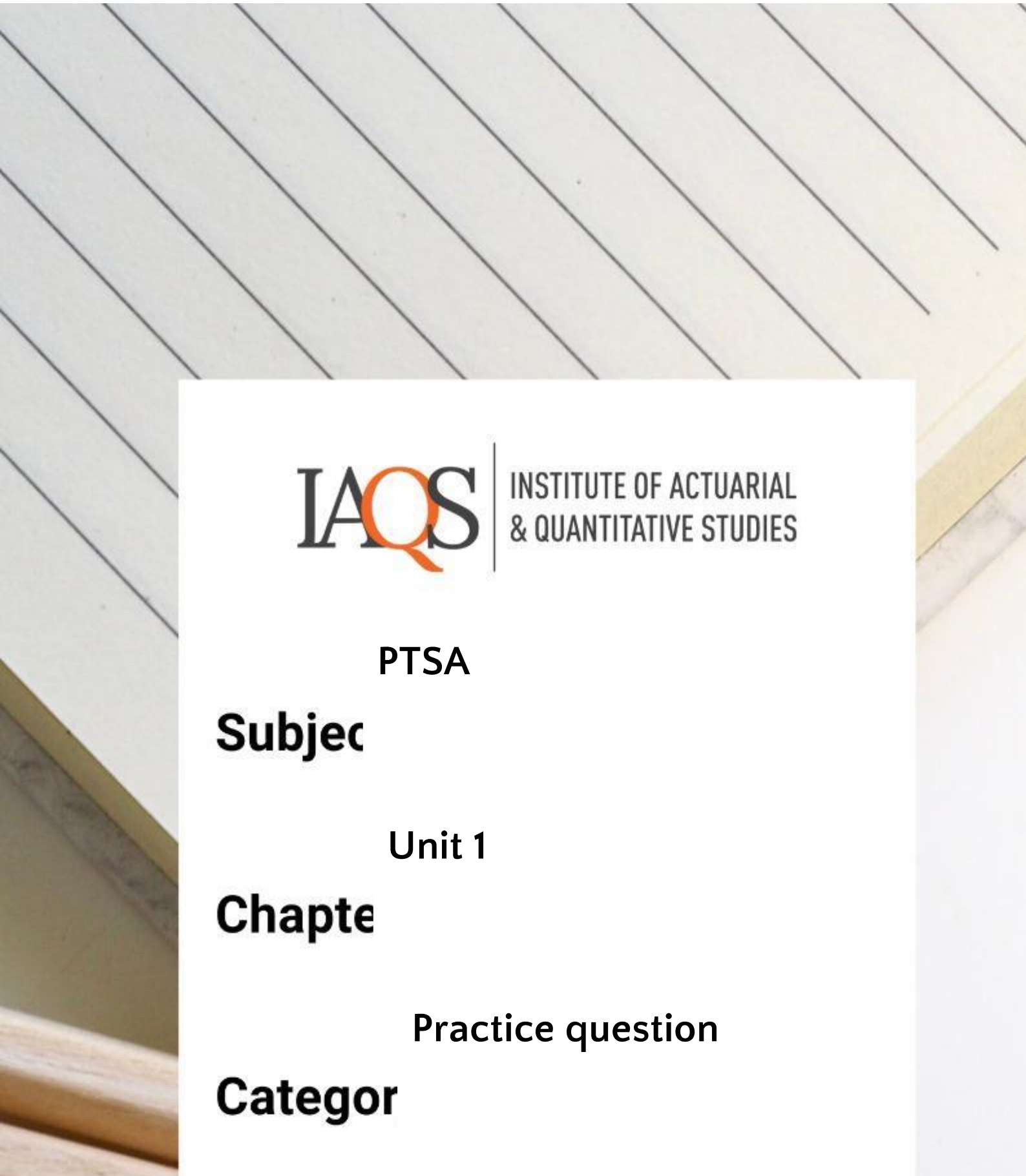

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PTSA

Subjec

Unit 1

Chapte

Practice question

Categor

1. CT8 April 2010 Q8

Outline the main points you would make in a discussion of the statement:

The efficient markets hypothesis states that the market price is always correct and therefore it is not possible for investors to make money from investing in shares.

