



Subject: Pricing & Reserving
for Life Insurance
Products - 2

Chapter: Unit 3

Category: Practice
Questions

1. CT5 September 2016 Q13

A life insurance company issues a 3-year endowment assurance policy to an unmarried life that offers the following benefits:

- On marriage, a return of 107.5% of total premiums paid.
- On surrender, a return of 50% of total premiums paid.
- On death, a benefit which is given by the formula:
 $10,000 \times (1 + t)$ $t = 0, 1$ and 2
 where t denotes the curtate duration in years since the inception of the policy.
- On survival, 30,000 is payable immediately.

The marriage, surrender and death benefits are payable at the end of the policy year of claim.

Premiums of 9,516 are payable annually in advance throughout the term of the policy or until earlier claim.

The policy ceases on payment of any benefit.

The company uses the following basis to profit test this policy:

Independent force of marriage 15%

Independent force of surrender 7.5% in years 1 and 2 only

Independent force of mortality 1%

Interest earned on cash flows 3.5% per annum

Expenses 1.5% of each premium paid

Reserves None held

The company assumes that:

- each force of decrement is independent and constant over each year of age.
- surrenders only occur in policy years 1 and 2.

(i) Determine for each policy year the dependent rates of mortality, marriage and surrender. [4]

(ii) Derive the expected cash flows for the policy for each policy year. [7]

(iii) Calculate, from part (ii) the expected present value of the profit or loss to the company for each policy year and in total. Use a risk discount rate of 4% per annum. [2]

(iv) Discuss the consequences for the company of the profit vector derived in part (iii). [2]
 [Total 15]

2. CT5 September 2015 Q12

A life insurance company issues a two year unit-linked endowment assurance policy to a male life aged 45 exact. Level premiums of 6,000 per annum are payable yearly in advance throughout the term of the policy or until earlier death with 98% of each premium being allocated to units. A policy fee of 50 is deducted from the bid value of units at the start of each policy year. The units are subject to a bid-offer spread of 6%. An annual management charge of 1.25% of the bid value of units is deducted at the end of each policy year.

If the policyholder dies during the term of the policy, the death benefit of 200% of the bid value of the units is payable at the end of the policy year of death.

The policyholder may only surrender the policy at the end of the first policy year. On surrender, the bid value of units less a surrender value penalty of 500 is payable.

On maturity, 100% of the bid value of the units is payable.

Management charges are deducted from the unit fund before death, surrender and maturity benefits are paid.

The company uses the following assumptions in carrying out profit tests of this contract:
Rate of growth on assets in the unit fund 5.0% per annum in year 1

4.5% per annum in year 2

Rate of interest on non-unit fund cash flows 3.0% per annum in both years 1 & 2

Mortality AM92 Select

Surrenders 2.5% of all policies in force at the end of policy year 1

Initial expense 225

Renewal expense 80 on the second premium date

Initial commission 7.5% of first premium

Renewal commission 2.5% of the second premium

Death claim expense 90

Maturity claim expense 55

Risk discount rate 6% per annum

(i) Calculate, showing all your workings, the non-unit fund cash flows in the first and second years of the policy if the policyholder:

(a) dies in the first year of the policy.

(b) surrenders in the first year of the policy.

(c) dies in the second year of the policy.

(d) survives to the end of the policy.

[7]

(ii) Derive the expected present value of profit for the policy in the event that the policyholder:

- (a) dies in the first year of the policy.
 - (b) surrenders in the first year of the policy.
 - (c) dies in the second year of the policy.
 - (d) survives to the end of the policy.
- [5]

(iii) Calculate, showing all your workings, the expected present value of the profit for the policy. [1]
[Total 13]

3. CT5 September 2014 Q10

A life insurance company issues a non-profit assurance policy for a term of n years to a life aged x exact.

For $t = 1, 2, \dots, n$:

- The level annual premium payable at the start of year t is P .
- The expense at the start of policy year t is Et .
- The benefits payable at the end of the t th policy year on death, surrender and survival are Dt , Bt and St respectively.
- The rate of interest earned on net cash flows during the t th policy year is it .
- The dependent rates of mortality and surrender at age $x+t$ are $(aq)_{x+t}^d$ and $(aq)_{x+t}^w$ respectively.

Assume that the insurance company **does not** set up a reserve for the policy.

