



Subject: Pricing & Reserving
for Life Insurance
Products - 2

Chapter: Unit 2

Category: Practice
Questions

1. CT5 September 2018 Q12

A life insurance company issues with-profits whole of life policies to lives aged 35 exact with the sum assured of 100,000 together with any attaching bonuses payable immediately on death of the life assured. Level premiums are payable monthly in advance to age 65 or until earlier death.

The company markets two versions of this policy as follows:

- Version A – assumed to provide compound bonuses of 4% of the sum assured vesting at the end of each policy year.
- Version B – assumed to provide simple bonuses of $b\%$ per annum of the sum assured, again vesting at the end of each policy year.

The death benefit under each version does not include any bonus relating to the policy year of death.

The following basis is used to price these contracts:

Mortality AM92 Select

Rate of Interest 4% per annum

Initial expenses 275

Renewal expenses 2.5% of the second and subsequent monthly premiums

Initial commission 40% of the total premiums payable in the first policy year, all incurred at the policy commencement date

Renewal commission 2.5% of the second and subsequent monthly premiums

Claim expenses 225 at payment of death claim

(a) Show that the monthly premium under version A of this policy is approximately 511.

(b) Calculate the level simple bonus rate b that can be supported each year under version B of this policy if the monthly premium calculated in part (a) is charged.

[10]

2. CT5 September 2018 Q13

A life insurance company issued 25-year decreasing term assurance policies on 1 January 2001 to lives then aged 40 exact. The death benefit, payable at the end of the year of death, is 500,000 in the first policy year, 480,000 in the second policy year thereafter reducing by 20,000 each year until the benefit is 20,000 in the twenty-fifth and final policy year. Premiums on the policies are payable annually in advance for 25 years or until earlier death.

(a) Show that the annual net premium per policy is approximately 643 using the basis below.

(b) Calculate the mortality profit or loss to the life insurance company during 2017 using the basis below if 1,527 policies were in force at the start of that year and 9 policyholders died during the year.

Basis:

Mortality AM92 Ultimate

Rate of Interest 4% per annum

[11]

3. CT5 April 2018 Q2

Describe three types of reversionary bonus that may be awarded to a with-profits contract. [4]

4. CT5 April 2018 Q8

A term assurance policy is issued on a life aged x for a term of 20 years.

Under this policy a sum assured, payable immediately on death, is 10,000 for the first 10 years increasing to 20,000 for the subsequent 10 years.

(i) Calculate the expected present value of the benefits. [3]

(ii) Determine the variance of the present value of the benefits. [5]

Basis:

Mortality $\mu_x = 0.03$ for all x

Force of Interest $\delta = 5\%$ throughout

[Total 8]

5. CT5 April 2018 Q12

On 1 January 2013, a life insurance company issued a with-profits whole-life policy to a life then aged 25 exact. Under the policy, the basic sum assured of 150,000 and attaching bonuses are payable immediately on death. The company declares simple reversionary bonuses at the start of each year (i.e. the death benefit includes the bonus relating to the policy year of death). Level premiums are payable annually in advance under the policy.

(i) Give an expression for the gross future loss random variable under the policy at the outset using the basis below. [3]

(ii) Calculate the gross annual premium, using the equivalence principle and the same basis.

Basis:

Mortality AM92 Select

Interest 6% per annum

Bonus loading 4% per annum simple

Expenses Initial 265

Renewal 5% of each premium payable in the second and subsequent years

Claim 315

[4]

On 31 December 2017 the policy is still in force. The actual bonuses added to the policy have been as follows:

<i>Year</i>	<i>Simple bonus per annum</i>
2013	4.0%
2014	4.0%
2015	3.75%
2016	3.5%
2017	3.0%

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(iii) Determine the gross premium prospective reserve for the policy as at 31 December 2017 using the following basis:

Mortality AM92 Ultimate

Interest 4% per annum

Bonus loading 3% per annum simple

Renewal expenses 5% of each premium

Claim expenses 325

[4]

[Total 11]

6. CT5 September 2017 Q13

On 1 January 2000, a life insurance company issued 25-year increasing term assurance policies to single lives aged 40 exact.

