IAQS | INSTITUTE OF ACTUARIAL & QUANTITATIVE STUDIES

Subject: Pricing & Reserving for Life Insurance Products - 2

Chapter: Unit 1

Category: Practice Questions

1. CM1A April 2019 Q6

A life insurance company issues a 20-year term assurance with additional permanent disability benefit. The benefits provided are:
• on death (whether the life was previously healthy or permanently disabled) a lump sum payment of £150,000 payable immediately
• on permanent disability a lump sum of £75,000 payable immediately.

(i) Draw a transition state model for this policy, labelling your diagram. [2]

(ii) Calculate the total expected present value of the benefits. [8]

Basis:
Force of mortality from healthy 0.03 for all ages
Force of mortality from permanent disability 0.08 for all ages
Force of permanent disability 0.001 for all ages
Force of interest 5% per annum
[Total 10]

2. CT5 April 2018 Q9

A life insurance company issues 20-year critical illness term assurance policies.

The benefits, payable during the policy term, are a lump sum of 50,000 payable immediately on diagnosis of a critical illness, or a benefit of 100,000 immediately on death if earlier.

Premiums are payable monthly in advance during the term of the policy, ceasing on any claim.

(i) Draw a transition state model for this policy labelling your diagram. [3]

A policy is issued to a life aged 45 exact, with an annual premium rate of 750.

(ii) Calculate the value of the policy to the company. [6]

Basis:
Mortality $mx = 0.004$ for all x
Critical Illness $sx = 0.002$ for all x
Force of Interest ∂ = 4% throughout
[Total 9]

3. CT5 September 2017 Q7

A population is subject to two modes of decrement, ✔ and 🚲 as defined below:

$$\mu_x^{\alpha} = 1/(110 - x) \qquad \text{for } 0 \le x < 110; \text{ and}$$

$$\mu_x^{\beta} = 0.03 \qquad \text{for } 0 \le x < 110$$

You are given that $\int_0^1 te^{-0.03t}dt = 0.490112$

Determine the value of $(aq)_{40}^{\beta}$.

[Total 7]

4. CT5 April 2015 Q3

Suppose α and β are the only two independent modes of decrement and $\mu_x^{\beta} = \frac{1}{4}\mu_x^{\alpha}$

Express $(aq)_x$ in terms of μ_x^{β}.

5. CT5 September 2015 Q6

The employees of a manufacturing company are subject to two modes of decrement, mortality and withdrawal from employment.

The independent forces of mortality and withdrawal for employees aged 50 and 51 are given in the following table:

Age	μ_x^d	μ_x^w
50	0.0010	0.15
51	0.0015	0.10

Calculate, showing all your workings, the probability that a new employee aged 50 exact will die as an employee at age 51 last birthday. State any assumptions that you make. [5]

6. CT5 September 2015 Q7

A critical illness scheme provides a benefit of 100,000 on death or earlier diagnosis of a critical illness.

(i) Draw and label the appropriate transition diagram. [3]

(ii) Set out an expression for the expected present value of this benefit. [3]
[Total 6]

7. CT5 April 2014 Q8

A double decrement table is to be constructed from two single decrement tables. The modes of decrement are α and β. The basis for each of the single decrement tables is shown below:

Basis:

In the table for single decrement α: $l_{x+t}^{\alpha} = l_x^{\alpha} - t^3 d_x^{\alpha}$ for $0 \le t \le 1$

In the table for single decrement β: $l_{x+t}^{\beta} = l_x^{\beta} - t^5 d_x^{\beta}$ for $0 \le t \le 1$

The l function represents the number of lives and the d function the number of decrements in the appropriate table.
(i) Show that

$$ {}_t p_x^{\beta} \mu_{x+t}^{\beta} = 5t^4 q_x^{\beta} \text{ for } 0 \le t \le 1 $$

(ii) Hence or otherwise show that

$$ (aq)_x^{\beta} = q_x^{\beta} \left(1 - \frac{5}{8} q_x^{\alpha} \right) $$

8. CT5 September 2013 Q20

The following is an extract of a decrement table assumed for a funeral plan, showing deaths (d) and withdrawals (w):

$Age\ x$	$(al)_x$	$(ad)_x^d$	$(ad)_x^w$
85	10,000	1,400	2,300
86	6,300	1,000	1,100
87	4,200		

It has been established that the independent rates of decrement of withdrawal are now only 50% of those assumed in the table above for the ages of 85 and 86. The underlying independent mortality rates are unchanged.