(i) Write down an expression for $(CF)_t$, the accumulation to the end of the t th policy year of the expected net cash flow arising during the t th policy year per policy in force at the start of that year. [2]

(ii) Derive an expression which could be used to calculate the level annual premium that the company should charge if the company requires the expected net present value of profit on the policy to be zero assuming a risk discount rate of $j\%$ per annum defining any notation used. [3]

Assume that the insurance company **does** set up a reserve ${}_{t-1}V$ for the policy at the start of the t th policy year.

(iii) Write down an expression for the expected profit at the end of the t th policy year for each policy in force at the start of that year. [2]
[Total 7]

4. CT5 September 2013 Q15

A three-year unit-linked endowment assurance policy is sold to a male life aged 40 exact. The profit signature for this policy, calculated using AM92 Select mortality and making no allowance for surrenders, is:
 (-209.80, 253.55, 109.85)

It is now assumed for the cash flows for this policy that 15% of all policies in force at the end of the first policy year are surrendered at that time. The surrender value payable at that time is the bid value of units at the end of the policy year less a surrender penalty of £500. There are no other changes to the policy.

- (a) Calculate the revised profit signature in the first policy year.
- (b) Comment on the impact on the profit signature in the second and third policy years.
- [4]

5. CT5 September 2012 Q14

A life insurance company issues a four-year policy to a male life aged 30 exact that offers the following benefits:

- On death during the term of the policy or on survival to the end of the term, a sum of £60,000.
- On redundancy during the term of the policy, a return of 100% of total premiums paid.
- On surrender during the term of the policy, a return of 50% of total premiums paid.

Premiums of £14,000 are payable annually in advance throughout the term of the policy or until earlier claim. The death, surrender and redundancy benefits are payable immediately on claim. The contract ceases on payment of any claim.

The company uses the following basis to profit test this contract:

Interest earned on cash flows 3% per annum

Expenses 5% of each premium paid

Reserves Ignore

The company has also calculated the following dependent rates of mortality, surrender and redundancy which are used to profit test this contract:

Year t	$(aq)_{[30]+t-1}^d$	$(aq)_{30+t-1}^s$	$(aq)_{30+t-1}^r$
1	.000447	.098727	.023744
2	.000548	.049361	.024368
3	.000602	.024680	.024680
4	.000636	0	0

Calculate the expected profit margin to the company on this policy using a risk discount rate of 5% per annum. [10]

6. CT5 September 2012 Q15

A life insurance company issues a three-year unit-linked endowment assurance policy to a male life aged 45 exact. The main features of the contract are:

- Premiums: £3,000 per annum are payable yearly in advance throughout the term of the policy or until earlier death
- Allocation rates: 75% of premium is allocated to units in the first policy year, 100% in the second and 105% in the third
- Policy fee: £35 is deducted from the bid value of units at the start of each policy year
- Death benefit: 150% of the bid value of the units is payable at the end of the policy year of death
- Maturity benefit: 100% of the bid value of the units is payable
- Bid-offer spread: 5%
- Annual management charge: 1.5% of the bid value of units is deducted at the end of each policy year (management charges are deducted from the unit fund before death and maturity benefits are paid).

The company uses the following assumptions in carrying out profit tests of this contract:

Rate of growth on assets in the unit fund 5.0% per annum in year 1

4.5% per annum in year 2

4.0% per annum in year 3

Rate of interest on non-unit fund cash flows 3.0% per annum

Mortality AM92 Select

Withdrawals None

Initial expenses £275

Renewal expenses £80 per annum on the second and subsequent premium dates

Initial commission 20% of first premium

Renewal commission 2.5% of the second and subsequent years' premiums

Rate of expense inflation 2.0% per annum

Risk discount rate 6.5% per annum

For renewal expenses, the amount quoted is at outset, and the increases due to inflation start immediately.

(i) Calculate the non-unit fund cash flows in each year of the contract and hence the expected present value of profit assuming that the policyholder dies in the third year of the contract. [9]

(ii) Calculate the expected present value of profit for the policy if the policyholder dies in the:

(a) first year of the contract.

(b) second year of the contract.

