INSTITUTE OF ACTUARIAL & QUANTITATIVE STUDIES

Subject: PTSA

Chapter:

Category: Assignment 1 solutions

Answer 1:

i) Expected cost of the gift: 1/3* 20,000 + +1/3*40,000+1/3*50,000 = 36666.67

Expected Utility: 1/3 * ($20000-15^{-7} \times 20000^2$) + 1/3 * ($50000 - 15^{-7} \times 50000^2$) +1/3 * ($40000 - 15^{-7} \times 40000^2$) = 36657.89

ii) Minimum cost S is equal to the certainty equivalent of the gift

$U(x) = S - 15^{-7} \text{ X } S^2 = 36657.89$

$-S+15^{-7}S^2 + 36657.89 = 0$

using $S = \frac{-1 \pm \sqrt{(-1)^2 - 4 * 15^{-7} * 36657.89}}{2*15^\wedge-7}$

we get (1+- 0.9996) / 2*15^-7

i.e Rs 34,172 or Rs 17,08,25,203.13

Conclusion: Minimum price of the gift should be Rs 34,172. Mr Z can buy gift of Rs 40,000 or Rs 50,000 both would please him. Gift of Rs 20,000 will not please Mr Y.

Answer 2:

Benchmark return = 3%

Parameters for binomial distribution: n = 3, p =0.5

Parameters for Poisson distribution: mu = 3

i)

a) Variance

Investment Y:

Mean = 3.5* np = 5.25%

Variance = 3.5^2 * npq = 9.1875%%

Investment X:

Mean = Variance = 2 * mu = 6

b) Downside semi-variance

Investment Y:

The binomial distribution is symmetric about mean. Therefore, semi variance = variance/ 2 = 4.59375

Investment X:

Downside semi variance
= exp(-3) *(3^2/2‰ (6 - 4)^2 + 3/1‰ (6 - 2)^2 + 1.(6 - 0)^2)
= 5.078%%;

[4]

ii) Maximising expected utility is equivalent to maximising semi-variance. Therefore, Investment B would maximise utility. [2]

Answer 3:

i) The market portfolio is (0.25, 0.45, 0.30), so

RM = (0.25RA+ 0.45RB + 0.30RC)

Thus

Cov(Ri, RM) = [0.25 Cov(Ri, RA) + 0.45Cov(Ri, RB) + 0.3 Cov(Ri, RC)] .

So,
Cov(RA, RM) = 0.25 Cov(RA, RA) + 0.45Cov(RA, RB) + 0.3 Cov(RA, RC) .
= 0.25 *.09 + 0.45*.03+ 0.3 *.0375 = 0.0473 .

Similarly, Cov(RB, RM) =.03330,

Cov(RC, RM) =0.0394, .

and

Variance M= [0.25 Cov(RM, RA) + 0.45 Cov(RM, RB) + 0.3 Cov(RM, RC)] = .0385.
Or variance M = .25* Var(a) + .45* Var(b) +.3* Var(c) + 2* cov(a,b) *.25*.45+ 2* cov (b,c)* .45*.3+2* cov(c,a) *0.3*0.25

We conclude that
Beta (AM) = 1.2281 ,
Beta (BM) = 0.8577 and
Beta (CM) = 1.0234.

Finally, solving
Ei = ro +beta *(RM- ro)
EA = 13.37%, EB =11.15%, EC = 12.14%

ii) Correlation co-efficient using beta:

Cor im = beta(im) * std(m)/ std(i) .

Cor AM= 0.80296, Cor BM = 0.84119, Cor CM= 0.80296

[Max

iii) The corresponding single index model is
Ri = alpha + beta* RM + ei where ei= error process follows standard normal with mean 0 and variance Vei

E (Ri) = alpha + beta* E(RM) (since Ei = 0)

Under CAPM,
E (Ri) = ro + beta* E(RM) - beta* ro

Giving
alpha = ro *(1- beta)

alpha (A) = .06 *(1-1.2281) = -0.013684

iv) Under the Single Index model ,

Total variance = beta^2 * V(M) + V(e)

Total variance = 0.09
Beta =1.2281
V(M) =.0385

Systematic risk = beta^2 * V(M) = 0.5803
Specific risk = V(e) = Total variance - beta^2 * V(M) =0.03197

v) Correlation under CAPM:

Cor = 0.5

Correlation under Single Index between A and B:
(beta(Am)* beta(BM) * V(M)) / (std(A) * std(B)) = 0.67543

Single Index Model is not compatible with CAPM

Answer 4:

Expected percentage return on the market = sum of the dividend yield of the market and its price appreciation [1]