The death benefit, payable at the end of the year of death, was 50,000 in the first policy year and increased at the beginning of each policy year at a rate of 1.92308% per annum compound. The first increase was at the start of the second policy year.

A return of premiums paid, with no interest, is payable on survival to the end of the term of the policy.

Level premiums on the policies are payable annually in advance for 25 years or until earlier death.

The company calculates its reserves on a net premium basis and negative reserves are permitted.

(i) Show that the annual net premium for each policy is approximately equal to 323 using the basis below. [4]

At the start of 2016, there were 1425 policies in force. 10 policyholders died during 2016.

(ii) Calculate the mortality profit or loss to the company during 2016 using the basis below. [7]

After an analysis of surplus investigation by the company, it found that it had made neither a profit nor a loss in 2016 in respect of the policies in part (ii).

(iii) Determine the rate of interest the company earned in 2016. [6]

Basis:

Mortality AM92 Select

Interest 6% per annum

Expenses Ignore

[Total 17]

7. CT5 April 2016 Q12

On 1 March 1997, a life insurance company issued a whole life with profit policy to a life then aged 45 exact. The basic sum assured was 150,000. The sum assured (together with any bonuses attaching) is payable immediately on death. Level premiums are payable monthly in advance to age 85 or until earlier death. Compound reversionary bonuses vest at the beginning of each policy year (i.e. the death benefit includes any bonus relating to the policy year of death).

The company calculates the premium on the following basis:

Mortality AM92 Select

Rate of interest 6% per annum

Initial expenses 70% of the first year's premium, incurred at the outset

Renewal expenses 5% of the second and each subsequent year's premium, incurred at the beginning of the respective policy years

Bonuses: 1.92308% per annum compound

(i) Show that the monthly premium is approximately 276. [7]

On 28 February 2015, the company alters the policy at the request of the policyholder to a paid-up policy with no future premiums payable. The sum assured under the policy is reduced, with no further bonuses payable.

The company calculates the reduced sum assured after alteration by equating prospective gross premium policy reserves immediately before and after alteration, allowing for an expense of alteration of 175.

The company calculates prospective gross premium policy reserves for the purpose of the alteration using the following basis:

Mortality AM92 Ultimate

Rate of interest 6% per annum

Expenses ignore

Future bonuses ignore

Bonuses have vested at a rate of 2% per annum compound at the beginning of each policy year from the date of issue of the policy.

(ii) Calculate, showing all your workings, the sum assured for the policy after alteration.

[6]

[Total 13]

8. CT5 September 2016 Q9

A life insurance company issues a 35-year with-profits endowment assurance policy to a life aged 30 exact. The sum assured of 125,000 plus declared reversionary bonuses are payable at the end of the year of death or on maturity.

Level premiums of 3,090 are payable annually in advance for 35 years or until earlier death.

A simple bonus, expressed as a percentage of the sum assured, is added to the policy at the end of each year. The death benefit does not include the bonus relating to the policy year of death.

The following basis is used to price this policy:

Mortality AM92 Select

Rate of interest 4% per annum

Initial expenses 325 plus 75% of the first annual premium, incurred at the policy commencement date

Renewal commission 2.5% of each premium from the start of the second policy year

Claim expense 375 at the point of claim payment

Calculate, showing all your workings, the level simple bonus rate that can be supported each year on this policy, using the principle of equivalence. [9]

9. CT5 September 2016 Q12

A life insurance company issues a 15-year decreasing term assurance policy to a life aged 50 exact. The initial sum assured is 450,000, decreasing by 30,000 at each policy anniversary (the first decrease taking place at the beginning of the second policy year). The death benefit is payable immediately on death.

Level annual premiums are payable in advance for 15 years, ceasing on earlier death.

