INSTITUTE OF ACTUARIAL & QUANTITATIVE STUDIES

Subject: Portfolio Theory & Security Analysis

Chapter: Unit 1 & 2

Category: Assignment Solution

EMH

1.
a. Mutual funds are good for investors as they can consistently give higher returns than Nifty Index returns due to their expertise in analysing companies' earnings, Balance sheet, etc. The market is either Weak form or inefficient. Under Semi-strong & Strong market, it's difficult to outperform the market using fundamental analysis in the long run.

b. A stock trader has made super normal profit for past 20 years using Bollinger-band technique (a well- known technical tool) The market is not even Weak form (hence, Inefficient) as technical analysis has given super normal profits.

c. A trust-worthy newspaper reports that XYZ Mining Corp has larger than expected coal reserve which would increase the Market capitalization by 5% and the share price increased by 5%. The market is Semi-strong as it has immediately reacted to the information coming to public.

d. A Television reporter traded in a stock and made money on the basis of information in his 'Interview' with the management (before it was broadcast) The market is not strong form and can be any of the other form as the reporter was able to make money on the basis of insider information.

2.
i) The strong form of EMH suggests that the market prices incorporate all information, both publically available and also that is available only to insiders. If this exists, then even with insider information, the investors won't be able to generate higher returns. Hence, any such rules pertaining to company employees and management over ban in stock trading would be unnecessary in a strong form of market.

ii) The semi strong form of market is said to be in existence if the market prices incorporate all publically available information. The technical analysis relies on making trading rules based on historical price data to generate higher investment returns. Since in semi strong form of market, the prices already incorporate all public information, technical analysis won't help investors generating any additional returns

iii) In semi strong form of market, prices already incorporate all publically available information. However, the extent of public information might vary from investor to investor. For instance, different stock exchanges having different disclosure requirements are expected to have different levels of public information and hence, efficiency. In addition, there could be additional costs involved in obtaining the public information accurately and quickly which otherwise would dilute the market efficiency.

Utility theory

1.

i) Utility functions having property of constant relative risk aversion are said to be iso-elastic functions.

ii)

$$U(x) = (x^{5\alpha} - 1)/10\alpha$$
$$U'(x) = 5\alpha * x^{5\alpha - 1} / 10\alpha$$
$$= 0.5 * x^{5\alpha - 1}$$

$$U''(x) = 0.5*(5\alpha - 1)* x^{5\alpha - 2}$$

Thus for the function to satisfy the principle of non-satiation and diminishing marginal utility of wealth, we require $\alpha < 1/5$

The absolute risk aversion is given by

$$A(x) = -U''(x) / U'(x) = -(5\alpha - 1) / x$$

$$A'(x) = (5\alpha - 1) / x^2 < 0$$

The relative risk aversion is given by

$$R(x) = x*-U''(x) / U'(x)$$
$$= -(5\alpha - 1)$$

$$R'(x) = 0$$

Thus the function exhibits the property of declining absolute risk aversion and constant relative risk aversion.
Hence, the function is iso-elastic.

iii)

The utility function of the individual is given by

$$U(x) = Log(x)$$

Let p be the insurance premium that the individual is willing to pay to protect against the Random Loss of x.
Therefore, we can write
$E[U(a - x)] = U(a - p)$, where a is the initial wealth of the individual.

We have,

X = 500, a = 1000

$E[U(a - x)]= 0.5*Log(1000-500) + 0.5*Log1000$

$=0.5*6.21 + 0.5 *6.91$

$=6.56$

Using the above equation, we have
$Log(1000 - p) = 6.56$

Or, $(1000 - p) = 707.11$

Or, $p = 292.89$

Insurance premium for the individual is 292.89.

2.

a) Your friend is more risk averse than yours we have a lower risk aversion coefficient . for you

$U(w) = \frac{w^{\gamma}-1}{\gamma}$

$U'(w) = w^{\gamma-1}$

$U''(w) = (\gamma-1)\, w^{\gamma-2}$

Substituting $\gamma = 1$

$R(w) = A(w) = 0$ i.e.