[4]

(iii) Hence calculate the expected present value of the contract allowing for the possibility that the policyholder survives to the end of the contract. [2]

[Total 15]

7. CT5 April 2011 Q11

A life insurance company issues a 4-year unit-linked endowment policy to a life aged 61 exact under which level premiums of £2,500 are payable yearly in advance throughout the term of the policy or until earlier death. In the first policy year 40% of the premium is allocated to units, while in the second and subsequent policy years 110% of the premium is allocated to units. The unit prices are subject to a bid-offer spread of 5%.

If the policyholder dies during the term of the policy, the death benefit of £10,000 or the bid value of the units, whichever is higher, is payable at the end of the policy year of death.

The policyholder may surrender the policy, in which case a value equal to a fixed percentage of the total premiums paid on the policy is payable at the end of the policy year of surrender. The percentage is based on the policy year of surrender as follows:

*Policy year % of total premiums payable
as a surrender value*

1	0
2	25
3	50
4	75

On maturity, 105% of the bid value of units is payable.

An annual management charge of 0.5% of the bid value of units is deducted at the end of each policy year before death, surrender and maturity benefits are paid.

The company uses the following assumptions in carrying out profit tests of this contract:

Rate of growth on assets in the unit fund 4.25% per annum

Rate of interest on non-unit fund cash-flows 3.5% per annum

Independent rate of mortality AM92 Select

Independent rate of surrender 6% per annum

Initial expenses £325

Renewal expenses £74 per annum on the second and subsequent premium dates

Initial commission 10% of first premium

Renewal commission 2.5% of the second and subsequent years' premiums

Risk discount rate 5.5% per annum

(i) Construct a multiple decrement table for this policy assuming that there is a uniform distribution of both decrements over each year of age in the single decrement table. [3]

(ii) Construct tables showing the growth of the unit fund and the non-unit fund. Include all commissions in the non-unit fund. [7]

(iii) Calculate the profit margin for this policy on the assumption that the company does not zeroise future expected negative cashflows. [3]

[Total 13]

8. CT5 April 2007 Q14

A life office issues a 4-year non-profit endowment assurance policy to a male life aged 61 exact for a sum assured of £100,000 payable on survival to the end of the term or at the end of the year of death if earlier. Premiums are payable annually in advance throughout the term of the policy.

There is a surrender benefit payable equal to a return of premiums paid, with no interest. This benefit is payable at the end of the year of surrender.

The life office uses the following assumptions to price this contract:

Mortality AM92 Select

Surrenders None

Interest 4% per annum

Initial expenses £500

Renewal expenses (on the second and subsequent premium dates) £50 per annum plus 2.5% of the premium

In addition, the company holds net premium reserves, calculated using AM92 Ultimate mortality and interest of 4% per annum.

In order to profit test this contract, the life office assumes the same mortality and expense assumptions as per the pricing basis above. In addition, it assumes it earns 5% per annum on funds and that 5% of all policies still in force at the end of 1, 2, and 3 years then surrender.

Calculate, using a risk discount rate of 8% per annum, the expected profit margin on this contract. [18]

9. CT5 April 2006 Q14

A life insurance company issues a 3-year unit linked endowment policy to a life aged 45 exact under which level premiums are payable yearly in advance. In the 1st year, 35% of the premium is allocated to units and 105% in the 2nd and 3rd years. The units are subject to a bid-offer spread of 5% and an annual management charge of 0.5% of the bid value of units is deducted at the end of each policy year. Management charges are deducted from the unit fund before death and surrender benefits are paid.

If the policyholder dies during the term of the policy, the death benefit of the bid value of the units is payable at the end of the year of death. The policyholder may surrender the policy only at the end of each year. On surrender or on survival to the end of the term, the bid value of the units is payable at the end of the year of exit.

The company uses the following assumptions in its profit test of this contract:

Rate of growth on assets in the unit fund 5% per annum

Rate of interest on non-unit fund cash flows 4% per annum

Independent rates of mortality AM92 Ultimate

Independent rates of withdrawal 5% per annum

Initial expenses £250

Renewal expenses £50 per annum on the 2nd and 3rd premium dates

Initial commission 20% of 1st premium

Renewal commission 2.5% of the 2nd and 3rd years premiums

The company sets premiums so that the net present value of the profit on the policy is 15% of the annual premium.

(i) Using a risk discount rate of 8% per annum, calculate the premium for the policy on the assumption that the company does not zeroise future expected negative cash flows. [12]

10. CT5 April 2005 Q1

Explain the difference between a profit vector and a profit signature. [2]