The life insurance company uses the following basis for calculating premiums and reserves:

Mortality AM92 Select

Rate of interest 4% per annum

Expenses Initial 275 plus 30% of the first premium

Renewal 5% of all premiums excluding the first plus 68 per annum inflating at 4% per annum compound at the start of the second and subsequent policy years.

Claim 315 inflating at 4% per annum compound

Inflation For renewal and claim expenses, the amounts quoted are at outset of the policy and the increases due to inflation start immediately.

(i) Write down the Gross Premium Future Loss Random Variable at the start of the policy. Use P for the annual premium. [4]

(ii) Calculate, showing all your workings, the premium, using the principle of equivalence. [8]

(iii) Calculate, showing all your workings, the gross premium prospective reserve after 14 years. [2]
[Total 14]

10. CT5 April 2015 Q12

An insurance company issues a 25-year with-profit endowment assurance policy to a life aged 40 exact. The sum assured of £75,000 plus declared reversionary bonuses are payable on survival to the end of the term or immediately on death if earlier.

The insurance company assumes that future reversionary bonuses will be declared at a rate of 3% of the sum assured, simple and vesting at the end of each policy year (i.e. the death benefit does not include any bonus relating to the policy year of death).

Premiums are payable in advance throughout the term of the policy or until earlier death.

Calculate the monthly premium.

Basis:

Mortality AM92 Select

Interest 6% per annum

Initial commission 115% of the first monthly premium

Initial expenses £210

Renewal commission 2.5% of each monthly premium payable excluding the first

Renewal expenses £85 per annum, inflating at 1.92308% per annum compound, at the start of the second and subsequent policy years.

Inflation For renewal expenses, the amount quoted is at outset, and the increases due to inflation start immediately.

[9]

11. CT5 September 2015 Q13

A life assurance company issues a policy to a male life aged 40 exact which provides the following benefits:

- An annuity of 30,000 per annum, payable annually in advance starting on the policyholder's 65th birthday and continuing for life thereafter. The annuity increases by 1,500 each year, with the first increase given on the policyholder's 66th birthday.

- A decreasing term assurance with a death benefit, payable immediately on death, which is given by the formula: $10,000 \times (25 - t)$, $t = 0, 1, 2, \dots, 24$ where t denotes the curtate duration in years since inception of the policy. Death benefit cover ceases at age 65.

The policy is paid for by level monthly premiums payable in advance from the date of issue for 25 years, but ceasing on earlier death.

The company uses the following premium basis for the policy:

Mortality AM92 Select

Interest 4% per annum

Initial commission 35% of the total premiums payable in the first policy year

Initial expenses 225

Renewal commission 5% of the second and subsequent monthly premiums

Renewal expense 55 per annum at the start of the second and subsequent policy years

Death benefit claim expense 275

Annuity payment expense 2.5% of each annuity payment

The renewal expense and the death benefit claim expense are both assumed to increase continuously at 4% compound per annum from inception of the policy and to cease at age 65, or earlier death.

Calculate, showing all your workings, the monthly premium for the policy. [13]

12. CT5 April 2014 Q13

On 1 January 2003, an insurance company issued a 35 year with profit endowment assurance policy to a life aged 30 exact for a sum assured of £60,000. The sum assured (together with any bonuses attaching) is payable at maturity or immediately on death, if earlier. Level premiums are payable annually in advance throughout the policy term or until earlier death. Compound reversionary bonuses vest at the end of each policy year (i.e. the death benefit does not include any bonus relating to the policy year of death).

The company calculates the premium on the following basis:

Mortality AM92 Ultimate

Interest 6% per annum

Initial expenses £250 plus 60% of the first year's premium, incurred at outset

Renewal expenses 2.5% of the second and each subsequent year's premium, incurred at the beginning of the respective policy years

Bonuses: 1.92308% per annum

(i) Show that the annual premium is approximately £1,146. [8]

(ii) Express, in stochastic form, the gross future loss random variable for this policy at duration t , where t is an integer and $0 < t < 35$.

