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Subject: Portfolio Theory & Security Analysis

Chapter: Unit 3

Category: Practice Questions

1. Subject CT8 April 2014 Question 2

(i) State the expression for the return on a security, i, in the single-index model, defining all terms used.

(ii) Explain the difference between the single-index model and the Capital Asset Pricing Model.

Suppose the market has expected return 6% and standard deviation 10%. Two securities have expected returns 8% and 10%, and standard deviations 15% and 20%. The correlation between these two securities and the market is 0.25 and 0.4 respectively. Assume the single-index model described in (i) holds.

(iii) Calculate the constant parameters in the expression for the return of these two securities.

(iv) Explain how a multi-index model would be expected to perform relative to the single-index model in terms of fitting data and predicting future security price moves.

[Total 10]

2. Subject CT8 April 2014 Question 6

(i) State the equation for the capital market line in the Capital Asset Pricing Model (CAPM), defining all the terms used.

In a market where the CAPM is assumed to hold, the expected annual return on the market portfolio is 12%, the variance is 4%% and the effective risk-free annual rate is 4%. An Agent wants an expected annual return of 18% on a portfolio worth £1,200,000.

(ii) Calculate the standard deviation of the return on the corresponding efficient portfolio.

(iii) Calculate the amount of money invested in each component of the Agent's portfolio.

[Total 8]

3. Subject CT8 April 2014 Question 9

Outline the evidence against normality assumptions in models of market returns.[8]

4. Subject CT8 April 2013 Question 2

Consider a mean-variance portfolio model with two securities, S_A and S_B, where the expected return and the variance of return for S_B are twice the corresponding values for S_A. Suppose the correlation between the returns on the two securities is ρ.

(i) (a) Determine the values of ρ which allow the possibility of constructing a zero-risk portfolio, by calculating the variance of the return on a portfolio with weights x_A and x_B invested in the two assets.
(b) Calculate the portfolio weights that lead to the most efficient zero-risk portfolio.
(c) Calculate the expected return on the portfolio in part (i)(b) in terms of the expected return on S_A.

(ii) Calculate the maximum expected return for an investor:
(a) if portfolio weights are unlimited.
(b) if the investor can short sell at most one unit of either security and the total he has to invest is one unit.

(iii) Calculate the expected return on the minimum variance portfolio if the covariance between the two securities is 60% of the variance of S_A.

5. Subject CT8 April 2013 Question 4

In a market where the CAPM holds there are five assets with the following attributes.

Asset	*A*	*B*	*C*	*D*	*E*	*Probability of being in state*
Annual return in						
State 1	3%	3%	3%	3%	3%	0.25
State 2	5%	7%	2%	8%	3%	0.5
State 3	7%	5%	8%	1%	3%	0.25
Market Capitalisation	10m	20m	40m	30m		

(i) Calculate the expected annual return on the market portfolio and σ_M, the standard deviation of the annual return on the market portfolio.
(ii) Calculate the market price of risk under CAPM.
(iii) Calculate the beta of each asset.
(iv) Outline the limitations of the CAPM.

6. Subject CT8 April 2015 Question 8

(i) State the main assumptions of mean-variance portfolio theory.

There are only three assets available on a stock exchange:

Asset 1, expected return 2%, standard deviation 4%
Asset 2, expected return 4%, standard deviation 12%
Asset 3, expected return 3%, standard deviation 8%

The correlation between the returns on assets 1 and 3 is 0.75. The return on asset 2 is uncorrelated with the returns on the other two assets.

An investor in this market wants to minimise the variance of his portfolio.

(ii) Determine the Lagrangian function that can be used to find the minimum variance portfolio for a given expected return.

Let x_i denote the weight of asset i (i = 1, 2, 3) in the minimum variance portfolio with an expected return of 4%.

(iii) Show, by taking partial derivatives of the Lagrangian function in part (ii), that:

$$x_1 = -0.45,\ x_2 = 0.55,\ x_3 = 0.9.$$

(iv) Comment on how the portfolio would change if short-selling was not allowed.

