Lecture 4

INSTITUTE OF ACTUARIAL & QUANTITATIVE STUDIES

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Chapter: Unit 4

Chapter Name: Profit Testing

Today's Agenda

0. Introduction

1. Pricing and Reserving Bases
    1. Different Bases

2. Calculating reserves for unit-linked contracts

3. Calculating reserves for conventional contracts using profit test

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Introduction

In the previous chapter, we saw how to project the cash flows for policies. In this chapter, we describe how such techniques can be used to set reserves for both unit-linked and conventional contracts, and how changes in the reserving and pricing assumptions affect profit.

1 Pricing & Reserving Bases

In this section we consider the different bases that may be used in the financial management of a life insurance contract.

The profit test, as described earlier, needs many assumptions to be made in order to compute the expected future profits of a contract for comparison with a stated profit criterion.
The assumptions will, in the first instance, be the insurer's best estimate of expected future experience. This can be termed the *experience basis*.

Assuming that the chosen risk discount rate reflects fully the uncertainties in the assumptions, no further margins would be taken.

Thus, **we arrive at the *pricing basis* for the contract, *ie* the insurer's realistic expected outlook that it chooses to use in setting premiums and/or charges.**

1 Pricing & Reserving Bases

In practice, the actual profits arising in the future will depend on the actual future experience values of the items for which assumptions have been made. The insurer must consider two implications of this:

(i) How will profits be affected by the actual values for assumptions turning out to be different from the pricing basis?; and

(ii) Might the actual experience give rise to a need for additional finance?

The answer to both questions lies in re-running the profit test with a different set of assumptions from the pricing basis.

1 Pricing & Reserving Bases

Policyholders will be prepared to pay higher premiums up to a point, in return for the increased security of the fund (and therefore obtaining greater certainty that the company will meet its future obligations to them).

On the other hand, if the cost of capital is too great, the company will lose customers and could go out of business.

The actuary therefore has to determine the level of reserve that leaves the company with an acceptably low probability (or risk) of insolvency occurring in the future, whilst at the same time imposing a cost of capital on the company that the policyholders are willing to pay for.

The result is that the reserving basis will be prudent. A significantly more pessimistic basis than ‘best estimate’ will be assumed, but it will not be beyond the realms of reasonable possibility (for example, we would not assume that all the company’s life assurance policyholders will die on the day after the valuation date!).

1 Pricing & Reserving Bases

This single *valuation (or reserving) basis* is set by an insurer's actuary to ensure that an adequate assessment of the reserves is made. In practice, the valuation basis chosen will have to satisfy any local legislation and professional guidance which exists to protect the interests of policyholders.

Once the actuary has decided on an appropriate level of reserves to be held by the insurer, the profit test would be finally re-run, still on the original pricing basis, but with the cash flows paid into reserves modelled explicitly. This may mean a reassessment of premiums, benefits, and charges with consequential reassessment of valuation bases. The approach of pricing a contract can therefore be iterative.

1.1 Different Bases

- A basis is a set of assumptions about quantities such as future investment returns, mortality rates, surrender rates and expenses.

- Assumptions that are our best estimates of the future give an experience basis.

- The basis we use to set premiums is called the pricing basis and can be the same as the experience basis, or slightly more prudent, or more risky.

- The basis we use to determine reserves – the valuation or reserving basis – is generally much more prudent than the experience basis. This will defer the emergence of profits from the contract. If the required rate of return is greater than the investment return on reserves, this will reduce the profitability of the contract.

2 Calculating reserves for unit-linked reserves

In this section we see how cashflow projections can be used to set reserves. This is particularly important for unit-linked products, where it is the only way of determining appropriate reserves, but we shall also see that we can apply the same methodology to conventional products.

We have already seen how to calculate the required amount of reserve prospectively, by valuing the stream of future benefit payments and future expenses and deducting future premiums. In other words we take each item of the expected cashflow and sum (with discounting and allowing for the probabilities of remaining in force) over all future years. We would get the same answer by summing each year's cashflow (again with discounting and allowing for the probabilities of remaining in force) and multiplying by –1, as we now explain below.