2. CT8 April 2010 Q9

An asset is worth 100 at the start of the year and is funded by a senior loan and a junior loan of 50 each. The loans are due to be repaid at the end of the year; the senior one with interest at 6% p.a. and the junior one with interest of at 8% p.a. Interest is paid on the loans only if the asset sustains no losses

Any losses of up to 50 sustained by the asset reduce the amount returned to the investor in the junior loan by the amount of the loss. Any losses of more than 50 mean that the investor in the junior loan gets 0 and the amount returned to the investor in the senior loan is reduced by the excess of the loss over 50.

The probability that the asset sustains a loss is 0.25. The size of a loss, L, if there is one, follows a uniform distribution between 0 and 100.

(i) Calculate the variances of return for the investors in the junior and senior loans.

(ii) Calculate the shortfall probabilities for the investors in the junior and senior loans, using the full return of the amounts of the loans as the respective benchmarks.

[(i) 388,110; (ii) 0.25,0.125]

3. CT8 September 2010 Q1

An investor holds an asset that produces a random rate of return, R, over the course of a year. The distribution of this rate of return is a mixture of normal distributions, i.e. R has a normal distribution with a mean of 0% and standard deviation of 10% with probability 0.8 and a normal distribution with a mean of 30% and a standard deviation of 10% with a probability of 0.2.

S is the normally distributed random rate of return on another asset that has the same mean and variance as R.

Unit 1

PRACTICE QUESTION

(i) Calculate the mean and variance of R.

(ii) Calculate the shortfall probabilities for R and for S using:
(a) a benchmark rate of return of 0%
(b) a benchmark rate of return of –10%

(iii) Comment on what the variance and shortfall probabilities at both benchmark levels illustrate about the asset returns, by referring to the calculations in (i) and (ii).

[(i) 6%, 0.1562^2, (ii) (a) 0.40027,0.3504, (b) 0.1269, 0.1528]

4. CT8 April 2011 Q4

(i) Outline the three forms of the efficient market hypothesis.

XYZ has just announced that its profits are up by 52% on last year. On the announcement XYZ shares fell in price by 20%. Analysts had been predicting a rise in profits of 65%. A friend says that this shows that the efficient markets hypothesis is false.

(ii) Comment on this statement.

5. CT8 April 2012 Q5

Let X be a random variable denoting the rate of return on the fund ABC. The distribution of X is $N(\mu,\sigma^2)$

(i) Define VaR_α (X) with $\alpha \in [0,1]$

(ii) Show that:

$$VaR_\alpha = -\left(\mu + \sigma\Phi^{-1}(\alpha)\right)$$

where Φ denotes the cumulative Normal distribution function.
(Hint: Consider the probability that X is less than VaR_α).

(iii) Derive an expression for $TailVaR_\alpha$ (X) given that:

$$TailVaR_\alpha = \frac{1}{\alpha}\mathbb{E}(X|X < VaR_\alpha).$$

An investor holds £350m invested in ABC, the expected return on the fund is 10% and the standard deviation of that return is 25%.

(iv) Calculate the VaR and TailVaR of this investment when α = 0.01.

[(iv) VaR = 168.56, TailVaR = 198.21]

6. CT8 September 2012 Q10

Let A and B be two investment portfolios taking values in [a,b] with cumulative probability distribution functions of returns F_A and F_B respectively, and let the investor's smooth utility function be U.

(i) Write down the equation that the function U satisfies if the investor prefers more to less.

7. CT8 April 2013 Q1

List the key advantages and disadvantages of the following measures of investment risk in the context of a portfolio of bonds subject to credit risk:

- Variance of return
- Downside semi-variance of return
- Shortfall probability
- Value at Risk
- Tail Value at Risk

8. CT8 September 2013 Q1

(i) (a) State the expected utility theorem.

(b) State the four axioms from which it can be derived.

(ii) Explain of the concepts of non-satiation and risk aversion, showing how they can be expressed in terms of a utility function.

A quadratic utility function is given by the equation $U(w) = w + bw^2$

The value of absolute risk aversion at a value of wealth of one unit is 0.25

(iii) Calculate the value of b and the range over which U'(w) satisfies the condition of non-satiation.

[(iii) b=-0.1, $-\infty<w<5$]

9. CT8 September 2013 Q3

(i) Outline the three forms of the Efficient Markets Hypothesis (EMH).

(ii) Discuss the following two scenarios in the light of the EMH:

Scenario 1: Company A's share price falls suddenly, immediately after news of an earthquake in the capital city of one of its major markets.