You should use T_x , K_x or both, together with the elements of the premium basis that are relevant.

Assume bonuses declared follow the assumptions stated in the premium basis.
[3]

On 31 December 2012, and just after the 10th bonus has been declared, the life wishes to surrender the policy. The insurance company calculates a surrender value equal to the gross prospective policy reserve, using the premium basis above.

(iii) Calculate the surrender value payable by the insurance company given that the total actual past bonus additions to the policy have followed the assumptions stated in the premium basis (including the bonus just vested). [6]
[Total 17]

13. CT5 April 2013 Q4

Describe the use of terminal bonus within the reversionary bonus system. [3]

14. CT5 April 2013 Q6

A life insurance company issues a 20-year increasing endowment assurance policy which provides a sum assured given by the formula:
£ $[50,000 + 1,500t]$ $t = 1, 2, \dots, 20$
where t denotes the policy year.

The sum assured is payable on maturity at age 50 exact or at the end of year of death if earlier. Premiums on the policy are payable annually in advance.

Write down an expression for:

- (a) the net premium for the policy.
 - (b) the net premium prospective policy reserve for the policy immediately before the tenth premium is paid.
- [4]

15. CT5 April 2013 Q13

A life insurance company issues 5,000 four-year decreasing term assurance policies on 1 January 2012 to a group of male lives aged 56 exact at that date.

Premiums are payable annually in advance on each policy. The initial annual gross premium P reduces to $.75P$, $.5P$ and $.25P$ at the beginning of the second, third and fourth policy year respectively.

The sum assured on each policy is payable at the end of year of death and is given by the formula:

$$£100,000 \times [1 - 0.25^t] \quad t = 0, 1, 2, 3$$

where t denotes the curtate duration in years since the inception of the policy.

- (i) Calculate the initial annual gross premium P for each policy using the basis below. [7]
- (ii) Determine the prospective gross premium reserve for each policy in force at the end of the first policy year using the same basis. [5]
- (iii) Calculate the mortality profit or loss for this portfolio of business for the calendar year 2012 given that 27 policyholders died during that year. [2]

Actual expenses incurred and interest earned by the company on this portfolio of business during 2012 was the same as that assumed in the premium basis.

- (iv) Derive the mortality profit or loss for the calendar year 2012 using the recursive relationship between the opening and closing prospective reserves in the first policy year. [2]

Basis:

Mortality AM92 Ultimate

Interest 6% per annum

Initial commission 25% of the first annual premium

Initial expenses £125

Renewal commission 3% of each annual premium excluding the first

Renewal expenses £35 per annum at the start of the second and subsequent policy years. The renewal expense is assumed to increase by

1.92308% compound per annum from inception of the policy.

[Total 16]

16. CT5 September 2013 Q24

A life insurance company issues a four-year with profits endowment assurance policy for a basic sum assured of £25,000 to a life aged 56 exact. Level premiums are payable annually in advance throughout the term of the policy.

Compound reversionary bonuses are added to the policy at the start of each year, including the first. The basic sum assured (together with any bonuses attaching) is payable at maturity or at the end of year of death, if earlier.

(i) Show that the annual premium is approximately £6,483 using the following premium basis:

Mortality AM92 Select

Interest 6% per annum

Initial expenses £100 plus 25% of the first premium (all incurred on policy commencement)

Renewal expenses 2.5% of the second and subsequent premiums plus £40 at the start of the second and subsequent policy years

Bonus rates A compound reversionary bonus will be declared each year at a rate of 1.92308% per annum

[5]

(ii) The insurance company holds net premium reserves using a rate of interest of 4% per annum and AM92 Ultimate mortality.

Calculate the expected profit margin on this policy using the following profit test basis:

Mortality 80% AM92 Select

Interest earned on funds 7.5% per annum

Initial expenses as per premium basis

Renewal expenses as per premium basis

Bonus rates as per premium basis

Risk discount rate 9.5% per annum

[13]

[Total 18]