2 Calculating reserves for unit-linked reserves

The calculation of a reserve involves summing all the elements in the following box, with suitable discounting for interest and allowing for staying in force:

Year	Discounted premium (positive)	...	Discounted expenses (negative)	Discounted claims (negative)	*etc* ...	Sum = discounted cashflow
1	$+P_1$		$-e_1$	$-C_1$		EPV at time 0 of premiums, expenses, *etc* in year 1
...						
t	$+P_t$		$-e_t$	$-C_t$		EPV at time 0 of premiums, expenses, *etc* in year t
...						
Sum	Sum of discounted premiums		Sum of discounted expenses			Sum of sums × −1 = reserve

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Calculating reserves for unit-linked contracts

Unit-linked contracts require a unit reserve, which is equal to the unit fund value at any particular time, and a non-unit reserve. The calculation of the non-unit reserve follows the procedure described below, which is sometimes referred to as *zeroising negative cashflows*.

It is a principle of prudent financial management that, once sold and funded at the outset, a product should be self-supporting. This implies that the profit signature has a single negative value (funds are provided by the insurance company) at policy duration zero. This is often termed 'a single financing phase at the outset'.

Many unit-linked products naturally produce profit signatures which have a single financing phase. However some products, particularly those with substantial expected outgo at later policy durations, can give profit signatures which have more than one financing phase. In such cases these later negative non-unit cashflows (financing phases) should be reduced to zero by establishing reserves in the non-unit fund at earlier durations. These reserves are funded by reducing earlier positive non-unit cashflows. Good financial management dictates that these reserves should be established as late as possible during the term of the contract.

2 Calculating reserves for unit-linked contracts

We can either sum the rows first and then sum the row totals, or sum the columns first and then sum the column totals. We then multiply by -1 so as to convert from:

‘EPV of future income less outgo’
to:
‘EPV of future outgo less income’

which is what we require in order to produce a (prospective) reserve value. So we can take our cashflow projections – these are our row totals above – and use these to calculate suitable reserves.

2 Calculating reserves for unit-linked contracts

General approach

A policy has a non-unit cashflow vector (profit vector without non-unit reserves) of $(NUCF)t$; t = 1, 2, 3, … determined using the methods described in the previous chapter.

Non-unit reserves (reserves in the cash fund) are to be set up so that there is only a single financing phase. The reserves to be established at policy duration t are tV .
The reserving basis interest rate is i_s , and the probability of a policy staying in force for one year at age x is $(ap)_x$. After establishing non-unit reserves the profit vector is (PRO)t , t = 1, 2, 3, … .

The equation of value at the end of policy year t , for cashflows in policy year t , per policy in force at time t -1, is:

$$(\mathrm{NUCF})_t + {}_{t-1}V(1+i_s) - (ap)_{x+t-1}\,{}_tV = (\mathrm{PRO})_t$$

2 Calculating reserves for unit-linked contracts

General approach

The process of establishing reserves begins at the greatest duration t for which $(NUCF)_t$ is negative. Let this be duration $t = m$. Non-unit reserves will not be required at durations $t \geq m$ because during these policy years the product is expected to be self-financing.
Hence we know that $_tV = 0$ for $t \geq m$.

For policy year m we can write:

$$(\text{NUCF})_m + {}_{m-1}V(1+i_s) - (ap)_{x+m-1} \times 0 = (\text{PRO})_m$$

where $(\text{NUCF})_m < 0$ and we wish to choose $_{m-1}V$ so that $(\text{PRO})_m = 0$.

Thus $_{m-1}V$ should be chosen to be:

$${}_{m-1}V = -\frac{(\text{NUCF})_m}{(1+i_s)} \qquad (*)$$

$(\text{NUCF})_m < 0$ has been 'turned into' $(\text{PRO})_m = 0$; the expected cashflows have been zeroised.

