INSTITUTE OF ACTUARIAL & QUANTITATIVE STUDIES

Subject: PTSA

Chapter: Unit 1 & 3

Category: Assignment Solutions

Answer 1:

i)

a. It makes information cheaper and more accessible thus making markets more efficient.

b. It is subject to new regulation thus marking markets less efficient.

c. It increases the volatility of security prices thus making markets less efficient.

d. It increases competition among brokers thus making markets more efficient.

ii) An excessive volatile market is one in which the changes in the market values of stocks (the observed volatility) are greater than can be justified by the news arriving. This is claimed to be evidence of market over-reaction, which is not compatible with efficiency.

Answer 2:

i)

a) $U'(w) = w^{-0.5} > 0$ for $w > 0$

b) $U''(w) = -0.5w^{-1.5} < 0$ for $w > 0$

ii) $R(w) = w*(-U''(w)/U'(w)) = 0.5$

$==-w*(-0.5w^{-1.5})/ w^{-0.5} =0.5$

iii) E[U] = 0.6U(1.69a) + 0.1U(6.25b) +0.3U(0)

= 0.6*2*((1.69a)^0.5-1) + 0.1*2*((6.25b)^0.5-1) – 0.6

= 1.2*(1.3a^0.5-1) + 0.2*(2.5b^0.5-1) – 0.6

= 1.56a^0.5 + 0.5b^0.5 – 1.4 – 0.6

= 1.56a^0.5 + 0.5(1000-a)^0.5 –2

dE[U]/da = 0.78a^-0.5 – 0.25(1000-a)^-0.5

Setting dE[U]/da = 0 gives 0.78a^-0.5 = 0.25(1000-a)^-0.5

=> 0.78 = 0.25a^0.5(1000-a)^-0.5 => 3.12

= (a/(1000-a))^0.5

Squaring both sides => 9.7344 = a/(1000-a)

=> 9,734.4 = 10.7344a or 8.7344a => a = 906.8 or 1,114.50

Rejecting the figure >£1,000 gives a = 906.80 and b=93.20

Checking the second derivative d^2E[U]/da^2

= -0.39a^-1.5 – 0.25(1000-a)^-1.5 < 0

hence this is a maximum

iv) E[U] = 0.6U(1.69a) + 0.1U(6.25b) +0.3U(0)

=1.56a^0.5 + 0.5(1000-a)^0.5 –2

Putting the value of a=906.80 in above equation we get

E[U}= 49.801

Answer 3:

i)

a) $X = (720000/800000)-1 = -10\%$
$P(X<-10\%) = P(Z<-3.09)=0.1\%$

b) $P(Z<(t-7\%)/5.5\%) = 0.005$
$(t-7\%)/5.5\% = -2.5758$
$t= -7.1669\%$
$800000*(1 - 7.1669\%) = 742664.8$

ii)

$P(X\leq-7.1\%) = P(Z\leq-2.56) = 0.00518$
$P(X>7\%) = 0.5$ (as 7% is the mean)
$P(-7.1\%<X\leq7\%) = 1-0.5-0.00518 = 0.49482$

Expected pay out = 730000*0.00518 + 750000*0.49482 + 962000*0.5 = 855896.40

iii) a) 0% . Payout is discrete and has 3 payouts all greater than 720000 and hence shortfall probability is 0
b) Probability payout is ≤ 730000 is 0.52% therefore the 99.5% VaR is 730000

iv) The expected return from investing in the index is 800000*1.07 =Rs. 856000
So the expected returns are very similar for each investment.
Based on the expected shortfall below Rs. 720000, the derivative is less risky as there is no possibility of this event occurring.
If the investor has a utility function with a discontinuity at the minimum required return then he may base his decision on this measure.
The 99.5% VaR is higher (i.e. a greater loss) for the derivative, so based on this measure the investor may prefer to invest in the stock index.
The pay off on the derivative is significantly higher than the index when the return is slightlyabove the mean, so the investor may prefer this.

Answer 4:
i) 1. Comparability
An investor can state a preference between all available certain outcomes.

