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Subject: Portfolio Theory & Security Analysis

Chapter: 1,2,3 (Unit 1)

Category: Practice Questions

1. Subject CT8 April 2013 Question 1

List the key advantages and disadvantages of the following measures of investment risk in the context of a portfolio of bonds subject to credit risk:

- Variance of return
- Downside semi-variance of return
- Shortfall probability
- Value at Risk
- Tail Value at Risk

2. Subject CT8 April 2015 Question 1

(i) State in words the four axioms of the Expected Utility Theorem.

(ii) State the conditions for an investor to be non-satiated and risk neutral in terms of their utility function, $U(w)$.

An investor makes investment decisions using utility function $U(w)$ = (w▪ − 1) / ▪ .

(iii) Derive the relative risk aversion function for $U(w)$.

(iv) Describe how the relative risk aversion of $U(w)$ changes with w.

3. Subject CT8 April 2015 Question 2

Consider an asset the annual return, X, on which has probability density function $f(x)$.

(i) Define the 5% Value at Risk for this asset.
(ii) Define the expected shortfall of the return on this asset below 2%.

Assume X has a Normal distribution with mean μ = 5% and variance ▪ 2 = 100%%.

(iii) Calculate the 5% Value at Risk.
(iv) Discuss the limitations of using Value at Risk to measure the downside risk in an investment portfolio.

4. Subject CT8 April 2016 Question 1

An investor measures the utility of her wealth using the utility function $U(w) = \ln(w)$ for $w > 0$.

(ii) Derive the absolute and relative risk aversions for this investor's utility function, and the first derivative of each.

(iii) Comment on what this tells us about the proportion of her assets that this investor will invest in risky assets.

The investor has £100 available to invest in two possible assets, Asset A and Asset B. The future value of Asset A depends on an uncertain future event.

- Every £1 invested in Asset A will be worth £1.30 with probability 0.75 and £0.40 with probability 0.25.
- Asset B is risk-free, so every £1 invested in Asset B will always be worth £1.

The investor does not discount future asset values when making investment decisions. She decides to invest a proportion a of her wealth in Asset A and the remaining proportion $1 - a$ in Asset B.

(iv) Express her expected utility of wealth in terms of a.

(v) Determine the amount that she should invest in each of Asset A and B to maximise her expected utility, using your result from part (iii).

5. Subject CT8 April 2016 Question 2

Consider an asset whose return follows the probability density function $f(x)$.

(i) Write down a formula for the variance of the return on the asset, defining any additional notation you use.

(ii) Write down a formula for the shortfall probability for the return on the asset below a level L.

The returns on an asset follow a Normal distribution with mean $\mu = 6\%$ per annum and variance $\sigma^2 = 23\%$ per annum. An investor buys €500 of the asset.

(iii) Determine the shortfall probability for the value of the asset in one year's time below a value of €480.

(iv) Explain what can be deduced about an investor's utility function if the investor makes decisions based on:

a. the variance of returns.
b. the shortfall probability of returns.

6. Subject CT8 April 2016 Question 5

(i) Define the three forms of the Efficient Markets Hypothesis
(ii) State two reasons why it is hard to test whether any of the three forms hold in practice.

7. Subject CT8 April 2017 Question 1

(i) State the expected utility theorem.

A risk averse investor makes decisions using a quadratic utility function:

$$U(w) = w + dw^2.$$

(ii) Derive an upper bound for d for this investor.

(iii) Explain why the investor can only use this utility function to make decisions over a limited range of wealth, w. Your answer should include a statement of this range.

The investor states that the upper limit of wealth where she can use this utility function is w = $1,000.

(iv) Determine the value of d in the investor's utility function.

The investor wins a prize of $250 in a gameshow. She is then offered the opportunity to exchange this prize for a larger prize of $600 if she can answer one more question correctly. However, she will receive no prize at all if she gets the question wrong.
She estimates her chances of answering the question correctly to be 50%.

(v) Determine whether the investor should take this opportunity to exchange.

8. Subject CT8 April 2008 Question 5

An investor is considering investing in one of two assets. The distribution of returns from each asset is shown below:

Asset 1		*Asset 2*	
Return (%)	*Probability (%)*	*Return (%)*	*Probability (%)*
-1	$8^1/_3$	0	50
11	$91^2/_3$	20	50

i. Calculate for each asset:

(a) the variance
(b) semi-variance
(c) and shortfall probability

Where necessary assume a benchmark return of 0%.

ii. Explain which asset an investor with a quadratic utility function would choose.

iii. State the reasons why variance of return is frequently used as a measure of risk.

9. Subject CT8 April 2009 Question 2

One of your colleagues says that the stock market is not efficient because some accounting ratios have been shown to have predictive powers.

(i) Explain which of the main forms of efficiency is most relevant to this situation.

(ii) Comment on whether you agree with your colleague.

