way, UWP operates in a very similar way to unit-linked contracts. A key difference is the way the unit price is calculated. Two example possibilities are:

Method (1) the unit price allows for guaranteed bonus interest increases only; the discretionary bonus is credited to the policy by awarding additional (bonus) units from time to time

Method (2) the unit price allows for both guaranteed and bonus interest increases.

UWP is usually subject to explicit charges, to cover expenses and any additional death benefit cost.



In this chapter we introduce the technique of profit testing. This is the process of projecting the income and outgo emerging from a policy, and discounting the results. The results can then be used for various different purposes, such as setting the premium for a life policy that will give us our required level of profitability.



Evaluating expected cash flows for various contract types

Profit testing begins with the projection of the expected cash flows of a hypothetical policy.

The standard approach is to divide the total duration of a contract into a series of consecutive non-overlapping time periods. The length of each time period is chosen so that it is reasonable to make simple assumptions about the cash flows within each period. These assumptions allow the expected cash flows during the period to be evaluated.

The expected cash flows, both positive and negative, are used to construct a projected revenue account (per contract in force at the start of the period) for each time period. The balancing item in the projected revenue account is the profit emerging at the end of the time period.



Evaluating expected cash flows for various contract types

In order to calculate the expected cash flows the following information is needed:

- premiums received and their times of payment
- expected expenses (from the basis) and their times of payment
- contingent benefits payable under the contract, *eg* death benefit, annuity payment, survival benefit for endowment, difference between guaranteed sum assured and value of unit fund for unit-linked endowment
- other benefits payable under the contract, eg surrender values
- other expected cash payments, eg taxes
- other expected cash receipts, eg management charges levied on a unit fund, and
- the reserves required for a contract, usually at the beginning and end of the time period, calculated using the valuation basis

together with the different probabilities of the various events leading to the payment of particular cash amounts. Any balance on the expected revenue account during the time period will be invested, and an assumption about the rate of investment return is needed. This allows the expected investment income during the period to be calculated and credited at the end of the period.



Example: Conventional whole life assurance

The contract is issued to a select life aged x and has a sum assured of S secured by level annual premiums of P payable in advance. The premium basis assumes initial expenses of I and renewal expenses of I. The valuation basis requires reserves of I for an inforce policy with sum assured I at policy duration I. The basis assumes that invested funds earn an effective rate I. The surrender value basis determines that a surrender value I will be paid to policies surrendered at policy duration I.

Note the difference between $(SV)_t$, where 'S' stands for surrender and $S_{t}V$, where 'S' stands for the sum assured.

The probabilities of events are determined from a multiple decrement table with decrements of death, d, and surrender, w, having dependent probabilities at age, x of, $(aq)_x^d$ and $(aq)_x^w$



Show the profit calculation for the policy



Show the profit calculation for the policy

The following shows the expected profit calculation for all policy years except the first year. In this formula time t is the beginning of the policy year, and time t+1 is the end of that year. This profit is the what we would expect each year in relation to a policy that is *in force* at the start of that year, *ie* for a policy that is in force at exact time t.

Income		
Premiums	P (from data)	
Interest on reserves	i.S. _t V	
Interest on balances	(P-e)i	t]
<u>Expenditure</u>		
Expenses	e (from data) (*	t]
Expected surrender value	$(aq)_{[x]+t}^{\mathbf{w}}.(SV)_{t+1}$	
Expected death claims	$(aq)_{[x]+t}^d$.S	
Transfer to reserves	$(ap)_{[x]+t}.S{t+1}V-S{t}V$	
Profit	Balancing item	



Projecting cashflows – unit-linked and unitised with-profits

Project unit fund at end of each year as:

- (+) Premium x allocation rate x (1-bid-offer spread) *
- (+) unit fund from end of previous year
- (+) expected unit fund growth
- (–) management charge
- = fund at end of year after charges

* 'Cost of allocation'

Per policy in force at start of each year, project expected amounts of:

- (+) premium less cost of allocation
- (-) expenses
- (+) interest on non-unit fund
- (-) non-unit benefit costs
- (+) non-unit surrender profit
- (+) management charge transferred from unit fund
- (+) other charges from unit fund, if applicable
- = non-unit cashflow vector (= profit vector in absence of any non-unit reserves)



Having now considered how to project the revenue accounts for a policy, we see how to use that projection.

The first step in the profit testing of a contract is the construction of the projected revenue accounts for the non-unit (cash) fund for each policy year.

For some contracts other funds, eg unit fund for unit-linked assurances, reserves for traditional assurances, provide cash flows to the non-unit fund.

Profit vector

The vector of balancing items in the projected revenue accounts for each policy year is called the profit vector; (PRO)t, t = 1, 2, 3, ...