For your friend $\gamma = 0.5$

$A(w) = \frac{1}{2w} > 0$

$R(w) = \frac{1}{2} > 0$

Hence your friend is strictly risk averse for all w>0

b) If your friend buys X then the expected utility
0.5(2((110)^0.5 -1) +2((92)^0.5-1)) = 18.08

However if he doesn't buy then expected utility is
2((100)^0.5 -1) = 18
Thus buying X has higher utility and hence he should buy X

3.

i) Marginal utility of X i.e MUx $= \frac{\partial\, U(X,Y)}{\partial X}$

$0.4\,X^{-0.6}\;Y^{0.6}$

Marginal utility of Y i.e MUy $= \frac{\partial\, U(X,Y)}{\partial Y}$

$0.6\,X^{0.4}\;Y^{-0.4}$

ii) $\frac{MUx}{Px} = \frac{MUx}{Py}$

$(0.4\,X^{-0.6}\;Y^{0.6})/2 = (0.6\,X^{0.4}\;Y^{-0.4})/6$

0.2Y = 0.1X
X = 2Y

iii) The consumer's budget constraint is
60 = 2X + 6Y
Substituting in the consumption for X in terms of Y we get

60 = 10Y ==> Y = 6 and X = 12

iv) Suppose the consumer has total 100

Consider the scenario where an individual X has initial wealth of 100. He is given a gamble where there is 45% chance that he will lose 50 and 55% chance that he will gain 50.

Let utility function of the X be U (W) = $\sqrt{W}$

Suppose X takes the gamble. Then, expected wealth of X is
E(W) = 55% (150) + 45% (50) = 102.5
However, expected utility of X is
E(U(W)) = 55% ($\sqrt{150}$) + 45% ($\sqrt{50}$) = 9.92
Initial expected utility of X = 10 So, if X were to maximise expected wealth, he should take the gamble, but if he were to maximise expected utility, he should not take the gamble.

Measures of risk

1.

i) Returns and solving for L and D:

For Luckworth's offer, returns are:

- \$(100-L) if coin toss outcome predicted correctly (0.5 probability)
- -\$L (i.e. loss of \$L) if coin toss outcome predicted incorrectly (0.5 probability)

This being a fair gamble, the expected return should be zero. Therefore, 0.5 * (100-L) + 0.5 * (-L) = 0.

Solving for L yields L = \$50

For Dewis's offer, returns are: \$(100s-D) where s is a random number generated uniformly from the interval [0,1].

This being a fair gamble, the expected return should be zero. E(100s-D) = 0 => 100*E(s) – D = 0.

E(s) = 0.5 as s is generated uniformly from [0,1]. Solving for D yields D = \$50.

ii) Risk measures:

a. Variance

For both L and D, Variance = $E[(X-\mu)^2] = E(X^2)$ since the mean return is zero (as both gambles are fair)

For L, variance = $0.5*50^2 + 0.5*(-50)^2 = 2500$

For D, variance = $\int_0^1 (100s-50)^2 ds$ = 2500/3 = 833.33 (approx)

b. Semi-variance

Downside semi-variance for a random variable X with mean μ is defined as:

$$\int_{-\infty}^{\mu} (\mu - x)^2 f(x)dx$$ for a continuous random variable (such as return for D)

$$\sum_{x<\mu} (\mu - x)^2 P(X = x)$$ for a discrete random variable (such as return for L)

For L, downside semi-variance = $0.5*(-50)^2 = 1250$

For D, variance = $\int_0^{0.5} (100s-50)^2 ds$ = 1250/3 = 416.67 (approx)

c. 90% Value-at-Risk (VaR):

90% VaR indicates the maximum level of loss with a 90% confidence, i.e. there is a 10% probability of a greater loss.

For L, since there is a 50% probability each of a $50 loss and a $50 profit, the 90% VaR will be $50.

For D, VaR = -t such that P(100s-50<t) = 0.1 => P(s<(t+50)/100) = 0.1 => (t+50)/100 = 0.1 => t = -40. Therefore, 90% VaR is $40.

d. Expected Shortfall (ES) at -$10 threshold:

For a random variable X and a threshold level K, Expected Shortfall = E[max(K-X,0)]

For L, since there is a 50% probability each of a $50 loss and a $50 profit, the ES will be 0.5*(-10-(-50)) = $20

For D, loss threshold K = -$10 corresponds to s = 0.4 (obtained by solving 100s – 50 = -10). Therefore, ES will be $\int_0^{0.4}(-10-(100s-50))ds = \$8$.

iii) Comments:

- Using all risk metrics, Luckworth's gamble is consistently riskier than Dewis's from the gambler's perspective even though both have the same expected return (i.e. $0) and range of returns (i.e. -$50 to $50).
- This is because Luckworth's gamble only allows for two extreme outcomes ($50 profit and loss with equal probabilities), while Dewis's gamble allows for a spectrum of intermediate outcomes
- Since both the gambles have symmetric payoffs around zero, the downside semi-variance is exactly half of the total variance in both cases

iv) Risk preferences:

- Both gambles are fair but risky. Since Steven doesn't choose either gamble, he should be risk-averse.
- Frank and Tony have opted for one of these gambles, so both of them should be risk-seeking.
- Since Frank has gone for the more risky option (Luckworth's gamble) of the two, he should be more risk-seeking than Tony.