Scenario 2: Company B's share price falls suddenly, when a long-awaited and publicly negotiated merger is completed.

10. CT8 April 2015 Q1

(i) State in words the four axioms of the Expected Utility Theorem.

(ii) State the conditions for an investor to be non-satiated and risk neutral in terms of their utility function, U(w).

An investor makes investment decisions using utility function $U(w) = (w^{\gamma} - 1)/\gamma$.

(iii) Derive the relative risk aversion function for U(w).

(iv) Describe how the relative risk aversion of U(w) changes with w.

[(iii) $1-\gamma$]

11. CT8 April 2015 Q2

Consider an asset the annual return, X, on which has probability density function f(x).

(i) Define the 5% Value at Risk for this asset.

(ii) Define the expected shortfall of the return on this asset below 2%.

Assume X has a Normal distribution with mean $\mu = 5\%$ and variance $\sigma^2 = 100\%\%$.

(iii) Calculate the 5% Value at Risk.

(iv) Discuss the limitations of using Value at Risk to measure the downside risk in an investment portfolio.

[(iii) 11.45%]

12. CT8 September 2015 Q2

An investor makes decisions using a quadratic utility function, $U(w) = a + bw + cw^2$.

(i) Write down the absolute and relative risk aversion for this utility function. The investor currently has wealth of £100, and using her utility function U(100) = 610.

The investor is offered a gamble with a profit of £20 with probability p, and a loss of £20 with probability (1 - p). She will accept this gamble only if $p \geq 0.55$.

(ii) Explain what this implies about the investor's risk aversion.

(iii) The investor accepts the gamble and wins. She now has wealth of £120.

The investor is offered the same gamble again, with a profit of £20 with probability p, and a loss of £20 with probability (1 - p). Based on her new wealth, she will now accept this gamble only if $p \geq 0.5625$.

Determine a, b and c.

(iv) Determine the maximum wealth for which the function U(w) satisfies the requirement of non- satiation.

[(iii) a =(17080-25*U(120))/3, b = (U(120)-610)/9, c = - (U(120) -610)/3600, (iv) 200]

13. CT8 April 2016 Q1

An investor measures the utility of her wealth using the utility function $U(w) = \ln(w)$ for $w > 0$.

(i) Derive the absolute and relative risk aversions for this investor's utility function, and the first derivative of each.

(ii) Comment on what this tells us about the proportion of her assets that this investor will invest in risky assets.

The investor has £100 available to invest in two possible assets, Asset A and Asset B. The future value of Asset A depends on an uncertain future event.

Every £1 invested in Asset A will be worth £1.30 with probability 0.75 and £0.40 with probability 0.25.

Asset B is risk-free, so every £1 invested in Asset B will always be worth £1.

The investor does not discount future asset values when making investment decisions. She decides to invest a proportion a of her wealth in Asset A and the remaining proportion 1 – a in Asset B.

(iii) Express her expected utility of wealth in terms of a.

(iv) Determine the amount that she should invest in each of Asset A and B to maximise her expected utility, using your result from part (iii).

[(iii) 0.75*ln(100+30a)+ 0.25*ln(100-60a), (iv) 0.4167]

14. CT8 April 2016 Q2

Consider an asset whose return follows the probability density function $f(x)$.

(i) Write down a formula for the variance of the return on the asset, defining any additional notation you use.

Unit 1

PRACTICE QUESTION

(ii) Write down a formula for the shortfall probability for the return on the asset below a level L.

The returns on an asset follow a Normal distribution with mean $\mu = 6\%$ per annum and variance $\sigma^2 = 23\%$ per annum. An investor buys €500 of the asset.

(iii) Determine the shortfall probability for the value of the asset in one year's time below a value of €480.

(iv) Explain what can be deduced about an investor's utility function if the investor makes decisions based on:
(a) the variance of returns.
(b) the shortfall probability of returns.

[(iii) 0.417]

15. CT8 April 2016 Q5

(i) Define the three forms of the Efficient Markets Hypothesis.

(ii) State two reasons why it is hard to test whether any of the three forms hold in practice.

16. CT8 September 2016 Q1

Consider an asset whose return follows the probability density function $f(x)$.

(i) Write down a formula for the Value at Risk for the asset, at confidence level p.

(ii) Write down a formula for the downside semi-variance of the return on the asset, defining any additional notation you use.