2. Transitivity
If A is preferred to B and B is preferred to C, then A is preferred to C.

3. Independence

If an investor is indifferent between two certain outcomes, A and B, then he is also indifferent between the following two gambles:

(i) A with probability p and C with probability (1 – p); and
(ii) B with probability p and C with probability (1 – p).

4. Certainty equivalence Suppose that A is preferred to B and B is preferred to C. Then there is a unique probability, p, such that the investor is indifferent between B and a gamble giving A with probability p and C with probability (1 – p). B is known as the certainty equivalent of the above gamble.

ii)

a) Wealth after the uncertain event will be either:

1,00,000 × (1.3a + (1 – a)) = 100000+ 30000a with probability 0.75

or:

100,000 × (0.4a + (1 – a)) = 100000 – 60000a with probability 0.25.

Thus expected utility of wealth is:

0.75 × ln(100000 + 30000a) + 0.25 × ln(100000 – 60000a).

b) Differentiate with respect to a:

30000 × 0.75/(100000 + 30000a) – 60000 × 0.25/(100000 – 60000a).

Set equal to zero:

30000 × 0.75 / (100000 + 30000a) – 60000 × 0.25 / (100000 – 60000a) = 0

30000× 0.75 × (100000– 60000a) = 60000 × 0.25 × (100000 + 30000a)

30-18a= 20+6a

10=24a

a = 0.4167

Check for maximum:

Differentiate with respect to a again:

– (30000)^2 × 0.75/(100000 + 30000a)^2 – (60000)^2 × 0.25/(100000 – 60000a)^2.

This must be negative because of the square terms, hence this is a local maximum.

Answer 5:

a) Maximum Expected Loss $= 0.9 * 5000000 = 4500000$

$P(X = 0) = 0.99$
$P(X = -450000) = 0.01$

VaR $= -t$ where $t = \max\{x: P(X<x) <= p\}$

But $P(X < -4500000) = 0$ and $P(X<0) = 0.01$.
Hence,
$t = \max\{x: P(X<x) <= 0.005\} = -4500000$

i.e. VaR $= -4500000$

b) we can assume that the blockchain contracts are expected to be independent in nature. Hence, we can assume a Binomial distribution for the losses.

Hence, Let X the total impact suffered by the company on these contracts, where $X \sim$ Binomial $(10, 0.01) * -4500000$
Then as per the definition of Var for discrete random variable we have:
$Var(X) = -t$ where $t = \max\{x: P(X<x) <= p\}$

If no contracts suffer losses then $X = 0$
Then probability of this is

$P(X = 0) = 10C0 * 0.01^{(0)} * (1-0.01)^{(10)} = 0.99^{100} = 0.9044$

And
$P(X < 0) = 1 - 0.9044 = 0.0956$

If exactly 1 contract suffers loss then $X = -450000$. The probability of this is:

$P(X = -4500000) = 10C1 * 0.01^{(1)} * (1-0.01)^{(9)} = 0.0914$

$P(X < -4500000) = 1 - 0.9044 - 0.0914 = 0.0042$

Thus,
$t = \max\{x: P(X< x) <= 0.005\} = -4500000$

hence, var is 4500000.

Answer 6:

Security market line

E1 = r + Beta *(Em –r)   - equation 1

Beta= rho * sigma 1/sigma m

Sigma 1 = std dev of risky asset 1

Sigma m = std dev of the market

E 1= .2* 0% + .25 * 15%+ .05* 2% + .5* 7% = 7.35%
Similary Em = 5.8%

sigma 1 = 0.2 2 * 0%2 + 0.25 * 15%2 + 0.05* 2%2 + 0.5* 7%2 - E1 2 = 0.051892

Similary sigma M = = 0.032187

Beta= 1.58494

Substituting all the information in equation 1

7.35% = r + Beta * EM – Beta *r

7.35% - 9.19264% = r * (1 -Beta) *r

r = 3.15 %

Answer 7:
I. weak form
II. Semi strong
III. strong form
IV. strong form.