2.

a) We have to find P (X < t) = 0.05

$X \sim N(5, 6^2)$

Standardizing gives

$P(Z < (t-5)/6) = \Phi((t-5)/6) = 0.05$

From the tables $\Phi(-1.65) = 0.05$

So t = -4.9

Since t is a percentage investment return per annum, the 95% value at risk over one year on a 100 cr portfolio is 100 x 4.9% = 4.9 cr. This means that, we are 95% certain that we will not lose more than 4.9 cr over the next year

b)

The mean return

E(x) = -7*0.04+5.5*0.96 = 5 i.e. 5% p.a

Var (X) = (5-(-7))^2 *0.04 + (5-5.5)^2 * 0.96 = 6%

We first find t where t = max (x:P(X<x)<= 0.05)

P(X<-7) = 0 and P(X<5.5) = 0.04

Therefore t = 5.5

The expected shortfall in returns below 5.5%is given by

$E(\max(5.5 - X), 0) = \sum_{x<5.5}(5.5 - x)P(X = x)$

=12.5 * 0.04 = 0.5

On a portfolio of 10 cr the 95% Tail VaR is 10 * 0.005 = 0.05 cr. This means the expected reduction in profit below INR 5.5 cr is INR 500,000/-

3.

i) The risk neutral probability for Gold, p_{gold} =

$1 = (1 - p_{gold}) * 1.06/1.04 + p_{gold} * 0$

$p_{gold} = 1.8868\%$

similarly, $p_{silver} = 3.7037\%$

ii) Now, **<u>for Investor X</u>**, 95% VaR (Value at Risk) is zero.

95% Tail VaR = $1.06\ p_{gold}/\ p_{gold} = 1.06$

For Investor Y, again, 95% VaR is zero.

95% Tail VaR = $1.08\ p_{silver}\ /\ p_{silver} = 1.08$

For Investor Z, either one can default or both can default and both cannot default. So the distribution of returns would be –

1.07 with probability $(1 - p_{gold})\ (1 - p_{silver}) = 0.94479$

0.54 with probability $p_{gold}\ (1 - p_{silver}) = 0.01817$

0.53 with probability $(1- p_{gold})\ p_{silver} = 0.03634$

0 with probability $p_{gold}\ p_{silver} = 0.00070$

So 95% VaR is 1.07 – 0.54 = 0.53

The 95% Tail VaR is

$$\frac{1.07 * p_{silver} * p_{silver} + 0.54\ (1- p_{silver})\ p_{silver}}{p_{silver}} = 0.55$$

iii)

Comments on results: Usually, investment in diversified portfolio leads to a lower dispersion of returns and hence lower risk. In the example above, Investor Z invested in a diversified portfolio compared to X and Y but his VaR is higher than for either X or Y. So the increase in VaR could correspond to a decrease in risk under such circumstances.
Further, zero VaR does not necessarily mean zero risk. As expected, the tail VaR for Investor Z is lower than the Investor X and Y.

Comments on appropriateness of VaR and Tail VaR as measures of investment risk:

VaR represents the maximum potential loss on a portfolio over a given future time period with a given degree of confidence. It is often calculated assuming that investment returns follow a normal distribution, which may not be an appropriate assumption. The usefulness of VaR in case of non-normal distributions depend on modelling skewed or fat-tailed distribution of returns. The further one gets into the “tails” of the distributions, the more lacking the data and hence, the more arbitrary the choice of the underlying probability distribution becomes.

TailVaR measures the expected loss in excess of the VaR, hence, relative to VaR, it provides much more information on how bad returns can be when benchmark level is exceeded. It has the same modeling issues as VaR in terms of sparse data, but captures more information on tail of the non-normal distribution.

4.

a) Variance = $\int_{-\infty}^{\infty} (\mu - x)^2 \ f(x)\,dx$

In the given case, Rx ~ exp(λ)

Given, Mean = 4, i.e. $1/\lambda = 4$

Hence, $\lambda = 0.25$

Variance = $1/\lambda^2 = 16$

b) Downside Semi Variance = $\int_{-\infty}^{\mu} (\mu - x)^2 \ f(x)\,dx$

(b)

Downside Semi Variance : $\int_{-\infty}^{4} (4-x)^2 f(x)\, dx$

$$= \int_0^4 (16 + x^2 - 8x)\, \lambda e^{-\lambda x}\, dx$$

Now, $\int_b^a x e^{-\lambda x}\, dx = \left. \frac{x e^{-\lambda x}}{-\lambda} - \frac{e^{-\lambda x}}{\lambda^2} \right|_b^a$