(iii) State the arguments for and against using semi-variance as a risk measure.

A farmer has a small apple tree which produces one harvest of apples per year. The number of apples the tree produces follows a Poisson distribution with a mean and variance of 8.

(iv) Determine the 10% Value at Risk level for the number of apples produced.

(v) Determine the expected shortfall below a harvest of 5 apples.

[(iv) 5, (v) 0.159]

17. CT8 April 2017 Q1

(i) State the expected utility theorem.

A risk averse investor makes decisions using a quadratic utility function:
$U(w) = w + dw^2$.

(ii) Derive an upper bound for d for this investor.

(iii) Explain why the investor can only use this utility function to make decisions over a limited range of wealth, w. Your answer should include a statement of this range.

The investor states that the upper limit of wealth where she can use this utility function is w = $1,000.

(iv) Determine the value of d in the investor's utility function.

The investor wins a prize of $250 in a gameshow. She is then offered the opportunity to exchange this prize for a larger prize of $600 if she can answer one more question correctly. However, she will receive no prize at all if she gets the question wrong. She estimates her chances of answering the question correctly to be 50%.

(v) Determine whether the investor should take this opportunity to exchange.

[(iv) -0.0005, (v) NO]

18. CT8 April 2018 Q1

Unit 1

PRACTICE QUESTION

A horse racing fan assesses her utility of wealth using the utility function
$U(w) = 2(w^{0.5}-1)$.

(i) Prove algebraically that the horse racing fan is:
(a) non-satiated
(b) risk averse.

(ii) Prove that the horse racing fan exhibits constant relative risk aversion.

The horse racing fan is attending a race and she intends to place bets on two horses. The table below shows the pay-out per £1 bet on each of these horses if it wins the race, and the investor's estimated probabilities of each horse winning the race. The pay-out is the total paid and is not in addition to the bet being returned.

Horse	*Winning pay-out per £1 bet*	*Probability of winning*
A	£1.69	60%
B	£6.25	10%

The horse racing fan has total wealth of £1,000 and she will bet all of her wealth on this race. Negative bets are not allowed.

(iii) Calculate the amount she should bet on each horse to maximise her expected utility of wealth.

(iv) Calculate the expected wealth resulting from the bets in part (iii).

(v) Explain how and why this differs from the utility of the horse racing fan's initial wealth.

[(iii) 906.8, 93.20, (iv) 49.801]

19. CM2A April 2019 Q1

(i) Describe the three forms of the Efficient Markets Hypothesis.

(ii) Describe the evidence against market efficiency in relation to:

(a) over-reaction of market prices to events
(b) under-reaction of market prices to events.

20. CM2A April 2019 Q8

A portfolio of derivatives is valued today at £1,000,000. The one-day return on the portfolio between today and tomorrow is normally distributed with parameters $\mu = 1\%$ and $\sigma = 20\%$.

(i) Calculate, in respect of the value of the portfolio tomorrow:
(a) the expected value of the portfolio
(b) the variance of the portfolio value
(c) the downside semi-variance of the portfolio value relative to the expected value
(d) the shortfall probability relative to £1,000,000.

(ii) Determine the 99% Value at Risk (VaR) over the one-day period, relative to the expected portfolio value.

(iii) Determine the minimum number of days that would need to pass such that the probability of a 99% one-day VaR event is at least 50%. You should assume that the returns each day are independent of each other.

[(i) a=10,10,000, b =$4*10^{10}$, c = $2*10^{10}$, d=0.48006, (ii) 465,260, (iii) n>69days]

21. CM2A September 2019 Q1

Consider a random variable X with probability density function f (x) and mean μ.

(i) Define algebraically the following risk measures for X:
(a) shortfall probability below a level L
(b) Value at Risk at the level p
(c) downside semi-variance.

An actuarial student is saving up to buy a car. He currently has £9,000 and he needs £10,000 to buy the car. There are three options for saving:

- Option 1: A bank account paying a guaranteed 10% per annum interest rate.

• Option 2: An equity fund whose returns are Gaussian with mean 15% p.a. and standard deviation 15% p.a.
• Option 3: A gamble that will return double the original investment with probability 65% or lose the entire investment otherwise.

Assume that the student can invest his entire savings in only a single investment option.