7. Subject CT8 April 2016 Question 4

In a market where the assumptions of the Capital Asset Pricing Model (CAPM) hold, there are a risk-free asset and two risky assets with the following attributes:

		Rate of return (per annum)		
State	*Probability*	*Asset 1*	*Asset 2*	*Asset 3*
1	0.2	5.0%	15.0%	26.0%
2	0.3	5.0%	22.0%	15.0%
3	0.1	5.0%	10.0%	24.0%
4	0.4	5.0%	28.0%	7.0%
Market capitalisation			30,000	70,000

(i) Determine the composition of the market portfolio.

(ii) Determine the market price of risk.

(iii) Calculate the beta of each risky asset.

(iv) State the limitations of the CAPM.

8. Subject CT8 April 2017 Question 9

Let R_i denote the return on security i in a two-factor model.

(i) Write down the return equation for this two-factor model, defining all additional notation that you use.

(ii) Describe the three main categories of multifactor models.

9. Subject CT8 April 2017 Question 10

In a market in which the Capital Asset Pricing Model (CAPM) holds, there are two securities with the following attributes (expressed per annum):

	security	*A*	*B*
$E(r_i)$		0.196	0.164
$\text{Cov}(r_i, r_j)$	A	0.05	0.01
	B	0.01	0.03

(i) Determine the composition of the market portfolio with expected return 18% per annum.

(ii) Calculate the beta of each security, under the assumption that the risk-free rate of interest is 10% per annum.

(iii) State the limitations of the CAPM.

10. Subject CT8 April 2008 Question 6

i. Outline the assumptions used in modern portfolio theory regarding investor behaviour that are necessary to specify efficient portfolios.

ii. An investor can construct a portfolio using only two assets X and Y. The statistical properties of the two assets are shown below:

	X	*Y*
Expected return	12%	8%
Variance of return	30%	15%
Correlation coefficient between assets *X* and *Y*	0.5	

Assuming that the investor cannot borrow to invest:

(a) Determine the composition of the portfolio which will give the investor the highest expected return.

(b) Calculate the composition of the portfolio which will give the investor the minimum variance.

iii. Explain and sketch how the investor would choose a utility maximizing portfolio.

11. Subject CT8 April 2008 Question 7

i. State the assumptions, additional to those used in modern portfolio theory, that allow the capital asset pricing model (CAPM) to be consistent with an equilibrium model of prices in the whole market.

ii. Explain why in the CAPM all investors should hold all risky assets in proportion to the market capitalisation of those assets.

In an investment market there are three risky assets available. The table below shows the returns each of the assets will earn in the three possible states of the world and the current market capitalisation of the assets. Assume a risk free rate of return of 4% is available.

States	*Probability*	*Asset 1*	*Asset 2*	*Asset 3*
1	0.4	5%	6%	7%
2	0.1	8%	2%	1%
3	0.5	3%	5%	4%
Market Capitalisation		30,000	50,000	30,000

iii. Calculate the market price of risk under the CAPM.

12. Subject CT8 April 2009 Question 3

(ii) (a) Discuss the extent to which a continuous time lognormal model of security prices can capture the statistical properties empirically observed or expected in the stock market.

(b) Outline other possible processes which may be used.

13. Subject CT8 April 2009 Question 4

(i) State how investors are assumed to make decisions in modern portfolio theory (MPT).

(ii) Define an efficient portfolio in the context of MPT.

An investor can invest in only three assets which are uncorrelated with one another. The assets have the following characteristics:

Asset A	*Expected Rate of Return*	*Standard Deviation*
A	9%	18%
B	5%	8%
C	4%	0%

iii. Calculate the efficient frontier for the investor taking into account the numbers provided in the table above.

iv. Explain how an investor with a quadratic utility function would select a portfolio from those making up the efficient frontier.

14. Subject CT8 April 2009 Question 5

(i) Define the market price of risk in the CAPM.

The table below gives the annual returns conditional on the state of the economy for all the assets in an investment market.

Economic State	*Asset*			*Probability*
	Stock	*Property*	*Bonds*	
Recession	0%	1%	2%	0.1
Normal	5%	3%	3%	0.7
Boom	10%	7%	3%	0.2
Value of asset (bn)	100	50	100	

(ii) Calculate the market price of risk given that the risk free annual rate of return is 2.5%.

(iii) Discuss the particular issue a young investor might face in using the CAPM.