2 Calculating reserves for unit-linked contracts

General approach

The reserve $_{m\text{-}1}V$ will be established at policy duration m - 1 out of the funds available at duration m - 1. A reserve of $_{m\text{-}1}V$ is required for every policyholder alive at the start of the mth policy year. The non-unit cashflow $(NUCF)_{m-1}$ at time m - 1 is $(NUCF)_{m-1}$ for every policyholder alive at the start of the (m - 1) th policy year. If we take the required reserve $_{m\text{-}1}V$ out of this cash flow, then the adjusted cashflow $(NUCF)'_{m-1}$ is given by:

$$(\mathrm{NUCF})'_{m-1} = (\mathrm{NUCF})_{m-1} - (ap)_{x+m-2}\ {}_{m-1}V$$

If $(\mathrm{NUCF})_{m-1} < 0$, then $(\mathrm{NUCF})'_{m-1}$ will be negative. However, if $(\mathrm{NUCF})_{m-1} > 0$, then $(\mathrm{NUCF})'_{m-1}$ may be positive or negative. If $(\mathrm{NUCF})'_{m-1} > 0$ then:

$$(\mathrm{PRO})_{m-1} = (\mathrm{NUCF})'_{m-1}$$

If $(\mathrm{NUCF})'_{m-1} < 0$, then we repeat the process establishing non-unit reserves $_{m-2}V$ at policy duration $m-2$.

2 Calculating reserves for unit-linked contracts

General approach

So we have:

$$(\text{NUCF})'_{m-1} + {}_{m-2}V(1+i_s) = (\text{PRO})_{m-1}$$

and choose ${}_{m-2}V$ so that $(PRO)_{m-1} = 0$, *ie*:

$${}_{m-2}V = -\frac{(\text{NUCF})'_{m-1}}{(1+i_s)}$$

The process then repeats as from (*) above, and continues in this way until all the necessary reserves at durations $t = 1, 2, 3, ..., m-1$ have been obtained.

Question

CT5 April 2012 Q5

A 10-year unit-linked policy has the following profit vector:
(–40, –12, –6, –1, 5, –4, 8, 20, 25, 30)

Determine the revised profit vector if reserves are set up to zeroise future negative cash flows on the following basis:

Mortality 0.5% per annum (i.e. probability of death at each age)
Interest 2.5% per annum

Solution

The reserves required at the beginning of policy years 6, 4, 3 and 2 are:

$$ {}_5V = \frac{4}{1.025} = 3.902 $$

$$ {}_3V = \frac{1}{1.025} = 0.976 $$

$$ {}_2V = \frac{1}{1.025}(6 + .995 \times {}_3V) = 6.801 $$

$$ {}_1V = \frac{1}{1.025}(12 + .995 \times {}_2V) = 18.309 $$

Revised cash flow in policy year 5 $= 5 - 0.995 \times {}_5V = 1.118$

Revised cash flow in policy year 1 $= -40 - 0.995 \times {}_1V = -58.218$

=> revised profit vector: $(-58.22, 0, 0, 0, 1.12, 0, 8, 20, 25, 30)$

3 Calculating reserves for conventional contracts using profit test

A profit test can also be used to determine the reserves for a conventional (*ie* non unit-linked) policy. We illustrate the procedures by using a without-profit endowment assurance with a term of n years, a sum assured of S payable at the end of the year of death or on survival to the end of the term, and a surrender value payable at the end of year t of U_t , which is secured by a level annual premium of P .

A basis is required for the projection of the cashflows and for calculating the required reserves. This will consist of an interest rate i , dependent probabilities and expenses per policy in force at time t of e_t .

$(CF)_t$, the expected cashflow at time t per policy in force at time t - 1, ignoring reserves, is:

$$(CF)_t = (P - e_{t-1})(1+i) - S(aq)^d_{x+t-1} - U_t (aq)^s_{x+t-1} \qquad t = 1, 2, 3, \ldots, n-1$$

$$(CF)_n = (P - e_{n-1})(1+i) - S$$

3 Calculating reserves for conventional contracts using profit test

If the contract is to be self-funding, then reserves must be established using the earlier positive cashflows. These reserves should be sufficient to pay the later expected negative. This requirement is exactly analogous to the need to establish reserves in the non-unit fund for a unit-linked contract. Reserves can be established for conventional assurances using the same procedures as those used to establish non-unit reserves.