The profit vector gives the expected profit at the end of each policy year per policy in force at the beginning of that policy year.



Profit signature

The vector of expected profits per policy issued is called the profit signature. This is obtained by multiplying the profit vector by the probability of a policy remaining in force from policy duration 0 to policy duration t - 1, ie:

$$(PS)_t = t - \mathbf{1}(ap)_x (PRO)t$$

where (PS)t denotes the t th entry in the profit signature and (PRO)t denotes the t th entry in the profit vector.



Summary measures of profit

Summary measures usually involve determining the present values of the expected cash flows. In some cases, this requires an assumption about the discount rate. This rate is chosen to equal the cost of capital plus a risk premium, and is called the risk discount rate, *id*.

The cost of capital is the rate at which funds can be borrowed, or the rate of return such funds would earn if invested elsewhere, (*ie* the 'opportunity cost').

The risk premium reflects the risks and uncertainties surrounding the cash flows to and from the policy.





CT5 April 2017 Q13

A life insurance company issues a 3-year unit-linked endowment assurance policy to a male life aged 60 exact. The details are:

- · Level premiums of 9,000 per annum are payable yearly in advance throughout the term of the policy or until earlier death.
- \cdot 80% of the premium is allocated to units in the first policy year and 100% in the second and third policy years.
- · A policy fee of 25 is deducted from the annual premium before the allocation to units.
- The units are subject to a bid-offer spread of 5%.
- An annual management charge of 1.5% of the bid value of the units is deducted at the end of each policy year.
- · Management charges are deducted from the unit fund before death, surrender and maturity benefits are paid.
- If the policyholder dies during the term of the policy, the death benefit of 125% of the bid value of the units is payable at the end of the policy year of death.
- On maturity, 100% of the bid value of the units is payable.
- · The policyholder may surrender the policy at any time during the first and second policy years.





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• On surrender, the bid value of the units less a surrender penalty is payable at the end of the policy year of exit as follows: *Year Penalty*

1 600 2 300

The company uses the following assumptions in carrying out profit tests of this contract:

Rate of growth on assets in the unit fund 4.5% per annum in year 1

4.0% per annum in year 2

3.5% per annum in year 3

Rate of interest on non-unit fund cash flows 2.0% per annum

Mortality AM92 Select

Initial expenses 220

Renewal expenses 75 per annum on the second and subsequent premium dates

Initial commission 30% of first premium

Renewal commission 1.5% of the second and subsequent years' premiums

Rate of expense inflation 2.0% per annum

Risk discount rate 6.5% per annum





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For renewal expenses, the amount quoted is at the commencement of the policy, and the increases due to inflation start immediately.

The company assumes that the force of decrement due to surrender is:

- 0.1 in policy year 1.
- · 0.05 in policy year 2.

It also assumes that each force of decrement is independent and constant over each year of age.

- (i) Determine for each policy the dependent rates of mortality and surrender. [4]
- (i) Calculate the profit margin for the policy. [11]



Annual premium	£9000.00	Allocation % (1st yr)	80.0%
Risk discount rate	6.5%	Allocation % (2nd yr)	100.0%
Interest on investments (1st yr)	4.5%	Allocation % (3rd yr)	100.0%
Interest on investments (2nd yr)	4.0%	B/O spread	5.0%
Interest on investments (3rd yr)	3.5%	Management charge	1.5%
Interest on non-unit funds	2.0%	Surrender penalty (1st yr)	£600
Death benefit (% of bid value of units)	125%	Surrender penalty (2nd yr)	£300
		Policy Fee	£25

	£	% prem
Initial expense	220	30.0%
Renewal expense	75	1.5%
Expense inflation	2.0%	



(i) Using $\mu_{[x]+t}^d = -\ln(1 - q_{[x]+t}^d)$ we have:

$$x$$
 t $q_{[x]+t}^d$ μ_{x+t}^s $\mu_{[x]+t}^d$ 0.10 0.005791 0.008680 0.05 0.008718 0.0080112 0.00 0.010163