(ii) Calculate for each of these three investment options, for a one-year time horizon:
(a) The shortfall probability below £10,000
(b) The Value at Risk at the 75% level.

(iii) Recommend, with reasons, an appropriate investment strategy for the student, given the results in part (ii).

[(ii) a = 100%, 40%,35%, b = 9,900, 9439, 9000]

22. CM2A September 2020 Q1

(i) State how the following economic characteristics of investors can be expressed mathematically in a utility function U(w) for wealth w, defining any further notation you use:
(a) non-satiation.
(b) risk aversion.
(c) increasing absolute risk aversion.
(d) decreasing relative risk aversion.

A new insurer uses a quadratic utility function, where $U(w) = w + dw^2$ for some $d < 0$.

(ii) Derive the absolute risk aversion of the insurer's utility function.

(iii) Demonstrate whether the insurer's absolute risk aversion is decreasing, constant or increasing, relative to wealth.

The insurer is about to write its first two contracts on a new policy. The contracts will be identical but independent, each with a 10% probability of a claim. Each claim costs £100 and there can be, at most, one claim per policy. The insurer's initial wealth is £250. The insurer uses the utility function with d = –0.001.

(iv) Calculate the premium, p, that the insurer should charge per policy such that the insurer's expected utility of wealth after writing the policies is equal to the insurer's current utility of wealth.

(v) Comment on whether the policy might be attractive to customers at this premium.

[(ii) -2d/(1+2dw), (iii) increasing, (iv) 11.81]

23. CM2A April 2021 Q1

An investor has $800 to invest, for a period of 1 year, and has identified two investment opportunities in which to invest. The first is a direct investment in a stock index for a period of 1 year. The annual return, X, on the index follows a Normal distribution with mean μ = 7% p.a. and standard deviation σ = 5.5% p.a.

(i) Calculate the following in respect of the investment at the end of 1 year:

(a) The shortfall probability below a value of $720

(b) The 99.5% value at risk.

The second opportunity is a derivative that offers the following payoff in 1 year's time based on the performance of the index during the year.

Payoff ($)	*Scenario*
730	when $X \leq -7.1\%$
750	when $-7.1\% < X \leq 7\%$
962	when $X > 7\%$

(ii) Calculate the expected payoff from the derivative at the end of the year.

(iii) Calculate the following in respect of the payoff from the derivative:

(a) The shortfall probability below a value of $720

(b) The 99.5% value at risk.

(iv) Comment on how the investor may choose between the two investments.

[(i) a= 0.1%, b= 742.66, (ii) 855.90, (iii) a= 0%, b= 730]

24. CM2A April 2021 Q3

Two assets are available for investment. Asset A returns 2X%, where X is a Binomial random variable with parameters n = 6 and p = 0.4. Asset B returns 1.5Y%, where Y is a Normal random variable with parameters $\mu = 3.2$ and $\sigma = 2$.

(i) Calculate the following separately for each of assets A and B:

(a) The variance of the return

(b) The shortfall probability below a return of 3%.

An investor with a quadratic utility function wishes to invest in either Asset A, or Asset B.

(ii) Explain which asset the investor should choose.

The investor is now considering whether to split their investment between both assets.

(iii) Explain whether the investor may decide to split the investment in each of the following circumstances:

(a) If the assets are independent

(b) If the assets exhibit some positive correlation.

[(i.a) V(A)=5.76%%, V(B)=9%%, (i.b) 0.2333, 0.27425, (ii) Asset A]

25. CM2A April 2021 Q5

A risk-averse, non-satiated investor is trying to determine their utility of wealth function, U(w). They have decided to use the utility function $U(w) = w + dw^2$, where $d < 0$ is a constant that the investor has chosen.

(i) Derive an upper bound in terms of d for the range of values of w over which U(w) can be used.

(ii) Explain why $d < 0$ is a necessary condition for U(w) to be a valid utility function.

The investor lives on a tropical island. On this island, root vegetables can be bought once a week for \$10 per box. The investor knows that they will be able to sell any vegetables they buy for \$30, \$12, \$10 or \$0.5 per box with equal probability. All boxes of vegetables sold at a given time will be sold for the same price. The investor's current

wealth is $100. If the investor were to buy seven boxes of vegetables, their expected utility of wealth after selling them would be 50.

(iii) Calculate the value of d.

(iv) Calculate the expected utility of wealth of the investor, if they do not buy any vegetables.