(iii) Explain the difference between active and passive fund management in terms of the concept of market efficiency.

10. Subject CT8 April 2010 Question 8

Outline the main points you would make in a discussion of the statement:

The efficient markets hypothesis states that the market price is always correct and therefore it is not possible for investors to make money from investing in shares.

11. Subject CT8 April 2010 Question 9

An asset is worth 100 at the start of the year and is funded by a senior loan and a junior loan of 50 each. The loans are due to be repaid at the end of the year; the senior one with interest at 6% p.a. and the junior one with interest of at 8% p.a. Interest is paid on the loans only if the asset sustains no losses.

Any losses of up to 50 sustained by the asset reduce the amount returned to the investor in the junior loan by the amount of the loss. Any losses of more than 50 mean that the investor in the junior loan gets 0 and the amount returned to the investor in the senior loan is reduced by the excess of the loss over 50.

The probability that the asset sustains a loss is 0.25. The size of a loss, L, if there is one, follows a uniform distribution between 0 and 100.

(i) Calculate the variances of return for the investors in the junior and senior loans.

(ii) Calculate the shortfall probabilities for the investors in the junior and senior loans, using the full return of the amounts of the loans as the respective benchmarks.

12. Subject CT8 April 2011 Question 4

(i) Outline the three forms of the efficient market hypothesis.

XYZ has just announced that its profits are up by 52% on last year. On the announcement XYZ shares fell in price by 20%. Analysts had been predicting a rise in profits of 65%. A friend says that this shows that the efficient markets hypothesis is false.

(ii) Comment on this statement.

13. Subject CT8 April 2012 Question 5

Let X be a random variable denoting the rate of return on the fund ABC. The distribution of X is $N\left(\mu, \sigma^2\right)$.

(i) Define $VaR_\alpha(X)$ with $\alpha \in [0,1]$.

(ii) Show that:

$$VaR_\alpha = -\left(\mu + \sigma\Phi^{-1}(\alpha)\right)$$

where Φ denotes the cumulative Normal distribution function.

(Hint: Consider the probability that X is less than VaR_α).

(iii) Derive an expression for $TailVaR_\alpha(X)$ given that:

$$TailVaR_\alpha = \frac{1}{\alpha}\mathbb{E}\left(X \mid X < VaR_\alpha\right).$$

An investor holds £350m invested in ABC, the expected return on the fund is 10% and the standard deviation of that return is 25%.

(iv) Calculate the VaR and TailVaR of this investment when $\alpha = 0.01$.

14. Subject CT8 September 2008 Question 1

Two assets are available for investment. Asset 1 returns a percentage 4*B*%, where *B* is a Binomial random variable with parameters $n = 3$ and $p = 0.5$. Asset 2 returns a percentage 2*P*%, where *P* is a Poisson random variable with parameter $\mu = 3$. Assume a benchmark return of 3%.

(i) Calculate the following three measures of investment risk for each asset:

(a) variance
(b) semi-variance and
(c) shortfall probability

An investor has £1,000 to invest in one of the assets.

(ii) Explain which asset the investor should choose assuming a utility function of the form:

$$u(x) = \begin{cases} -(1{,}060 - x)^2 & : x < 1{,}060; \\ 0 & : x > 1{,}060. \end{cases}$$

15. Subject CT8 September 2008 Question 4

One of your colleagues tells you that the work of Shiller conclusively proves that the stock-market overreacts.

(i) Outline the nature of Shiller's findings.

(ii) Discuss whether you agree with your colleague.

16. Subject CT8 September 2009 Question 3

A small bank wishes to improve the performance of its investments by investing £1m in high returning assets. An investment bank has offered the bank two possible investments:

Investment A: A diversified portfolio of shares and derivatives which can be assumed to produce a return of £R_1 million where $R_1 = 0.1 + N$, where N is a normal $N(1,1)$ random variable.

Investment B: An over-the-counter derivative which will produce a return of £R_2 million where the investment bank estimates:

$$R_2 = \begin{cases} 1.5 \text{ with probability } 0.99 \\ -5.0 \text{ with probability } 0.01. \end{cases}$$

The chief executive of the bank says that if one investment has a better expected return and a lower variance than the other then it is the best choice.

(i) (a) Calculate the expected return and variance of each investment A and B.

(b) Discuss the chief executive's comments in the light of your calculations.

(ii) Calculate the following risk measures for each of the two investments A and B:

(a) semi-variance of return
(b) shortfall probability of the returns falling below 0
(c) shortfall probability of the returns falling below 2

(iii) (a) Define other suitable risk measures that could be calculated.
(b) Discuss what the risk measures in (iii) (a) would show.

(iv) Compare the merits of the two investments A and B.