15. Subject CT8 April 2011 Question 3

A securities market has only three risky securities, A, B and C with the following annual return attributes:

	Asset A	*Asset B*	*Asset C*
Market capitalisation	£100bn	£150bn	£250bn
Annual expected return	4%	r_B	6%

Assume that:

- the assumptions of the Capital Asset Pricing Model hold
- the market price of risk is 10% per annum
- the risk free rate is 3.3% per annum
- the expected annual return on the market portfolio is 5.3% per annum.

i. Calculate σ_M, the standard deviation of the annual return on the market portfolio. Quote any results that you use.

ii. Calculate r_B, the expected annual return on asset *B*.

iii. Calculate the covariance of the annual returns on each asset with the annual return on the market portfolio. State any further results that you use.

16. Subject CT8 April 2012 Question 2

In a market where the CAPM holds there are five risky assets with the following attributes per year.

Asset number	*1*	*2*	*3*	*4*	*5*
Expected return	6%	5%	8%	13%	11%
Market capitalisation (in $)	2.6m	3.9m	5.2m		1.3m
Beta				1.5	

The risk-free rate is r = 1% p.a.

(i) Calculate the expected return on the market portfolio.

(ii) Deduce the market capitalisation of asset 4 and the betas of all the other assets.

(iii) Calculate the beta of a portfolio P which is equally weighted in the five assets and the risk-free asset.

Explain whether or not this portfolio P lies on the Capital Market Line.

17. Subject CT8 September 2008 Question 3

(i) (a) Define Beta in the Capital Asset Pricing Model (CAPM)
(b) Explain why Beta is used in pricing securities.

In a market where the CAPM holds the following parameters are known:

Risk-free rate of interest = 6% Expected
market rate of return = 12%
Standard deviation of an efficient portfolio's returns = 0.50
Standard deviation of the market returns = 0.7

ii. Calculate the expected return on the portfolio.

iii. An investor is evaluating the risk and expected return of a portfolio of *N* securities.

Explain how many parameters need to be estimated if:

a. the evaluation is made using mean-variance portfolio theory without any assumed cross-sectional structure in the variances of the securities.
b. the CAPM is assumed to hold.

18. Subject CT8 September 2009 Question 4

An investor invests a proportion x_i of the assets in his portfolio in the ith of N securities.

(i) State the expected return and variance of his portfolio. Define any notation you use.

Securities with the properties in the table below are available to an investor. The statistics in the table refer to the next year.

	A	*B*
Expected return	4%	3%
Variance of return	16%%	4%%
Correlation coefficient between assets	$r_{AB} = 1$	

The investor combines the securities to form a portfolio.

(ii) Calculate the relative amount which should be invested in each security to give a portfolio with the minimum possible variance. (Note: you may assume that short selling securities is allowable.)

iii. Show that if it is possible to borrow at the rate of 1% p.a. over the next year, it is possible for the investor to make a risk free profit over the year without using any of his own capital.

19. Subject CT8 September 2012 Question 3

(i) State the three main assumptions of Modern Portfolio Theory.

Assume Modern Portfolio Theory holds true.

ii. Write down equations for the expected return, E and variance, V of a portfolio of N securities, defining any notation used.

iii. Describe how an efficient portfolio can be found.

20. Subject CT8 September 2012 Question 9

Consider a market where there are two risky assets A and B and a risk free asset. Both risky assets have the same market capitalisation.

Assume that all the assumptions of the CAPM hold.

(i) State the composition of the market portfolio.

(ii) Derive the expressions for the variance of the market portfolio and for the beta of each asset, in terms of the variance of each asset and of their covariance.

Assume now that the risk-free rate is r_f = 10%, the expected return of the market portfolio is r_M=18%, the variance of asset A is 4%, the variance of asset B is 2% and their covariance is 1%.

(iii) Derive the value for the expected return on asset A and asset B.

(iv) An investor wants an expected return of 20%.

(v) Calculate the composition of the corresponding portfolio.

(vi) Derive the corresponding standard deviation using the Capital Market Line.

21. Subject CT8 September 2013 Question 2

(i) Describe the single-index model of security returns, defining any terms used.

The single-index model is to be used in a particular market.

(ii) Determine the following results:
   (a) the expected return on a security
   (b) the variance of returns on a security; and
   (c) the covariance of returns between two securities in the market, using the parameters described in part (i).