Construct a revised decrement table to reflect this change. [7]

9. CT5 April 2012 Q10

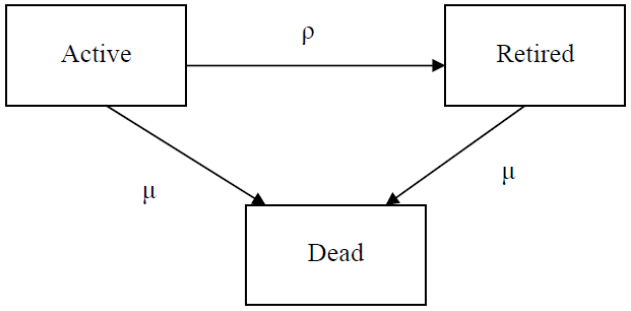
An insurance company writes policies that provides benefits of £1,000 in the event of becoming disabled due to accident and £10,000 on death.

(a) Construct a multiple state transition model for these policies.

(b) Give a formula for the expected present value of the benefits.
[6]

10. CT5 April 2011 Q5

A pension scheme uses the following model to calculate probabilities, where the transition intensities are $\mu = 0.05$ and $\rho = 0.08$.

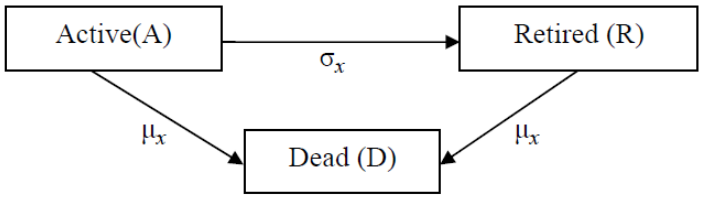


Calculate:
(a) the dependent probability of retirement
(b) the independent probability of death from active service

using the Kolmogorov equations. [5]

11. CT5 April 2010 Q9

A life insurance company models the experience of its pension scheme contracts using the following three-state model:



(i) Derive the dependent probability of a life currently Active and aged x retiring in the year of age x to $(x + 1)$ in terms of the transition intensities. [2]

(ii) Derive a formula for the independent probability of a life currently Active and aged x retiring in the year of age x to $(x + 1)$ using the dependent probabilities. [4]
[Total 6]

12. CT5 April 2010 Q10

The decrement table extract below is based on the historical experience of a very large multinational company's workforce.

Age (x)	*Number of employees* $(al)_x$	*Deaths* $(ad)_x^d$	*Withdrawals* $(ad)_x^w$
40	10,000	25	120
41	9,855	27	144
42	9,684		

Recent changes in working conditions have resulted in an estimate that the annual independent rate of withdrawal is now 75% of that previously used.

Calculate a revised table assuming no changes to the independent death rates, stating your results to one decimal place. [7]

13. CT5 April 2009 Q5

A population is subject to two modes of decrement α and β where

$$q_x^{\beta} = \frac{1}{3} + \frac{1}{4} q_x^{\alpha}$$

Derive from first principles $(aq)_x^{\beta}$

State clearly any assumptions you make. [5]

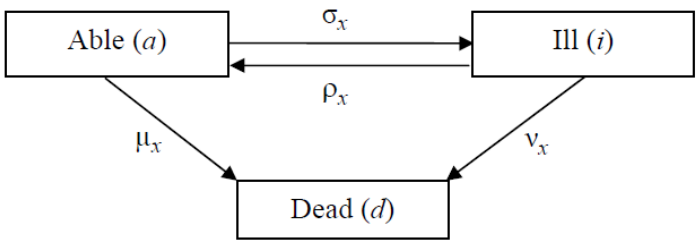
14. CT5 April 2009 Q9

A life insurance company sells a policy with a 10 year term to a healthy life aged 55 exact. The policy provides the following benefits:

• £25,000 payable immediately on death

• £1,000 per annum payable continuously during illness

The company prices the policy using the following multiple state model:

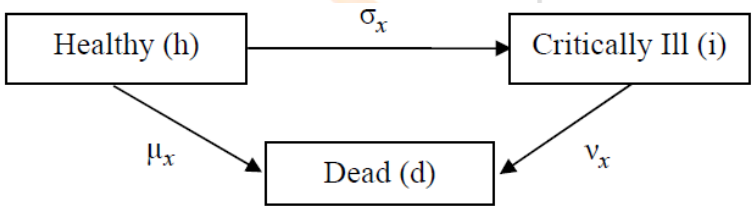


Give a formula for the expected present value of the benefits under the policy. [5]

15. CT5 April 2008 Q4

A life insurance company sells a term assurance and critical illness policy with a 20 year term to a life aged 40 exact. The policy provides a benefit of £50,000 payable immediately on death or earlier diagnosis of critical illness. No further benefit is paid in the event of death within the term after a prior critical illness claim has been paid.

The company prices the policy using the following multiple state model:



Calculate the expected present value of the benefits under the policy.
Basis: i = 5% per annum
μx = 0.005 at all ages
νx = 0.006 at all ages
σx = 0.003 at all ages [5]