IAQS | INSTITUTE OF ACTUARIAL & QUANTITATIVE STUDIES

Subject: PTSA

Chapter: Unit 1 & 3

Category: Assignment 1

1. Mr. Y has a quadratic utility function of the form $U(w) = w - 15^{-7} w^2$. It is Mr. Y's birthday today, he has invited Mr. Z to the party. Mr. Z is now looking for the birthday gift that would please Mr. Y. He is not sure how much amount he should budget for the gift. In the gift shop he likes three gifts. The cost of the gifts are Rs. 20,000, Rs. 40,000 and Rs. 50,000. Mr. Z likes all the gifts equally.

i) Calculate the expected cost of the gift and expected utility offered by those gifts to Mr. Y.

ii) You know Mr. Y and his utility function, using this information which gift would you suggest to Mr. Z, such that Mr. Y is pleased.

2. There are two investments X and Y. Investment Y provides a return 3.5B% where B is a Binomial random variable with parameters n=3, p=0.5. Investment X provides a return 2P% where P is a Poisson random variable with mean 3%. The benchmark return is 3% p.a.

i) Calculate the following:

a) Variance

b) Downside semi-variance

ii) Without performing any further calculation, determine which investment an investor should choose, if their utility function is

$$u(x) = \{-(1060 - x)^2, \quad x < 1060$$

$$0 \qquad\qquad elsewhere$$

3. A market consists of three companies shares A, B and C with capitalisations of 2500, 4500 and 3000 crores respectively. Annual returns on the three shares have the following characteristics:

Return on company	Standard deviation
A	30%
B	20%
C	25%

The expected rate of return on the market portfolio is 12% p.a.

The correlation between the returns on each pair of distinct securities is 0.5.

The risk-free rate of return is 6% p.a. No adjustments to an investor's portfolio is possible within the year.

i) Calculate the expected returns on Company A, B and C if the CAPM is assumed to hold.

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ii) Calculate the covariance co-efficient of each of the securities with the Market.

An investor wants to invest in Company A. She uses the Single index factor model (with the index equal to RM , the random return of the market portfolio) with the same expected returns and variances as in the CAPM.

iii) Estimate the value of Alpha in the single factor model if the expected return of A is same that Expected return of A as per CAPM.

iv) Calculate the systematic and the specific risk that the investor would face if she invests in A.

v) Compare the correlation between A and B using the single index model & the CAPM and comment.

4. Toggle Corporation's share is selling for Rs. 80 per share and its dividend next year is expected to be Rs. 2. Nifty 50 index is 14370 at present and it is expected to go up to 15500 after one year. The average dividend yield for the Nifty 50 index is 1.52% and the risk-free rate is 5.14%. If the beta of Toggle Corporation's share price is 1.14, find the expected share price of Toggle Corporation after one year using CAPM.

5.

i) What is the result of widespread usage of the Internet with regards to efficient markets?

ii) Explain what is meant by an excessive volatile market?

6. A teenaged cricket fan, fond of betting on HPL cricket game assesses his utility of wealth using the utility function $U(w) = 2(w^{0.5} - 1)$.

i) Prove algebraically that the cricket fan is:

a) Non-satiated.

b) Risk averse.

ii) Prove that the cricket fan exhibits constant relative risk aversion.

The cricket fan is intended to bet on two teams. The table below shows the pay-out per Rs. 1 bet on each of these teams if it wins the match, and the investor's estimated probabilities of each team winning the match. The pay-out is the total paid and is not in addition to the bet being returned.

Team	Winning payout per Rs. 1 bet	Probability of winning
A	1.69	60%
B	6.25	10%

The cricket fan has total wealth of Rs. 1,000 and he will bet all of his wealth on this match. Negative bets are not allowed.

iii) Calculate the amount he should bet on each team to maximize his expected utility of wealth.

iv) Calculate the expected wealth resulting from the bets in part (iii).

7. An investor has Rs. 8,00,000 to invest, for a period of 1 year, and has identified two investment opportunities in which to invest.
The first is a direct investment in a stock index for a period of 1 year. The annual return, X, on the index follows a Normal distribution with mean $\mu = 7\%$ p.a. and standard deviation $\sigma = 5.5\%$ p.a.

i) Calculate the following in respect of the investment at the end of 1 year:
a) The shortfall probability below a value of Rs. 7,20,000.
b) The 99.5% value at risk.

The second opportunity is a derivative that offers the following payoff in 1 years' time based on the performance of the index during the year.

Payoff (Rs)	Scenario
7,30,000	$X \leq -7.1\%$
7,50,000	$-7.1\% < X \leq 7\%$
9,62,000	$X > 7\%$

ii) Calculate the expected payoff from the derivative at the end of the year.

iii) Calculate the following in respect of the payoff from the derivative:
a) The shortfall probability below a value of Rs. 7,20,000.
b) The 99.5% value at risk.

iv) Comment on how the investor may choose between the two investments.

8. Assuming CAPM holds, find the risk free rate using the following information

State	Probability	Risky Asset 1	Market of risky assets
1	0.2	0%	2%
2	0.25	15%	11%
3	0.05	2%	3%
4	0.5	7%	5%

Correlation between risky asset 1 and Market of risky assets= 0.9831

9. A market consists of shares of 3 companies A, B and C with capitalization of Rs 10,000 crores, Rs 15,000 crores and Rs 10,000 crores respectively.
Annual returns on the three shares (RA, RB and RC) have the following characteristics:

Company	*Standard deviation*
A	30%
B	20%
C	10%

The expected rate of return on the market portfolio is 12% p.a. The correlation between the returns on each pair of shares is 0.5.
The risk-free rate of return is 7% p.a.

i) Find the expected returns on A, B and C if the CAPM is assumed to hold.

ii) Split the variances of returns into systematic and specific risk for every company using the values in the question & the values obtained in (i) above.

10. There are three assets in a market with the following characteristics:

Asset	Expected Return	Volatility
1	4%	6%
2	6%	12%
3	8%	18%

The correlation between Asset 1 and 2 is 0.5 while the correlation between Asset 3 and other two assets is zero.

i) State the Lagrangian function that can be minimised to find the minimum variance portfolio associated with a given expected return, defining any notation used.

ii) By taking five partial derivatives of this function, calculate the minimum variance portfolio which yields an expected return of 7%.

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