1.52 % dividend has to be added to appreciation to get expected market return [0.5]

E(Rm) = 0.0152 + (15500 – 14370)/14370 = .09384 [1]

Using CAPM,

E(Ri) = 0.0514 + 1.14*(.09384 – 0.0514) = .09978 [1.5]

If x is the expected price of the stock next year, then the stock return should follow;

X = (1 +.09978) * 80 -2 (where 2 is the dividend amount) [1. 5]

This gives x = Rs 85.98 [0.5]

[Max 6 Marks]

Answer 5:

i)

a. It makes information cheaper and more accessible thus making markets more efficient.

b. It is subject to new regulation thus marking markets less efficient.

c. It increases the volatility of security prices thus making markets less efficient.

d. It increases competition among brokers thus making markets more efficient.

ii) An excessive volatile market is one in which the changes in the market values of stocks (the observed volatility) are greater than can be justified by the news arriving. This is claimed to be evidence of market over-reaction, which is not compatible with efficiency.

Answer 6:

i)

a) $U'(w) = w^{-0.5} > 0$ for $w > 0$

b) $U''(w) = -0.5w^{-1.5} < 0$ for $w > 0$

ii) $R(w) = w*(-U''(w)/U'(w)) = 0.5$

$==-w*(-0.5w^{-1.5})/ w^{-0.5} =0.5$

iii) E[U] = 0.6U(1.69a) + 0.1U(6.25b) +0.3U(0)
= 0.6*2*((1.69a)^0.5-1) + 0.1*2*((6.25b)^0.5-1) – 0.6
= 1.2*(1.3a^0.5-1) + 0.2*(2.5b^0.5-1) – 0.6
= 1.56a^0.5 + 0.5b^0.5 – 1.4 – 0.6
= 1.56a^0.5 + 0.5(1000-a)^0.5 –2
dE[U]/da = 0.78a^-0.5 – 0.25(1000-a)^-0.5
Setting dE[U]/da = 0 gives 0.78a^-0.5 = 0.25(1000-a)^-0.5
=> 0.78 = 0.25a^0.5(1000-a)^-0.5 => 3.12
= (a/(1000-a))^0.5
Squaring both sides => 9.7344 = a/(1000-a)
=> 9,734.4 = 10.7344a or 8.7344a => a = 906.8 or 1,114.50
Rejecting the figure >£1,000 gives a = 906.80 and b=93.20
Checking the second derivative d^2E[U]/da^2
= -0.39a^-1.5 – 0.25(1000-a)^-1.5 < 0
hence this is a maximum

iv) E[U] = 0.6U(1.69a) + 0.1U(6.25b) +0.3U(0)
=1.56a^0.5 + 0.5(1000-a)^0.5 –2
Putting the value of a=906.80 in above equation we get
E[U}= 49.801

Answer 7:

i)

a) X = (720000/800000)-1 = -10%
P(X<-10%) = P(Z<-3.09)=0.1%

b) P(Z<(t-7%)/5.5%) =0.005
(t-7%)/5.5% =-2.5758
t= -7.1669%
800000*(1 – 7.1669%) = 742664.8

ii)

P(X≤-7.1%) = P(Z≤-2.56) =0.00518
P(X>7%) = 0.5 (as 7% is the mean)
P(-7.1%<X≤7%) = 1-0.5-0.00518 = 0.49482

Expected pay out = 730000*0.00518 + 750000*0.49482 + 962000*0.5 = 855896.40

iii) a) 0% . Payout is discrete and has 3 payouts all greater than 720000 and hence shortfall probability is 0
b) Probability payout is ≤ 730000 is 0.52% therefore the 99.5% VaR is 730000

iv) The expected return from investing in the index is 800000*1.07 =Rs. 856000

ASSIGNMENT 1

So the expected returns are very similar for each investment.
Based on the expected shortfall below Rs. 720000, the derivative is less risky as there is no possibility of this event occurring.
If the investor has a utility function with a discontinuity at the minimum required return then he may base his decision on this measure.
The 99.5% VaR is higher (i.e. a greater loss) for the derivative, so based on this measure the investor may prefer to invest in the stock index.
The pay off on the derivative is significantly higher than the index when the return is slightlyabove the mean, so the investor may prefer this.