$\therefore$,

$$DSV = \left[-16 e^{-\lambda x} - 8\lambda \left(\frac{x e^{-\lambda x}}{-\lambda} - \frac{e^{-\lambda x}}{\lambda^2} \right) + \lambda \left(\frac{x^2 e^{-\lambda x}}{-\lambda} - \int \frac{2x e^{-\lambda x}}{-\lambda}\, dx \right) \right]_0^4$$

$$= \left[-16 e^{-\lambda x} + 8x e^{-\lambda x} + \frac{8 e^{-\lambda x}}{\lambda} - x^2 e^{-\lambda x} + 2\left(\frac{x e^{-\lambda x}}{-\lambda} - \frac{e^{-\lambda x}}{\lambda^2} \right) \right]_0^4$$

$$= 16 - 32 e^{-1}$$

(c) Shortfall probability

$$\int_0^k \lambda e^{-\lambda x}\, dx = \left[-e^{-\lambda x} \right]_0^k$$

$$= 0.39347$$

(d) Expected Shortfall

$$\int_0^k x \lambda e^{-\lambda x}\, dx = \left[-x e^{-\lambda x} - \frac{e^{-\lambda x}}{\lambda} \right]_0^k$$

$$= -k e^{-\lambda k} - \frac{e^{-\lambda k}}{\lambda} + \frac{e^{-0}}{\lambda}$$

Shortfall given default prob,

$$= k - \frac{\frac{1}{\lambda} - e^{-\lambda k}\left(k + \frac{1}{\lambda}\right)}{1 - e^{-\lambda k}}$$

$$= 2 - \frac{4 - 6e^{-0.5}}{1 - e^{-0.5}} = 1.083$$

Portfolio management overview

1.

a. The expected cash flow is: (0.5 × \$70,000) + (0.5 × 200,000) = \$135,000

With a risk premium of 8% over the risk-free rate of 6%, the required rate of return is 14%. Therefore, the present value of the portfolio is:

\$135,000/1.14 = \$118,421

b. If the portfolio is purchased for \$118,421, and provides an expected cash inflow of \$135,000, then the expected rate of return [E(r)] is derived as follows:

\$118,421 × [1 + E(r)] = \$135,000

Therefore, E(r) = 14%. The portfolio price is set to equate the expected rate or return with the required rate of return.

c. If the risk premium over T-bills is now 12%, then the required return is:

6% + 12% = 18%

The present value of the portfolio is now:

\$135,000/1.18 = \$114,407

2.

Expected return = (0.7 × 18%) + (0.3 × 8%) = 15%

Standard deviation = 0.7 × 28% = 19.6%

3.

Expected return for equity fund = T-bill rate + risk premium = 6% + 10% = 16%

Expected return of client's overall portfolio = (0.6 × 16%) + (0.4 × 6%) = 12%

Standard deviation of client's overall portfolio = 0.6 × 14% = 8.4%

4. Since we do not have any information about expected returns, we focus exclusively on reducing variability. Stocks A and C have equal standard deviations, but the correlation of Stock B with Stock C (0.10) is less than that of Stock A with Stock B (0.90). Therefore, a portfolio comprised of Stocks B and C will have lower total risk than a portfolio comprised of Stocks A and B.

5. Subscript OP refers to the original portfolio, ABC to the new stock, and NP to the new portfolio

i. $E(r_{NP}) = w_{OP}\, E(r_{OP}) + w_{ABC}\, E(r_{ABC}) = (0.9 \times 0.67) + (0.1 \times 1.25) = 0.728\%$

ii. $Cov = r \times \sigma_{OP} \times \sigma_{ABC} = 0.40 \times 2.37 \times 2.95 = 2.7966 \cong 2.80$

iii. $\sigma_{NP} = [w_{OP}^{2}\, \sigma_{OP}^{2} + w_{ABC}^{2}\, \sigma_{ABC}^{2} + 2\, w_{OP}\, w_{ABC}\, (Cov_{OP\,,\,ABC})]^{1/2}$

$= [(0.9^{2} \times 2.37^{2}) + (0.1^{2} \times 2.95^{2}) + (2 \times 0.9 \times 0.1 \times 2.80)]^{1/2}$

$= 2.2673\% \cong 2.27\%$

6.

$\sigma_P = 30 = y\sigma = 40y \Rightarrow y = 0.75$

$E(r_P) = 12 + 0.75(30 - 12) = 25.5\%$