Answer 8:
i) Utility functions exhibiting constant relative risk aversion are said to be isoelastic Iso-elastic means that the elasticity of the marginal utility of wealth is constant with respect to wealth. Comment :- Single line definition is fine. Highlighted words are critical

ii) Only (III) does not exhibit iso-elasticity as its relative risk aversion measures are given by $R'(w) = -2d/(1+2dw)^2$ is not constant

iii)

a) U'(w)= 1+2dw and U''(w)= 2d, Because the contestant is risk averse, we must have U''(w)<0, so we must have d<0

The condition of non-satiation requires U'(w)>0
Hence 1+2dw>0, and w< -1/2d

So the quadratic utility function can only satisfy the condition of non satiation over a limited range of w i.e -∞<w<-1/2d
Now 10,00,000= -1/2d, so d= -0.0000005

Comment:
U'(W) – ½
U''(W) – ½
Condition –1/2
Quadratic utility satisfaction range – 1/2

b) $U(250{,}000) = 250{,}000 - 0.0000005*(250{,}000)^2 = 218{,}750$

$E[U(\text{exchange})] = 0.5 \times U(600{,}000) + 0.5 \times U(0)$
$= 0.5 \times (600{,}000 - 0.0000005 \times 600000^2) = 210{,}000$

So the contestant should not accept the opportunity to exchange because the expected utility of the exchange opportunity is lower than that of the prize.

Answer 9:
The market portfolio is (2/7, 3/7, 2/7),
so RM = (2RA + 3RB + 2RC) / 7.

Thus
Cov(Ri, RM) = [2 Cov(Ri, RA) + 3Cov(Ri, RB) + 2 Cov(Ri, RC)] / 7

So,
Cov(RA, RM) = $[2 * 0.3^2 + 3*0.5*0.3*0.2 + 2*0.5*0.3*0.1] / 7 = 0.042857143$

Similarly,
Cov(RB, RM) = 0.028571 ,
Cov(RC, RM) = 0.011429 and
Var(M) = [2 Cov(RM, RA) + 3 Cov(RM, RB) + 2 Cov(RM, RC)] / 7 = 0.027755

We conclude that βA = 1.5441, βB = 1.0294 and βC = 0.4118.

EM = 12%, Ro= 7%

Finally, solving

Ri - *R*0 = β*i** (*EM* - *R*0), we get *RA* = 14.7206% , *RB* = 12.1471% and *RC* = 9.0588%

ii) Using $V_i = \beta^2_i\ V_M + V_{\varepsilon i}$ in which the first term of RHS is Systematic risk & the second Specific risk

Company	Specific	Systematic	Vi
A	0.02382	0.06618	0.09
B	0.01059	0.02941	0.04
C	0.00529	0.00471	0.01

Answer 10:

i)

For Government Bonds:

Mean = 6.64%

Variance = $\frac{\sum(x-Mean)2}{n}$ = 0.14 %%

For Corporate Bonds:

Mean = 5.14%

Variance = $\frac{\sum(x-Mean)2}{n}$ = 2.65 %%

For Equities:

Mean = 13.61%

Variance = $\frac{\sum(x-Mean)2}{n}$ = 433.68 %%

For the Portfolio:

Mean = 7.43%

Variance = $\frac{\sum(x-Mean)2}{n}$ = 15.76 %%

ii) Variance of return

Merits:

· Variance is mathematically tractable.

· Variance fits neatly with a mean-variance portfolio construction framework.

De-merits:

· Variance is a symmetric measure of risk. The problem of investors is really the downside part of the distribution.

· Not suitable for returns with asymmetric distribution or fat tails. · Neither skewness or kurtosis of returns is captured by a variance measure.

Shortfall probability Merits:

· It gives an indication of the possibility of loss below a certain level.

· It allows a manager to manage risk where returns are not normally distributed.
De-merits:

· The choice of benchmark level is arbitrary.

· For a portfolio of Equities, the shortfall probability will not give any information on:
nor upside returns above the benchmark level o nor the potential downside of returns when the benchmark level is exceeded.

Value at Risk (VaR)
Merits:

· VaR generalises the likelihood of underperformance by providing a statistical measure of downside risk.
De-merits:

· Equity Portfolios may exhibit non-normal distributions. The usefulness of VaR in the above situation depends on modelling skewed or fat-tailed distributions of returns. The further one gets out into the "tails" of the distributions, the more lacking the data and, hence, the more arbitrary the choice of the underlying probability distribution becomes.

· No attention is paid to the distribution of outperformance above the benchmark.