The dependent rates of decrement are calculated for each policy year using:

$$(aq)_x^j = \frac{\mu^j}{\mu^d + \mu^s} \left[1 - e^{-(\mu^d + \mu^s)} \right]$$

where d denotes mortality and s surrender



$$(aq)_{60}^{d} = \frac{\mu^{d}}{\mu^{d} + \mu^{s}} \left[1 - e^{-(\mu^{d} + \mu^{s})} \right] = \frac{0.005791}{0.105791} \left[1 - e^{-(0.105791)} \right] = 0.005495$$

$$(aq)_{60}^{s} = \frac{\mu^{s}}{\mu^{d} + \mu^{s}} \left[1 - e^{-(\mu^{d} + \mu^{s})} \right] = \frac{0.1}{0.105791} \left[1 - e^{-(0.105791)} \right] = 0.094892$$

$$(aq)_{61}^{d} = \frac{\mu^{d}}{\mu^{d} + \mu^{s}} \left[1 - e^{-(\mu^{d} + \mu^{s})} \right] = \frac{0.008718}{0.058718} \left[1 - e^{-(0.058718)} \right] = 0.008467$$

$$(aq)_{61}^{s} = \frac{\mu^{s}}{\mu^{d} + \mu^{s}} \left[1 - e^{-(\mu^{d} + \mu^{s})} \right] = \frac{0.05}{0.058718} \left[1 - e^{-(0.058718)} \right] = 0.048560$$

$$(aq)_{62}^{d} = \frac{\mu^{d}}{\mu^{d} + \mu^{s}} \left[1 - e^{-(\mu^{d} + \mu^{s})} \right] = \left[1 - e^{-(0.010163)} \right] = 0.010112$$



Multiple decrement table:

x	$(aq)_x^d$	$(aq)_x^s$	(ap)	$_{t-1}(ap)$	
60	0.005495	0.094892	0.899613	1.000000	
61	0.008467	0.048560	0.942973	0.899613	
62	0.010112	0.000000	0.989888	0.848310	
ii)	Unit f	and (per policy	at start of year)		
			yr 1	yr 2	yr 3
value	e of units at sta	art of year	0.000	7021.026	15926.629
alloc			7180.000	8975.000	8975.000
B/O			359.000	448.750	448.750
inter	est		306.945	621.891	855.851
mana	agement charg	;e	106.919	242.538	379.631
value	e of units at ye	ear end	7021.026	15926.629	24929.099



Cash flows (per policy at start of year)

	yr 1	yr 2	yr 3
unallocated premium + pol fee	1820.000	25.000	25.000
B/O spread	359.000	448.750	448.750
expenses	2920.000	211.500	213.030
interest	-14.820	5.245	5.214
man charge	106.919	242.538	379.631
extra death benefit	9.645	33.712	63.021
surrender penalty	56.935	14.568	0.000
end of year cashflow / profit vector	-601.611	490.888	582.545



probability in force	1	0.899613	0.848310
discount factor	0.938967	0.881659	0.827849
expected p.v. of profit	233.56		
premium signature	9000.000	7602.362	6731.287
expected p.v. of premiums	23333.649		
profit			
margin	1.00%		



Net present value (NPV)

This is the present value of the profit signature determined using the risk discount rate.

NPV =
$$\sum_{t=1}^{t=\infty} (1 + i_d)^{-t} (PS)_t$$

The NPV can be interpreted as the EPV of the future profits from the policy, for a single policy as at the start date of the contract.

Profit Margin

This is the NPV expressed as a percentage of the EPV of the premium income. If the premium paid at the beginning of the t th policy year is Pt, this is:

$$\frac{\sum_{t=1}^{t=\infty} (1+i_d)^{-t} (PS)_t}{\sum_{t=1}^{t=\infty} (1+i_d)^{-(t-1)} t - 1(ap)_x P_t}$$

In other words, the profit margin is:

NPV/EPV premiums

where the risk discount rate is used to do the discounting in both the numerator and the denominator.



The internal rate of return (IRR)

This has been described earlier in this subject in relation to non-contingent cashflows, but it can also be calculated for contingent cashflows, as here. It is defined as the discount rate that would make the NPV of the contract equal to zero.

The internal rate of return does not always exist. It does exist where the profit signature has a single financing phase, *ie* where the first profit flow is negative and the profit flows in all subsequent years are positive. This situation commonly occurs, however, and so the IRR is often a useful measure in practice.



Using present value random variable approach

Alternatively, the NPV can be calculated using:

 $\sum_{all\ events} \{Present\ value\ of\ profit | Event\ occurs\} \ x\ P\{Event\ occurs\}$



2.3 Pricing using a profit test

- The actual profitability is not known until each respective contract terminates and we know the actual
 experience.
- When products are designed, the expected level of profit will usually be specified as an objective and the features of the product can be set to achieve this objective.

Profit criterion

- The objective specified for the expected level of profit is termed the 'profit criterion'. Careful choice of a profit criterion is central to the actuarial management of the company selling the products.
- Examples of the profit criterion are:
 NPV = 40% of Initial Sales Commission
 Profit Margin = 3% of EPV of premium income
- We can set the premiums for a product to give a desired level of profitability by projecting cashflows under a certain set of assumptions, deciding on a risk discount rate and profit criterion, and then varying the premium amount until the profit criterion is satisfied.