3

Calculating reserves for conventional contracts using profit test

For policy year m, we write:

$$(CF)_m + {}_{m-1}V(1+i) - (ap)_{x+m-1} \times 0 = (PRO)_m$$

where $(CF)_m < 0$ and we wish to choose ${}_{m-1}V$ so that $(PRO)_m = 0$.

This requires that ${}_{m-1}V$ is chosen to be:

$$ {}_{m-1}V = \frac{-(CF)_m}{(1+i_r)} \qquad (**)$$

We set up this reserve from $(CF)_{m-1}$ and determine the adjusted cashflow:

$$(CF)'_{m-1} = (CF)_{m-1} - (ap)_{x+m-2}\ {}_{m-1}V$$

If $(CF)'_{m-1} > 0$, then ${}_{m-2}V = 0$. If $(CF)'_{m-1} < 0$, then we calculate:

$$ {}_{m-2}V = \frac{-(CF)'_{m-1}}{(1+i)}$$

3 Calculating reserves for conventional contracts using profit test

The process then repeats as from (**) above, and continues in this way until all the necessary reserves at durations t = 1, 2, 3,..., m-1 have been obtained.

For conventional assurances it is usually the case that reserves are needed at all policy durations. So the calculation begins with $(CF)_n$ and concludes with $(CF)_1$.

Question

CT5 September 2017 Q11

A life insurance company issues a large number of 4-year unit-linked endowment assurance policies to lives aged 65 exact. Level premiums are payable annually in advance until maturity or earlier death.

The company has performed a profit test on these policies and the profit vector per policy sold, ignoring surrenders, is as follows: **(185.21, –121.52, –5.28, 12.95)**

(i) Calculate the profit signature per policy sold if negative non-unit fund cash flows are zeroised. [3]

The company now wishes to allow for surrenders in its calculations. It assumes that at the end of the first and second policy years only, 3% of the surviving policyholders will surrender. Surrender values are equal to the bid value of units held (after deduction of the fund management charge) less a surrender penalty of 50.

(ii) Calculate the revised profit signature per policy sold after allowing for surrenders if negative non-unit cash flows are zeroised. [6]

(iii) Calculate the net present value of the revised profit signature in part (ii), using a risk discount rate of 8% per annum. [1]

Basis:

Mortality AM92 Ultimate

Interest earned on non-unit cash flows 5% per annum fund

Expenses Ignore [Total 10]

JAQS

Solution

(i)

t	*Profit vector*	${}_{t-1}p_{65}$	*Profit signature*
1	185.21	1	185.21
2	–121.52	0.985757	–119.79
3	–5.28	0.970044	–5.12
4	12.95	0.952754	12.34

Let X be the reserve required at t =1 in order to zeroise negative cash flows at t = 2 and t = 3.

Then:

$X = 119.79v + 5.12v^2$ at 5% $= 118.73$ [1]

Revised cash flow at t = 1 is 185.21 – 118.73 = 66.48

Hence profit signature is: (66.48, 0, 0, 12.34) [½]

Solution

(ii) Multiple decrement table – although deaths can be assumed to be uniformly distributed over the year, surrenders occur only at the year end. Therefore:

$$(aq)_x^d = q_x^d \text{ and } (aq)_x^s = q_x^s(1 - q_x^d)$$

x	q_x^d	q_x^s	$(aq)_x^d$	$(aq)_x^s$	$(ap)_x$	$_{t-1}(ap)_x$
65	0.014243	0.03	0.014243	0.029573	0.956184	1
66	0.015940	0.03	0.015940	0.029522	0.954538	0.956184
67	0.017824	0.00	0.017824	0.00	0.982176	0.912714
68	0.019913	0.00	0.019913	0.00	0.980087	0.896446

Solution

t	*Revised profit vector*	${}_{t-1}(ap)_{65}$	*Revised profit signature*
1	$185.21+50\left(aq\right)^{s}_{65}=186.69$	1	186.69
2	$-121.52+50\left(aq\right)^{s}_{66}=-120.04$	0.956184	–114.78
3	–5.28	0.912714	–4.82
4	12.95	0.896446	11.61