Tail Value at Risk (TailVaR)
Merits:

· Relative to VaR, TailVaR provides much more information on how bad returns can be when the benchmark level is exceeded.
De-merits:

· It has the same modelling issues as VaR in terms of sparse data, but captures more information on tail of the non-normal distribution.

· Like VaR, no attention is paid to the distribution of outperformance above the benchmark

iii)
Using the confidence level of 95%, Risk Metrics is 1.645 as the z-score for 95%.
For Equities:

Var at 95% = 13.61% - 1.645 * $\sqrt{0.04337}$ = - 20.65 % [1]

The employee is 95% confident that she would not lose more than 20.65% in Equity in the following year. [½]

For the portfolio:

Var at 95% = 7.43% - 1.645 * $\sqrt{0.00158}$ = 0.904 % [1]

The employee is 95% confident that her return would not be less than 0.904% in the following year.

Answer 11:

i)

Under the theory of MPT, variance of the portfolio is expressed as:

$$V = Var[R_P] = \sum_{j=1}^{N}\sum_{i=1}^{N} xi\ xj\ Cij$$

This can be rewritten as:

$$V = \sum_{i=1}^{N} Vi\ xi^2 + \sum_{j=1}^{N}\sum_{\substack{i=1 \\ i\neq j}}^{N} xi\ xj\ Cij$$

In case of all independent assets, the covariance between them is zero and the formula for variance becomes:

$$V = \sum_{i=1}^{N} xi^2\ Vi$$

Now if there are N assets and equal amount is invested in each of the N assets, the proportion invested in each is 1/N. Hence,

$$V = \sum_{i=1}^{N}\left(\frac{1}{N}\right)^2 Vi = (1/N)\sum_{i=1}^{N}\left(\frac{1}{N}\right)Vi$$

$$= \frac{\overline{V}}{N}$$

Where $\overline{V}$ represents the average variance of the stocks in the portfolio. As N gets larger and larger, the variance of the portfolio approaches zero. In other words, in the presence of enough independent assets, a lower variance i.e. a lower risk can be achieved.

ii) However, in case of not so independent assets, i.e. when the correlation coefficient and the covariance between assets is positive, the formula for the variance of the portfolio becomes:

$$V = \sum_{i=1}^{N} (1/N)^2\ Vi + \sum_{j=1}^{N}\sum_{\substack{i=1 \\ i\neq j}}^{N}\left(\frac{1}{N}\right)\left(\frac{1}{N}\right) Cij$$

$$= (1/N)\sum_{i=1}^{N} (1/N)\ Vi + \frac{(N-1)}{N}\sum_{j=1}^{N}\sum_{\substack{i=1 \\ i\neq j}}^{N}\frac{Cij}{N(N-1)}$$

Replacing variances and covariances with their averages $\overline{V}$ and $\overline{C}$,

$$V = \frac{\overline{V}}{N} + \frac{N-1}{N}\overline{C}$$

As N gets very large, the contribution to the portfolio variance of the variances of individual securities goes to zero. However, the contribution of the covariance terms approaches the average covariance as N gets large. So the individual risk of securities can be diversified away but the contribution to the total risk caused by the covariance terms cannot be diversified away.

Answer 12:

i) Let the proportion invested in Asset i be xi, with expected return Ei, Variance Vi and correlation as ρ12. Assume E to be the return on the portfolio of three assets and let λ and μ be the Lagrange multipliers. Then the Lagrangian function W satisfies:

$$W = \Sigma_{i=1 \text{ to } 3}\ x_i^2V_i + 2\,\rho_{12}\sigma_{12}x_1x_2 - \lambda(E_1x_1 + E_2x_2 + E_3x_3 - E) - \mu(x_1 + x_2 + x_3 - 1)$$

$$= 36\,x_1^2 + 144\,x_2^2 + 324\,x_3^2 + 72x_1x_2 - \lambda\,(4x_1 + 6x_2 + 8x_3 - E) - \mu(x_1 + x_2 + x_3 - 1)$$

ii)

(b)

$\frac{\partial W}{\partial x_1} = 0 \Rightarrow 72x_1 + 72x_2 - 4\lambda - \mu = 0$ (1)