CT5 April 2010 Q13

A life insurance company issues a 3-year unit-linked endowment assurance policy to a male life aged 45 exact.

Level premiums of £4,000 per annum are payable yearly in advance throughout the term of the policy or until earlier death. 95% of the premium is allocated to units in the first policy year, 100% in the second and 105% in the third. A policy fee of £50 is deducted from the bid value of units at the start of each year. The units are subject to a bid-offer spread of 5% on purchase. An annual management charge of 1.75% of the bid value of units is deducted at the end of each policy year.

Management charges are deducted from the unit fund before death, surrender and maturity benefits are paid.

If the policyholder dies during the term of the policy, the death benefit of 125% of the bid value of the units is payable at the end of the policy year of death. On maturity, 100% of the bid value of the units is payable.

The policyholder may surrender the policy only at the end of the first and second policy years. On surrender, the bid value of the units less a surrender penalty is payable at the end of the policy year of exit. The surrender penalty is £1,000 at the end of the first policy year and £500 at the end of the second policy year.





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The company uses the following assumptions in carrying out profit tests of this contract:

Rate of growth on assets in the unit fund 5.5% per annum in year 1

5.25% per annum in year 2

5.0% per annum in year 3

Rate of interest on non-unit fund cash flows 4.0% per annum

Mortality AM92 Select

Initial expenses £200

Renewal expenses £50 per annum on the second and third premium dates

Initial commission 15% of first premium

Renewal commission 2.0% of the second and third years' premiums

Rate of expense inflation 2.0% per annum

Risk discount rate 7.0% per annum

For renewal expenses, the amount quoted is at outset and the increases due to inflation start immediately. In addition, you should assume that at the end of the first and second policy years, 12% and 6% respectively of all policies still in force then surrender immediately.





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- (i) Calculate the profit margin for the policy. [13]
- (ii) Calculate the expected present value of profit for the policy if the company assumed that there were no surrenders at the end of each of the first and second policy years. [3] [Total 16]



Annual premium		£4000.00	Allocation % (1st yr)	95.0%
Risk discount rate		7.0%	Allocation % (2nd yr)	100.0%
Interest on investment	s (1st yr)	5.5%	Allocation % (3rd yr)	105.0%
Interest on investment	s (2nd yr)	5.25%	B/O spread	5.0%
Interest on investment	s (3rd yr)	5.0%	Management charge	1.75%
Interest on non-unit fu	nds	4.0%	Surrender penalty (1st yr)	£1000
Death benefit (% of bid value of units)		ts) 125%	Surrender penalty (2nd yr)	£500
			Policy Fee	£50
	£	% prem		
Initial expense	200	15.0%		
Renewal expense	50	2.0%		
Expense inflation	2.0%			



(i) Multiple decrement table:

```
q_x^d
                       q_x^s
\boldsymbol{x}
      0.001201
45
                    0.12
46
      0.001557
                    0.06
47
      0.001802
                    0.00
        (aq)_x^d
                      (aq)_x^s
                                   (ap)
                                                _{t-1}(ap)
\boldsymbol{x}
                    0.11986
                                0.878943
45
      0.001201
                                              1.000000
46
      0.001557
                    0.05991
                                0.938536
                                              0.878943
                                              0.824920
47
      0.001802
                    0.00000
                                0.998198
```



Unit fund (per policy at start of year)

	yr 1	yr 2	yr 3
value of units at start of year	0.000	3690.074	7693.641
Alloc	3800.000	4000.000	4200.000
B/O	190.000	200.000	210.000
policy fee	50.000	50.000	50.000
Interest	195.800	390.604	581.682
management charge	65.727	137.037	213.768
value of units at year end	3690.074	7693.641	12001.554



Cash flows (per policy at start of year)

	yr 1	yr 2	yr 3
unallocated premium + pol fee	250.000	50.000	-150.000
B/O spread	190.000	200.000	210.000
expenses	800.000	131.000	132.020
Interest	-14.400	4.760	-2.881
man charge	65.727	137.037	213.768
extra death benefit	1.108	2.995	5.407
surrender penalty	119.856	29.953	0.000
end of year cashflow	-189.926	287.755	133.461
probability in force	1	0.878943	0.824920
discount factor	0.934579	0.873439	0.816298
expected p.v. of profit	133.280		
premium signature	4000.000	3285.769	2882.069
expected p.v. of premiums	10167.837		
profit Margin	1.31%		



(ii) Revised profit vector (-309.781, 257.802, 133.461)
 Revised profit signature (-309.781, 257.492, 133.093)

Revised PVFNP = -289.515 + 224.904 + 108.643 = 44.032