17. Subject CT8 September 2010 Question 1

An investor holds an asset that produces a random rate of return, *R*, over the course of a year. The distribution of this rate of return is a mixture of normal distributions,

i.e. *R* has a normal distribution with a mean of 0% and standard deviation of 10% with probability 0.8 and a normal distribution with a mean of 30% and a standard deviation of 10% with a probability of 0.2.
S is the normally distributed random rate of return on another asset that has the same mean and variance as *R*.

(i) Calculate the mean and variance of *R*.

(ii) Calculate the shortfall probabilities for *R* and for *S* using:
(a) a benchmark rate of return of 0%
(b) a benchmark rate of return of–10%

(iii) Comment on what the variance and shortfall probabilities at both benchmark levels illustrate about the asset returns, by referring to the calculations in (i) and (ii).

18. Subject CT8 September 2012 Question 10

Let A and B be two investment portfolios taking values in [a,b] with cumulative probability distribution functions of returns F_A and F_B respectively, and let the investor's smooth utility function be U.

(i) Write down the equation that the function U satisfies if the investor prefers more to less.

(ii) Explain what it would mean for portfolio A to first order stochastically dominate portfolio B.

(iii) Prove, by considering the expected utility of investments in either A or B, that if portfolio first order stochastically dominates portfolio B, then the investor prefers A to B.

19. Subject CT8 September 2013 Question 1

(i) (a) State the expected utility theorem.
(b) State the four axioms from which it can be derived.

(ii) Explain of the concepts of non-satiation and risk aversion, showing how they can be expressed in terms of a utility function.

A quadratic utility function is given by the equation $U(w) = w + bw^2$ The value of absolute risk aversion at a value of wealth of one unit is 0.25

(iii) Calculate the value of b and the range over which $U(.)$ satisfies the condition of non-satiation.

20. Subject CT8 September 2013 Question 3

(i) Outline the three forms of the Efficient Markets Hypothesis (EMH).

(ii) Discuss the following two scenarios in the light of the EMH:

Scenario 1: Company A's share price falls suddenly, immediately after news of an earthquake in the capital city of one of its major markets.

Scenario 2: Company B's share price falls suddenly, when a long-awaited and publicly negotiated merger is completed.

21. Subject CT8 September 2014 Question 1

Outline consumer choice theory.

22. Subject CT8 September 2015 Question 1

Describe the definitions, assumptions and key results of consumer choice theory.

23. Subject CT8 September 2015 Question 2

An investor makes decisions using a quadratic utility function, $U(w) = a + bw + cw^2$.

(i) Write down the absolute and relative risk aversion for this utility function. The investor

currently has wealth of £100, and using her utility function
$U(100) = 610$.

The investor is offered a gamble with a profit of £20 with probability p, and a loss of £20 with probability $(1 - p)$. She will accept this gamble only if $p \geq 0.55$.

(ii) Explain what this implies about the investor's risk aversion.

(iii) The investor accepts the gamble and wins. She now has wealth of £120.

The investor is offered the same gamble again, with a profit of £20 with probability p, and a loss of £20 with probability $(1 - p)$. Based on her new wealth, she will now accept this gamble only if $p \geq 0.5625$.

(iv) Determine a, b and c.

(v) Determine the maximum wealth for which the function $U(w)$ satisfies the requirement of non-satiation.

24. Subject CT8 September 2015 Question 5

An actuary plans to retire in five years' time, and hopes to celebrate retirement with a round-the-world cruise. The cruise will cost €20,000. The actuary chooses to save for the cruise by buying non-dividend paying shares with price S_t governed by the Stochastic Differential Equation:

$$dS_t = S_t(\mu dt + \sigma dZ_t)$$

where:

- Z_t is a standard Brownian motion.
- $\mu = 10\%$.
- $\sigma = 20\%$.
- t is the time from now measured in years; and
- $S_0 = 1$.

The instantaneous, constant, continuously compounded risk-free rate of interest is 4% p.a.

(i) Calculate the amount, A, that the actuary will need to invest in the shares to give a 40% probability of having savings of at least €20,000 in five years' time.

(ii) Calculate the following risk measures at $t = 5$ applied to the difference between the value of the share holding and €20,000, if the actuary invests €10,000 at $t = 0$:

(a) standard deviation

(b) 95% Value at Risk relative to €20,000

25. Subject CT8 September 2016 Question 1

Consider an asset whose return follows the probability density function $f(x)$.

(i) Write down a formula for the Value at Risk for the asset, at confidence level p.

(ii) Write down a formula for the downside semi-variance of the return on the asset, defining any additional notation you use.

(iii) State the arguments for and against using semi-variance as a risk measure.
A farmer has a small apple tree which produces one harvest of apples per year. The number of apples the tree produces follows a Poisson distribution with a mean and variance of 8.

(iv) Determine the 10% Value at Risk level for the number of apples produced.

(v) Determine the expected shortfall below a harvest of 5 apples.