$\frac{\partial W}{\partial x_2} = 0 \Rightarrow 288x_2 + 72x_1 - 6\lambda - \mu = 0$ (2)

$\frac{\partial W}{\partial x_3} = 0 \Rightarrow 648x_3 - 8\lambda - \mu = 0$ (3)

$\frac{\partial W}{\partial \lambda} = 0 \Rightarrow 4x_1 + 6x_2 + 8x_3 = 7$ (4)

$\frac{\partial W}{\partial \mu} = 0 \Rightarrow x_1 + x_2 + x_3 = 1$ (5)

from (1), $\mu = 72x_1 + 72x_2 - 4\lambda$ (6)

Putting (6) into (2),

$288x_2 + 72x_1 - 6\lambda - (72x_1 + 72x_2 - 4\lambda) = 0$

$\Rightarrow 216x_2 - 2\lambda = 0$

or $\lambda = 108x_2$ (7)

Putting (6) & (7) into (3)

$648x_3 - 8(108x_2) - (72x_1 + 72x_2 - 4(108x_2)) = 0$

$\Rightarrow 648x_3 - 504x_2 - 72x_1 = 0$ (8)

Also, $x_3 = 1 - x_1 - x_2$ (9)

Solving (4),

$4x_1 + 6x_2 + 8(1 - x_1 - x_2) = 7$

$\Rightarrow x_2 = 0.5 - 2x_1$ (10)

Substituting (10) & (9) into (8),

$648(1 - x_1 - (0.5 - 2x_1)) - 504(0.5 - 2x_1) - 72x_1 = 0$

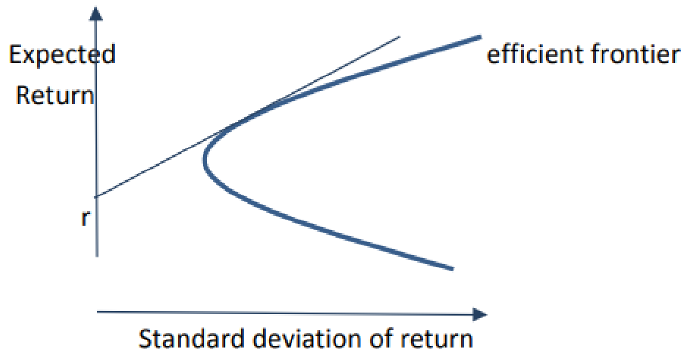
$\Rightarrow x_1 = -0.04545$

Hence, $x_2 = 0.59091$

$x_3 = 0.45454$

Answer 13:

i) The set of efficient portfolios in E – V space is known as the efficient frontier. A portfolio is efficient if the investor cannot find a better one in the sense that it has either a higher expected return and the same (or lower) variance or a lower variance and the same (or higher) expected return.



The portfolios which lie below the efficient frontier are sub-optimal as they do not provide enough return to compensate for the underlying risk.

ii) The security market line for any portfolio P is defined as

$E_P = r + (E_M - r)\beta_P$,

Where,

E_P is the expected return on Portfolio P

r is the risk free rate of return

E_M is the expected return on the market portfolio

β_P is the beta of the portfolio with respect to market portfolio

The capital market line for any portfolio P is defined as

$E_P - r = (E_M - r)\,\sigma_P / \sigma_M$

Where,

E_P is the expected return on Portfolio P on the efficient portfolio

r is the risk free rate of return

E_M is the expected return on the market portfolio

σ_P is the standard deviation of the return on Portfolio P

σ_M is the standard deviation of the return on market Portfolio

The capital market line relationship only holds for efficient portfolios which are a combination of the risk-free asset and the market portfolio whereas the security market line applies to any portfolios as well as individual securities. In other words, any security or a portfolio, whether efficient or not, would lie on the security market line

iii) a) False

b) False

iv) As per CAPM, the market capitalization for Script A in the market portfolio would be 5%, i.e. in the proportion it is held by the investors.

