INSTITUTE OF ACTUARIAL & QUANTITATIVE STUDIES

Subject: Portfolio Theory and Security Analysis

Chapter: Unit 3 & 4

Category: Assignment 2 solutions

Solution 1:

i) The market portfolio is (2/7, 3/7, 2/7),

so RM = (2RA + 3RB + 2RC) / 7.

Thus Cov(Ri, RM) = [2 Cov(Ri, RA) + 3Cov(Ri, RB) + 2 Cov(Ri, RC)] / 7

So, Cov(RA, RM) = [2 * 0.32+ 3*0.5*0.3*0.2+ 2*0.5*0.3*0.1] / 7 = 0.042857143

Similarly,

Cov(RB, RM) = 0.028571 ,

Cov(RC, RM) = 0.011429

and

Var(M) = [2 Cov(RM, RA) + 3 Cov(RM, RB) + 2 Cov(RM, RC)] / 7 = 0.027755

We conclude that βA = 1.5441, βB = 1.0294 and βC = 0.4118.

EM = 12%, Ro= 7%

Finally, solving

Ri - R0 = βi* (EM - R0), we get RA = 14.7206% , RB = 12.1471% and RC = 9.0588%

ii) Using $V_i = \beta^2{}_i \times V_M + V_{\varepsilon i}$ in which the first term of RHS is Systematic risk & the second Specific risk

Company	Specific	Systematic	Vi
A	0.02382	0.06618	0.09
B	0.01059	0.02941	0.04
C	0.00529	0.00471	0.01

Solution 2:

i) The set of efficient portfolios is called as efficient frontier. A portfolio is efficient if the investor can't find a better portfolio in the sense that it has either a higher expected return and the same (or lower) variance or a lower variance and the same (or higher) expected return.

The efficient frontier is a straight line which is tangent to the efficient frontier (of risky assets) and passes through the point in (S.D., return) space corresponding to the risk-free asset.

Initially, we need to find the portfolio using A and B that maximises

(expected return – 3%)/standard deviation

Assume proportion x of assets in A and (1 – x) in B

Expected return of risky portfolio is 7% *x + 4% *(1 – x)

Standard deviation of the risky portfolio is

$[(15\%*x)^2 + (7\%*(1-x))^2]^{0.5}$

We need to find x that maximises the function

$[7\% *x + 4\% *(1-x) - 3\%] / [(15\%*x)^2 + (7\%*(1-x))^2]^{0.5}$

Taking log, we get

$Ln[0.07x + 0.04 - 0.04x - 0.03] - 0.5*Ln[(0.15x)^2 + (0.07(1-x))^2]$

$=Ln[0.03x + 0.01] - 0.5*Ln[0.0225x^2 + 0.0049(1 - 2x + x^2)]$

$= Ln[0.03x + 0.01] - 0.5*Ln[0.0274x^2 - 0.0098x + 0.0049]$

Differentiate and set to zero,

$0.03/[0.03x + 0.01] - 0.5*[0.0548x - 0.0098] / [0.0274x^2 - 0.0098x + 0.0049] = 0$

$0.03*[0.0274x^2 - 0.0098x + 0.0049] - 0.5*[0.0548x - 0.0098]* [0.03x + 0.01] = 0$

$[0.000822x^2 - 0.000294x + 0.000147] - [0.000822x^2 + 0.000274x - 0.000147x - 0.00049] = 0$

Solving we get x = 0.46556

When x = 0.46556,

Expected return of the risky portfolio = 0.054

Standard deviation of the risky portfolio = 0.0792

Thus, the efficient frontier is the straight line passing through (0.03,0) and (0.054,0.0792)

ii) The portfolio would be corresponding to the point where the utility indifference curve of the investor touched the efficient frontier.

Solution 3:

Under the theory of MPT, variance of the portfolio is expressed as:

$$V = Var[R_P] = \sum_{j=1}^{N} \sum_{i=1}^{N} xi\ xj\ Cij$$

This can be rewritten as:

$$V = \sum_{i=1}^{N} Vi\ xi^2 + \sum_{j=1}^{N} \sum_{\substack{i=1 \\ i \neq j}}^{N} xi\ xj\ Cij$$

In case of all independent assets, the covariance between them is zero and the formula for variance becomes:

$$V = \sum_{i=1}^{N} xi^2 \; Vi$$

Now if there are N assets and equal amount is invested in each of the N assets, the proportion invested in each is 1/N. Hence,

$$V = \sum_{i=1}^{N} (\tfrac{1}{N})^2 \; Vi \; = (1/N) \sum_{i=1}^{N} \left(\tfrac{1}{N}\right) Vi$$

$$= \frac{\overline{V}}{N}$$

Where V represents the average variance of the stocks in the portfolio. As N gets larger and larger, the variance of the portfolio approaches zero. In other words, in the presence of enough independent assets, a lower variance i.e. a lower risk can be achieved.

ii) However, in case of not so independent assets, i.e. when the correlation coefficient and the covariance between assets is positive, the formula for the variance of the portfolio becomes:

$$V = \sum_{i=1}^{N} \; (1/N)^2 \; Vi + \sum_{j=1}^{N} \sum_{\substack{i=1 \\ i \neq j}}^{N} (\tfrac{1}{N}) \; (\tfrac{1}{N}) \; Cij$$

$$= (1/N) \sum_{i=1}^{N} \; (1/N) \; Vi + \frac{(N-1)}{N} \sum_{j=1}^{N} \sum_{\substack{i=1 \\ i \neq j}}^{N} \frac{Cij}{N(N-1)}$$

Replacing variances and covariances with their averages V and C,

$$V = \frac{\overline{V}}{N} + \frac{N-1}{N} \overline{C}$$

As N gets very large, the contribution to the portfolio variance of the variances of individual securities goes to zero. However, the contribution of the covariance terms approaches the average covariance as N gets large.

