IAQS | INSTITUTE OF ACTUARIAL & QUANTITATIVE STUDIES

Subject: PTSA

Chapter: Unit 1 & 3

Category: Assignment 1

1.

i) What is the result of widespread usage of the Internet with regards to efficient markets?

ii) Explain what is meant by an excessive volatile market?

2. A teenaged cricket fan, fond of betting on HPL cricket game assesses his utility of wealth using the utility function $U(w) = 2(w^{0.5} - 1)$.

i) Prove algebraically that the cricket fan is:

a) Non-satiated.

b) Risk averse.

ii) Prove that the cricket fan exhibits constant relative risk aversion.

The cricket fan is intended to bet on two teams. The table below shows the pay-out per Rs. 1 bet on each of these teams if it wins the match, and the investor's estimated probabilities of each team winning the match. The pay-out is the total paid and is not in addition to the bet being returned.

Team	Winning payout per Rs. 1 bet	Probability of winning
A	1.69	60%
B	6.25	10%

The cricket fan has total wealth of Rs. 1,000 and he will bet all of his wealth on this match. Negative bets are not allowed.

iii) Calculate the amount he should bet on each team to maximize his expected utility of wealth.

iv) Calculate the expected wealth resulting from the bets in part (iii).

3. An investor has Rs. 8,00,000 to invest, for a period of 1 year, and has identified two investment opportunities in which to invest.

The first is a direct investment in a stock index for a period of 1 year. The annual return, X, on the index follows a Normal distribution with mean μ = 7% p.a. and standard deviation σ = 5.5% p.a.

i) Calculate the following in respect of the investment at the end of 1 year:

a) The shortfall probability below a value of Rs. 7,20,000.

b) The 99.5% value at risk.

The second opportunity is a derivative that offers the following payoff in 1 years' time based on the performance of the index during the year.

Payoff (Rs)	Scenario
7,30,000	$X \leq -7.1\%$
7,50,000	$-7.1\% < X \leq 7\%$
9,62,000	$X > 7\%$

ii) Calculate the expected payoff from the derivative at the end of the year.

iii) Calculate the following in respect of the payoff from the derivative:
a) The shortfall probability below a value of Rs. 7,20,000.
b) The 99.5% value at risk.

iv) Comment on how the investor may choose between the two investments.

4. i) State in words the four axioms of the Expected Utility Theorem.

ii) An investor measures the utility of his wealth using the utility function $U(w) = \ln(w)$ for $w > 0$. He has Rs.100,000 available to invest in two possible assets, Asset A and Asset B.

The future value of Asset A depends on an uncertain future event.
· Every Rs.1 invested in Asset A will be worth Rs.1.30 with probability 0.75 and Rs. 0.40 with probability 0.25.
· Asset B is risk-free, so every Rs.1 invested in Asset B will always be worth Rs.1.

The investor does not discount future asset values when making investment decisions. He decides to invest a proportion a of his wealth in Asset A and the remaining proportion 1 – a in Asset B.
a) Express his expected utility of wealth in terms of a.
b) Determine the amount that he should invest in each of Asset A and B to maximise his expected utility, using your result from part (ii)

5. The company conducted a consumer survey to understand the marketability of the Blockchain Technology Insurance product. In the consumer response, it assessed that the expected business from the product would be 10 policies in the first year. Each policy will have a sum insured of Rs. 50,00,000 and the company is expected to pay 90% of the sum insured in case there is a loss event. The company expects that there is one hundredth chance of suffering the loss.

a) Calculate the 99.5% Value at risk from one policy.
b) Calculate the 99.5% Value at risk from 10 policies.

6. Assuming CAPM holds, find the risk free rate using the following information

State	Probability	Risky Asset 1	Market of risky assets
1	0.2	0%	2%
2	0.25	15%	11%
3	0.05	2%	3%
4	0.5	7%	5%

Correlation between risky asset 1 and Market of risky assets= 0.9831

7. i) Give one word on which form of efficient markets hypothesis is represented by each of following statements:

I. It should not be possible to consistently profit by selling winners and hanging on to losers.

II. It should not be possible to consistently profit by trading on any public information, such as that found on the Internet or in the financial newspaper.

III. It should not be possible to consistently profit by trading on private information, such as that obtained from a thorough analysis of the company and its industry.

IV. It should not be possible to consistently profit by trading on inside information.

8.

i) Define iso-elastic.

ii) Which utility function does not exhibit iso-elasticity and why?

I. $U(w)=\ln(w),\ w>0$

II. $U(w)= (w^x-1)/x\ ,\ w>0$

III. $U(w)= a+bw+cw^2$

IV. $U(w)= -e^{-aw}\ ,\ \ a>0$

iii) One of the risk averse contestants in the game show "Kaun Banega Crorepati" has utility function as $U(w)= w+dw^2$. The contestant states that the upper limit of wealth where she can use this utility function is w = Rs 10,00,000

a) Determine the value of d in the contestant's utility function.