Answer 14:

i)

Asset	Expected Return	Standard Deviation
1	.06	.10
2	.08	.15
3	.10	.20

Correlation matrix is

$$\begin{bmatrix} 1 & .5 & .5 \\ .5 & 1 & .5 \\ .5 & .5 & 1 \end{bmatrix}$$

Variance and covariance matrix can be determined as

$$\begin{bmatrix} .01 & .0075 & .01 \\ .0075 & .0225 & .015 \\ .01 & .015 & .04 \end{bmatrix}$$

Where

$C_{ij} = \rho_{ij}\sigma_i\sigma_j$

Lagrangian function satisfies

$W = \sum_{j=1}^{3}\sum_{i=1}^{3} x_i x_j C_{ij} - \lambda(\sum_{i=1}^{3} x_i E_i - E) - \mu(\sum_{i=1}^{3} x_i - 1)$

$= (.01x_1^2 + .0225\, x_2^2 + .04\, x_3^2) + 2*(.0075\, x_1 x_2 + .015\, x_2 x_3 + .01 x_3 x_1)$

$- \lambda(.06\, x_1 + .08 x_2 + .10 x_3 - .09) - \mu\,(x_1 + x_2 + x_3 - 1)$

Where λ and μ are Lagrangian multipliers, x_i are the proportion of assets, E_i is expected return on each asset and E is expected return on the portfolio

ii) Equating partial derivative of W w.r.t. x_i to 0, we get

$\frac{\partial W}{\partial x_1}$ = .02 x_1+.015x_2 +.02 x_3- .06λ- μ = 0

⇒ .06λ+ μ = .02 x_1+.015x_2 +.02 x_3 ----------- (A)

$\frac{\partial W}{\partial x_2}$ = .045x_2 +.015 x_1+.03 x_3- .08λ- μ = 0

⇒ .08λ+ μ = .015 x_1+.045x_2 +.03 x_3 ----------- (B)

$\frac{\partial W}{\partial x_3}$ = .08 x_3+.03x_2 +.02 x_1- .1λ- μ = 0

⇒ .1λ+ μ = .02x_1+.03x_2 +.08x_3 ----------- (C)

$\frac{\partial W}{\partial \lambda}$ = .06x_1+.08x_2 +.10x_3- .09=0----------- (D)

$\frac{\partial W}{\partial \mu}$ = x_1+x_2 +x_3- 1=0----------- (E)

iii) Corner portfolio where $x1$ = 0. Equations become
x_2 +x_3=1 => x_3=1- x_2

.06λ+ μ = .015x_2 +.02 x_3 = .015x_2 +.02(1- x_2)---------- (A)
.06λ+ μ = .02-.005 x_2 ---------- (I)

.08λ+ μ = .045x_2 +.03 x_3= .045x_2 +.03(1- x_2)----------- (B)

.08λ+ μ = .03+.015x_2 ---------- (II)

.1λ+ μ = .03x_2 +.08(1-x_2) ----------- (C)

.1λ+ μ = .08 -.05x_2---------- (III)

Solving (I), (II) and (III) we get
x_2 =.4706 i.e. 47.06%
x_3 =.5294 i.e. 52.94%

Answer 15:

i) Within the context of CAPM, the market price of risk is defined as:

$$\text{Market- price of risk} = \frac{(E_M - r)}{\sigma_M}$$

Where

E_M = the expected return on market portfolio

r = the risk free rate of return

σ_M = the standard deviation of market portfolio returns

It is the additional expected return that the market requires in order to accept an additional unit of risk, as measured by the portfolio standard deviation of return.

It is equal to the gradient of the capital market line in E – σ space

ii) **a)** $E_P = 18\%$

$\sigma^2_M = 4\%\% \Rightarrow \sigma_M = 2\%$

$r = 4\%$

$E_M = 12\%$

$$Ep - r = \sigma_p \frac{(E_M - r)}{\sigma_M}$$

$$18 - 4 = \sigma_p \frac{(12 - 4)}{2}$$

$\sigma_p = 3.5\%$

b) The efficient portfolio is a mix of the market portfolio and the risk-free asset. If the weights (which sum to 1) are x_M and x_0, then the expected return is

$x_M E_M + x_0 r = Ep$

$x_M * 12 + (1 - x_M) * 4 = 18$

⇨ $x_M = 1.75$

⇨ $x_0 = -0.75$

Thus the efficient portfolio has Rs. 2,100,000 in the market portfolio and is short Rs. 900,000 in cash.