So the individual risk of securities can be diversified away but the contribution to the total risk caused by the covariance terms cannot be diversified away.

Solution 4:

i) Within the context of CAPM, the market price of risk is defined as:

Market- price of risk = $\dfrac{(E_M - r)}{\sigma_M}$

Where

E_M = the expected return on market portfolio

r = the risk free rate of return

σ_M = the standard deviation of market portfolio returns

It is the additional expected return that the market requires in order to accept an additional unit of risk, as measured by the portfolio standard deviation of return.

It is equal to the gradient of the capital market line in E – σ space

ii)

a) $E_P = 18\%$

$\sigma^2_M = 4\%\% \Rightarrow \sigma_M = 2\%$

r = 4%

$E_M = 12\%$

$E_p - r = \sigma_p \dfrac{(E_M - r)}{\sigma_M}$

$18 - 4 = \sigma_p \dfrac{(12 - 4)}{2}$

$\sigma p = 3.5\%$

b) The efficient portfolio is a mix of the market portfolio and the risk-free asset. If the weights (which sum to 1) are xM and x0, then the expected return is

$x_M E_M + x_0 r = E_p$

$x_M * 12 + (1 - x_M)*4 = 18$

⇨ $x_M = 1.75$

⇨ $x_0 = -0.75$

Thus the efficient portfolio has Rs. 2,100,000 in the market portfolio and is short Rs. 900,000 in cash.

Solution 5:

i) Variance of return on security i as per single index model is given as

$V_i = \beta^2_i V_M + V\varepsilon_i$

i.e $\sigma^2_i = \beta^2_i \sigma^2_M + \sigma^2\varepsilon_i$

$\sigma^2_A = (.08^2 * .20^2) + .25^2 = 0.0881$

$\sigma^2_B = (1.0^2 * .20^2) + .10^2 = .05$

$\sigma^2_C = (1.2^2 * .20^2) + .20^2 = .0976$

ii) If there are infinite number of assets with identical characteristics, then a well- diversified portfolio of each type will have only systematic risk since the non- systematic risk will approach zero with large n:

Well diversified $\sigma^2_{A} = (0.8^2 * .20^2) + 0^2 = .0256$ -------(I)
Well diversified $\sigma^2_{B} = (1.0^2 * .20^2) + 0^2 = .04$ -------(II)
Well diversified $\sigma^2_{C} = (1.2^2 * .20^2) + 0^2 = .0576$ -------(III)

The mean will equal to that of the individual (identical) stocks.

$E_A = 10\%$

$E_B = 12\%$

$E_C = 14\%$

a) $E_A = 10\%$, $\sigma^2_A = 0.0256$

b) Portfolio consists 55% of Type B

45% of Type C

Return on the portfolio

$R_p = \Sigma x_i R_i$

$E(R_p) = E(\Sigma x_i R_i) = \Sigma x_i\, E(R_i) = .55*12\% + .45*14\%$

$= 12.9\%$

$Var(R_p) = \beta^2_p\, V_M + V\varepsilon_p$

In case of well diversified portfolio $V\varepsilon_p = 0$

& $\beta_p = x_i * \beta_i = .55 * \beta_B + .45 * \beta_C = .55*1.0 + .45*1.2 = 1.09$

$Var(R_p) = 1.09^2 * (\sigma^2_M)$

$= 1.09^{2} * (0.2)^2$

$= .047524$

c) Arbitrage is the risk free trading profit. In this market, there is no arbitrage opportunity because on a well diversified portfolio all plot on the security market line (SML).

Solution 6:

A is correct. Environmental considerations should be incorporated in the industry analysis of Company A. It operates in a sector that has high exposure to greenhouse gas emissions, as well as water management and waste and hazardous materials management, which could incur additional costs to running businesses in this sector.

Solution 7:

Low-cost strategy:

Companies strive to become the low-cost producers and to gain market share by offering their products and services at lower prices than their competition while still making a profit margin sufficient to generate a superior rate of return based on the higher revenues achieved. Low-cost strategies may be pursued defensively to protect market positions and returns or offensively to gain market share and increase returns. Pricing also can be defensive (when the competitive environment is one of low rivalry) or aggressive (when rivalry is intense). In the case of intense rivalry, pricing may even become predatory—that is, aimed at rapidly driving competitors out of business at the expense of near-term profitability. The hope in such a strategy is that having achieved a larger market share, the company can later increase prices to generate higher returns than before. For example, the ride-sharing industry has produced fierce competition among companies that seek to capture market share by offering incentives and discount pricing for rides. Companies seeking to follow low-cost strategies must have tight cost controls, efficient operating and reporting systems, and appropriate managerial incentives. In addition, they must commit themselves to painstaking scrutiny of production systems and their labor forces and to low-cost designs and product distribution. In some cases, they must be able to invest in productivity-improving capital equipment and to finance that investment at a low cost of capital.