The contestant wins a prize of Rs 2,50,000 in that gameshow. She is then offered the opportunity to exchange this prize for a larger prize of Rs 6,00,000 if she can answer one more question correctly. However, she will receive no prize at all if she gets the question wrong. She estimates her chances of answering the question correctly to be 50%.

b) Determine whether the contestant should take this opportunity to exchange.

Unit 1 & 3

ASSIGNMENT QUESTION

9. A market consists of shares of 3 companies A, B and C with capitalization of Rs 10,000 crores, Rs 15,000 crores and Rs 10,000 crores respectively.
Annual returns on the three shares (RA, RB and RC) have the following characteristics:

Company	*Standard deviation*
A	30%
B	20%
C	10%

The expected rate of return on the market portfolio is 12% p.a. The correlation between the returns on each pair of shares is 0.5.
The risk-free rate of return is 7% p.a.

i) Find the expected returns on A, B and C if the CAPM is assumed to hold.

ii) Split the variances of returns into systematic and specific risk for every company using the values in the question & the values obtained in (i) above.

10. A group of employees working in a Private company contribute part of their salary towards a Pension Scheme. The Pension Scheme invests in 3 asset classes, namely, Government securities, corporate bonds and Equities in the proportion 40%, 40% and 20% respectively. Employees have been contributing for the past 5 years.
The returns in each of the financial years are given in the below table:

FY ending March	Government	Corporate bonds	Equities
2014	6.2%	8.0%	5.7%
2015	7.1%	4.2%	-15.0%
2016	6.8%	3.1%	35.1%
2017	6.9%	5.2%	2.25%
2018	6.2%	5.2%	40.0%

Note: Return in each asset class include capital gains/losses and income

i) Find the Variance of the return of each of the asset classes and the portfolio (Note: Round up to 5 digits after decimal at each step, Use 'n' in the denominator)

ii) List the key advantages and disadvantages of the following measures of investment risk in the context of equity:
- Variance of return
- Shortfall probability
- Value at Risk
- Tail Value at Risk

Unit 1 & 3

ASSIGNMENT QUESTION

iii) Find the Value at risk for Equities and the portfolio for the next 1 year, if the next 1 year return of Equities and the portfolio follow normal distribution with Mean and Variance as obtained from the Table. Interpret the results.

11. Under Mean Variance Portfolio theory, derive how variance can be reduced using diversification as the principal tool, under the given two scenarios:
i) Independent assets
ii) Non-independent assets (where assets can be assumed to have positive co-variance)

12. There are three assets in a market with the following characteristics:

Asset	Expected Return	Volatility
1	4%	6%
2	6%	12%
3	8%	18%

The correlation between Asset 1 and 2 is 0.5 while the correlation between Asset 3 and other two assets is zero.

i) State the Lagrangian function that can be minimised to find the minimum variance portfolio associated with a given expected return, defining any notation used.

ii) By taking five partial derivatives of this function, calculate the minimum variance portfolio which yields an expected return of 7%.

13.i) Define efficient market frontier along with a diagram.

ii) Explain the difference between capital market line and security market line.

iii) In the context of CAPM, state whether the following statements are True or False:
a) Risk attitude for given set of securities is identical for all investors
b) Only efficient portfolios lie on the security market line

iv) What will be the market capitalization of script A in Market portfolio as per CAPM, if half of the investors hold 5% of their respective portfolio in script A and rest half of them hold only risk-free assets, given that market consists of rational and irrational investors

14. Suppose there are only three assets available on stock exchange:

Asset	Expected Return	Standard Deviation
1	.06	.10
2	.08	.15
3	.10	.20

Correlation matrix is given by:

$$\begin{bmatrix} 1 & .5 & .5 \\ .5 & 1 & .5 \\ .5 & .5 & 1 \end{bmatrix}$$

An investor in this market wants to minimise the variance of this portfolio.

i) Determine the Lagrangian function from the above information that can be used to find the minimum variance portfolio for a given expected return. Define any notation used.

ii) By deriving the partial derivatives of the function in (i) state the five equations that could be solved to determine the minimum variance portfolio associated with an expected return of 9%.

iii) Determine the composition of the corner portfolio where asset 1 is not present

15. i) Within the context of Capital Asset Pricing Model (CAPM), explain what is meant by the "market price of risk".

ii) In the market where CAPM is assumed to hold, the expected annual return on the market portfolio is 12%, the variance is 4%% and the effective risk free annual rate is 4%. An Agent wants an expected annual return of 18% on a portfolio worth of Rs.12,00,000/-.

a) Calculate the standard deviation of return on the corresponding efficient portfolio.

b) Calculate and explain the amount of money invested in each component of Agent's.