Then:

$$Y=114.78v+4.82v^2 \text{ at } 5\%=113.69$$

Revised cash flow at $t=1$ is 186.69 – 113.69 = 73.00

Hence revised profit signature is: (73.00, 0, 0, 11.61)

Solution

(iii) NPV of revised profit signature $= 73.00v + 11.61v^4$ at $8\% = 76.13$

Question

CT5 April 2018 Q13

A life insurance company issues a three-year unit-linked endowment assurance contract to a male life aged 62 exact under which level annual premiums of 6,000 are payable in advance throughout the term of the policy or until earlier death. 90% of each year's premium is invested in units at the offer price.

There is a bid-offer spread in unit values, with the bid price being 95% of the offer price.

There is an annual management charge of 1% of the bid value of units. Management charges are deducted at the end of each year, before death or maturity benefits are paid.

On the death of the policyholder during the term of the policy, the benefit, payable at the end of the year of death, is 12,000, or the bid value of the units if greater. The policyholder may surrender the policy only at the end of each year immediately before a premium is payable. On surrender, the bid value of the units is payable at the end of the year of exit. On maturity, 110% of the bid value of the units is payable.

The company holds unit reserves equal to the full bid value of the units. It sets up non-unit reserves to zeroise any negative non-unit fund cash flows, other than those occurring in the first year.

Question

The company uses the following assumptions in carrying out profit tests of this contract:
Rate of growth on assets in the unit fund – 5% per annum
Rate of interest on non-unit fund cash flows – 3% per annum
Mortality – AM92 Ultimate
Surrenders – 10% at the end of the first policy year, 5% at the end of the second policy year based on policies in force at that time
Initial expenses 225 plus 5% of the first premium (all incurred on policy commencement)
Renewal expenses 65 at the start of each of the second and third policy years plus 2.5% of the second and third premiums
Risk discount rate 7% per annum

Calculate the profit margin on the contract. [15]

Solution

x	q_x^d	q_x^s
62	0.010112	0.100
63	0.011344	0.050
64	0.012716	0.000

x	$(aq)_x^d$	$(aq)_x^s$	$(ap)_x$	${}_{t-1}(ap)_{62}$
62	0.010112	0.09899	0.890899	1.000000
63	0.011344	0.04943	0.939223	0.890899
64	0.012716	0.00000	0.987284	0.836753

Solution

Unit fund

Year	*1*	*2*	*3*
Fund at the start of the year	0	5332.635	10875.909
Premium allocation	5400.000	5400.000	5400.000
B/O spread	270.00	270.00	270.00
Interest	256.500	523.132	800.295
Management charge	53.865	109.858	168.062
Fund at the end of the year	5332.635	10875.909	16638.142

[½ mark for each line]
[Total 3]

Solution

Non-unit fund before reserves

Year	*1*	*2*	*3*
Unallocated premium + B/O spread	870.00	870.00	870.00
Expenses	525.00	215.00	215.00
Interest	10.350	19.650	19.650
Extra death cost	67.420	12.752	0
Extra maturity cost	0	0	1642.657
Management charge	53.865	109.858	168.062
End of year cash flow	341.795	771.756	–799.945

Solution

Reserve required at the start of year 3 = 799.945 / 1.03 =776.646 [1]

Reduced profit at the end of year 2 = 771.756 – 776.646 × $(ap)_{63}$ = 42.312 [1]

Revised profit vector: $(341.795, 42.312, 0)$ [½]

$$\text{Net present value} = \frac{341.795}{1.07} + \frac{42.312 \times (ap)_{62}}{1.07^2} = 352.359$$ [1½]

$$\text{Present value of premiums} = 6000 \times \left(1 + \frac{(ap)_{62}}{1.07} + \frac{{}_2(ap)_{62}}{1.07^2}\right) = 15{,}380.812$$

[1½]

$$\text{Profit margin} = \frac{352.359}{15380.812} = 2.29\%$$ [½]