Answer 8:

Security market line

E1 = r + Beta *(Em –r)   - equation 1

Beta= rho * sigma 1/sigma m

Sigma 1 = std dev of risky asset 1

Sigma m = std dev of the market

E 1= .2* 0% + .25 * 15%+ .05* 2% + .5* 7% = 7.35%
Similary Em = 5.8%

sigma 1 = 0.2 2 * 0%2 + 0.25 * 15%2 + 0.05* 2%2 + 0.5* 7%2 - E1 2 = 0.051892

Similary sigma M = = 0.032187

Beta= 1.58494

Substituting all the information in equation 1

7.35% = r + Beta * EM – Beta *r

7.35% - 9.19264% = r * (1 -Beta) *r

r = 3.15 %

Answer 9:
The market portfolio is (2/7, 3/7, 2/7),
so RM = (2RA + 3RB + 2RC) / 7.

Thus
Cov(Ri, RM) = [2 Cov(Ri, RA) + 3Cov(Ri, RB) + 2 Cov(Ri, RC)] / 7

So,
Cov(RA, RM) = [2 * 0.3^2+ 3*0.5*0.3*0.2+ 2*0.5*0.3*0.1] / 7 = 0.042857143

Similarly,
Cov(RB, RM) = 0.028571 ,
Cov(RC, RM) = 0.011429 and
Var(M) = [2 Cov(RM, RA) + 3 Cov(RM, RB) + 2 Cov(RM, RC)] / 7 = 0.027755

We conclude that βA = 1.5441, βB = 1.0294 and βC = 0.4118.

EM = 12%, Ro= 7%
Finally, solving
Ri - *R0* = β*i** (*EM* - *R0*), we get *RA* = 14.7206% , *RB* = 12.1471% and *RC* = 9.0588%

ii) Using $V_i = \beta^2_i V_M + V_{\varepsilon i}$ in which the first term of RHS is Systematic risk & the second Specific risk

Company	Specific	Systematic	Vi
A	0.02382	0.06618	0.09
B	0.01059	0.02941	0.04
C	0.00529	0.00471	0.01

Answer 10:
i) Let the proportion invested in Asset i be xi, with expected return Ei, Variance Vi and correlation as ρ12. Assume E to be the return on the portfolio of three assets and let λ and μ be the Lagrange multipliers. Then the Lagrangian function W satisfies:

$$W = \Sigma i{=}1 \text{ to } 3\ x_i^2 V_i + 2\,\rho_{12}\sigma_{12}x_1x_2 - \lambda(E_1x_1 + E_2x_2 + E_3x_3 - E) - \mu(x_1 + x_2 + x_3 - 1)$$

$$= 36\,x_1^2 + 144\,x_2^2 + 324\,x_3^2 + 72x_1x_2 - \lambda\,(4x_1 + 6x_2 + 8x_3 - E) - \mu(x_1 + x_2 + x_3 - 1)$$

ii)

(b)
$$\frac{\partial W}{\partial x_1} = 0 \Rightarrow 72x_1 + 72x_2 - 4\lambda - \mu = 0 \quad (1)$$

$$\frac{\partial W}{\partial x_2} = 0 \Rightarrow 288x_2 + 72x_1 - 6\lambda - \mu = 0 \quad (2)$$

$$\frac{\partial W}{\partial x_3} = 0 \Rightarrow 648x_3 - 8\lambda - \mu = 0 \quad (3)$$

$$\frac{\partial W}{\partial \lambda} = 0 \Rightarrow 4x_1 + 6x_2 + 8x_3 = 7 \quad (4)$$

$$\frac{\partial W}{\partial \mu} = 0 \Rightarrow x_1 + x_2 + x_3 = 1 \quad (5)$$

from (1), $\mu = 72x_1 + 72x_2 - 4\lambda$ (6)

Putting (6) into (2),

$$288x_2 + 72x_1 - 6\lambda - (72x_1 + 72x_2 - 4\lambda) = 0$$

$$\Rightarrow 216x_2 - 2\lambda = 0$$

or $\lambda = 108x_2$ (7)

Putting (6) & (7) into (3)

$$648x_3 - 8(108x_2) - (72x_1 + 72x_2 - 4(108x_2)) = 0$$

$$\Rightarrow 648x_3 - 504x_2 - 72x_1 = 0 \quad (8)$$

Also, $x_3 = 1 - x_1 - x_2$ (9)

Solving (4),

$$4x_1 + 6x_2 + 8(1 - x_1 - x_2) = 7$$

$$\Rightarrow x_2 = 0.5 - 2x_1 \quad (10)$$

Substituting (10) & (9) into (8),

$$648(1 - x_1 - (0.5 - 2x_1)) - 504(0.5 - 2x_1) - 72x_1 = 0$$

$$\Rightarrow x_1 = -0.04545$$

Hence, $x_2 = 0.59091$

$x_3 = 0.45454$