Differentiation strategies:

Companies attempt to establish themselves as the suppliers or producers of products and services that are unique either in quality, type, or means of distribution. To be successful, their price premiums must be above their costs of differentiation and the differentiation must be appealing to customers and sustainable over time. Corporate managers who successfully pursue differentiation strategies tend to have strong market research teams to identify and match customer needs with product development and marketing. Such a strategy puts a premium on employing creative and inventive people.

Solution 8:

1. Present value models (synonym: discounted cash flow models).

These models estimate the intrinsic value of a security as the present value of the future benefits expected to be received from the security. In present value models, benefits are often defined in terms of cash expected to be distributed to shareholders (dividend discount models) or in terms of cash flows available to be distributed to shareholders after meeting capital expenditure and working capital needs (free-cash-flow-to-equity models). Many models fall within this category, ranging from the relatively simple to the very complex. In Sections 4–8, we discuss in detail two of the simpler models, the Gordon (constant) growth model and the two-stage dividend discount models.

2. Multiplier models (synonym: market multiple models).

These models are based chiefly on share price multiples or enterprise value multiples. The former model estimates intrinsic value of a common share from a price multiple for some fundamental variable, such as revenues, earnings, cash flows, or book value. Examples of the multiples include price to earnings (P/E, share price divided by earnings per share) and price to sales (P/S, share price divided by sales per share). The fundamental variable may be stated on a forward basis (e.g., forecasted EPS for the next year) or a trailing basis (e.g., EPS for the past year), as long as the usage is consistent across companies being examined. Price multiples are also used to compare relative values. The use of the ratio of share price to EPS—that is, the P/E multiple—to judge relative value is an example of this approach to equity valuation.

3. Enterprise value (EV)

These multiples have the form (Enterprise value)/(Value of a fundamental variable). Two possible choices for the denominator are earnings before interest, taxes, depreciation, and amortization (EBITDA) and total revenue. Enterprise value, the numerator, is a measure of a company's total market value from which cash and short-term investments have been subtracted (because an acquirer could use those assets to pay for acquiring the company). An estimate of common share value can be calculated indirectly from the EV multiple; the value of liabilities and preferred shares can be subtracted from the EV to arrive at the value of common equity.

4. Asset-based valuation models.

These models estimate intrinsic value of a common share from the estimated value of the assets of a corporation minus the estimated value of its liabilities and preferred shares. The estimated market value of the assets is often determined by making adjustments to the book value (synonym: carrying value) of assets and liabilities. The theory underlying the asset-based approach is that the value of a business is equal to the sum of the value of the business's assets

Solution 9:

Step 1 – Find the present value of dividends for years 1 and Year 2.

- PV (year 1) = $20/((1.15)^1)
- PV(year 2) = $20/((1.15)^2)
- In this example, they come out to be $17.4 and $16.3, respectively, for 1st and 2nd-year dividends.

Step 2 – Find the present value of the future selling price after two years.

- PV(Selling Price) = $333.3 / (1.15^2)

Step 3 – Add the present value of dividends and the present value of selling price

- $17.4 + $16.3 + $252.0 = $285.8

Solution 10:

i) The Gordon (constant) growth model (Gordon, 1962) is a simple and well- recognized DDM. The model assumes dividends grow indefinitely at a constant rate.

The Gordon growth model is particularly appropriate for valuing the equity of dividend- paying companies that are relatively insensitive to the business cycle and in a mature growth phase.

$$V_0 = \sum_{t=1}^{\infty} \frac{D_0(1+g)^t}{(1+r)^t} = D_0\left[\frac{(1+g)}{(1+r)} + \frac{(1+g)^2}{(1+r)^2} + \ldots + \frac{(1+g)^\infty}{(1+r)^\infty}\right]$$

With a constant growth assumption, where g is the constant growth rate:

If required return r is assumed to be strictly greater than growth rate g, then the square- bracketed term in above formula is an infinite geometric series and sums to [(1 + g)/(r – g)].

$$V_0 = \frac{D_0(1+g)}{r-g} = \frac{D_1}{r-g}$$

The assumptions of the Gordon model are as follows:

1. Dividends are the correct metric to use for valuation purposes.
2. The dividend growth rate is forever: It is perpetual and never changes.
3. The required rate of return is also constant over time.
4. The dividend growth rate is strictly less than the required rate of return.

ii) By using the stock – PV with constant growth formula, we get –

- $P_0 = Div_1 / (r – g)$
- Or, P_0 = \$40,000 / (8% – 4%)
- Or, P_0 = \$40,000 / 4%
- Or, P_0 = \$40,000 * 100/4 = \$10, 00,000.