(iii) Show that investors can diversify away specific risk in this model by holding equal weights in an increasing number of securities.

(iv) State the potential impact of adding additional indices to the model:
   (a) in terms of explaining historic data
   (b) in terms of forecasting security returns.

22. Subject CT8 September 2014 Question 2

An investor wishes to allocate her capital between a service company *S* and a manufacturing company *M*. The investor believes that returns on shares in *S* have mean 10% and variance 16%% while returns on shares in *M* have mean 8% and variance 25%%. The correlation between the two companies is 0.3.

Assume the investor chooses their investments according to mean-variance portfolio theory.

(i) Explain which company's share she would prefer.

Assume the investor's preferences are described by a standard quadratic utility function.

(ii) State which assumption of the mean-variance portfolio theory can be relaxed.

(iii) Calculate the expected return and the standard deviation of a portfolio which is invested three quarters in *S* and one quarter in *M*.

(iv) Calculate the minimum variance portfolio.

A new study suggests that in the future, *S* will make more sales to *M*, when *M* is delivering strong profits.

(v) Describe the effect this will have on portfolios composed of *M* and *S*, including the minimum-variance portfolio.

23. Subject CT8 September 2015 Question 3

(i) Define an "efficient portfolio" in the context of mean-variance portfolio theory.
(ii) State the assumptions required for the existence of efficient portfolios.

Suppose an investor invests his wealth in N securities, $i = 1, ..., N$, with x_i denoting the proportion of wealth invested in security i.

iii. Write down a formula for the expected return on this portfolio.

iv. Write down a formula for the variance of the return on this portfolio.

Now suppose the investor invests in only two securities, A and B.

v. Derive the proportion x_A that should be invested in security A to minimise the portfolio variance.

24. Subject CT8 September 2016 Question 2

(i) State the main assumptions of mean-variance portfolio theory.

Consider a mean-variance portfolio model with two securities, with respective returns S_A and S_B, where the expected return $E[S_B] = 0.25E[S_A]$ and the variance of return $V[S_B] = 0.25V[S_A]$.

Let the correlation between the returns on the two securities be ρ.

(ii) Determine, in terms of $E[S_A]$, the expected return on the minimum variance portfolio if:

(a) ρ = 0
(b) ρ = 1

(iii) (a) Calculate the variance of the return on the minimum variance portfolio for part (ii)(b).

(b) Comment on the risk in this portfolio.

25. Subject CT8 September 2016 Question 3

In a market where the assumptions of the Capital Asset Pricing Model hold, there are two risky assets with the following attributes:

Security	*A*	*B*
Expected return (p.a.)	20%	16%

(i) Determine the composition of the market portfolio with expected return 18% per annum.
(ii) Calculate the beta of each security under the assumption that the risk-free rate of interest is 10% per annum.

26. Subject CT8 September 2016 Question 5

i. (State the key arguments against modelling market returns using a Gaussian random walk.

ii. Describe the difference in time series modelling between a cross-sectional property and a longitudinal property, including their dependence on the initial conditions imposed on the model.

27. Subject CT8 September 2017 Question 2

(i) Define in the context of mean-variance portfolio theory:

(a) an inefficient portfolio
(b) an efficient portfolio

(ii) State the two assumptions about investor behaviour that are needed for the existence of efficient portfolios.

An investment universe includes two assets, A and B, with expected return on asset i of r_i and variance v_i as set out below:

Asset i	*Expected return* r_i	*Variance of return* v_i
A	$r_A = 0.05$	$v_A = 0.16$
B	$r_B = 0.07$	$v_B = 0.25$

The correlation of returns is $c_{AB} = -0.2$.

In an efficient portfolio, let a be the proportion which is held in asset A.

iii. Express the portfolio variance V in terms of a quadratic function in a, showing your workings.

Let R be the expected return on the portfolio.

iv. Express the portfolio variance V in terms of a quadratic function in R, using your result from part (iii) and showing your workings. [Your expression should not include a.]

The expression in part (iv) represents the efficient frontier.

An investor uses a utility function that gives rise to an indifference curve $V = 16R - 200R^2$.

v. Determine the two portfolios on the efficient frontier that also lie on the investor's indifference curve.

vi. Comment on the implications for part (v) if short selling is not allowed in the market.