The investor has decided they want to buy seven boxes of vegetables.

(v) Discuss whether U(w) is appropriate for the investor.

[(i) w<-1/2d, (iii)-0.003518, (iv)64.82]

26. CM2A September 2021 Q1

An investor makes decisions using the utility function $U(w) = \ln(w)$ where $w > 0$.

The investor is going to invest $100 now for a period of 1 year, and has identified the following two assets to invest in:

- Asset A is risk-free and will not change in value over the year.
- Asset B will increase in value by 50% over the year with probability 0.6 or decrease in value by 50% over the year with probability 0.4.

The investor does not make any allowance for discounting when making investment decisions. They are going to invest a proportion, x, of their wealth in Asset A and the remaining proportion, (1 – x), in Asset B.

(i) Construct a formula, in terms of x, for their expected utility at the end of the year.

(ii) Determine, using your result from part (ii), the amount that the investor should invest in each asset to maximise their expected utility.

[(i)0.6*ln(150-50x)+0.4*ln(50+50x), (ii) A=60,B=40]

27. CM2A September 2021 Q2

Consider an exponential distribution with parameter $\lambda = 2$, and a lognormal distribution with parameters $\mu = -1.04$, $\sigma = 0.833$.

(i) Calculate for each distribution:

(a) the mean.

(b) the variance.
(c) the 99th percentile.

(ii) Comment on your answers to part (i) in the context of choosing a distribution for financial modelling.

(iii) Comment on why the lognormal distribution may be preferred to the exponential distribution for modelling a security price.

[(i)a = 0.5,0.5, b = 0.25, 0.25, c = 2.30259, 2.45]

28. CM2A September 2021 Q8

Consider a random variable X with probability density function $f_X(x)$.

(i) Write down the formula for the following in terms of $f_X(x)$:
(a) Variance
(b) Downside semi-variance
(c) Expected shortfall relative to a level.

A trader has built a Value at Risk (VaR) model of a security that fits a distribution to underlying historical data. The modelled 1-day 99% VaR for this security is \$L, meaning that there is a 1% chance that the trader loses more than \$L in 1 working day by holding this security.
The trader is examining the effectiveness of their model over a month with 20 working days in it. They have assumed that day-on-day movements of the security are all independent from one another.

(ii) Demonstrate that, assuming the VaR model is correct, the probability the trader loses more than \$L on at least 3 working days in the month is 0.001.

Over this month, a market crash occurs. On each of 10 separate working days during the month, the security generates losses in excess of \$L per day.

(iii) Discuss, without any further calculations, the effectiveness of VaR as a risk measure, given this information and your answer to part (ii).

[(ii) 0.001]

29. CM2A September 2021 Q11

Academic studies have shown that lemurs (primates from the island of Madagascar), are risk averse and non-satiated. A zoologist is trying to determine an appropriate utility function, U(w), to model their behaviour in an experiment.

(i) Determine, for each of the following functions, whether the zoologist could use it as a valid utility function:

a) U(w) = $w + w^2 \; for - \infty < w \leq 3$

b) $U(w) = \frac{(w^\gamma - 1)}{\gamma} \; for \; 0 < w < \infty \; and \; \gamma < 1$

c) $U(w) = w - 2w^2 \; for - \infty < w \leq 3$

The zoologist has chosen the function U(w) =ln(1 + w). The zoologist now carries out an experiment, and presents the lemurs with two options:

- Scenario A: w = 3 or w = 0 with equal probability
- Scenario B: w = 1.1 with certainty.

The zoologist finds that the lemurs prefer Scenario B. Assume that the initial wealth of the lemurs is w = 0, and that they behave rationally in the experiment.

(ii) Show that the utility function the zoologist has chosen is consistent with the behaviour of the lemurs in the experiment

[(i) a = NO, b = YES, c = NO]

30. CM2A April 2022 Q7

An investor makes decisions based on the utility function U(w) = w – 6w^2, where w is the investor's wealth in millions of dollars ($m).

(i) Demonstrate that the investor has both increasing absolute and relative risk aversion.

The investor has $50,000 to invest over a 1-year period and has no other wealth. They have three options:

A. Invest in a risk-free account. There will be no change in the value of the investment over 1 year.

B. Invest in an asset that will give a 60% return over 1 year with probability 0.2, a 20% return with probability 0.7 and a –40% return with probability 0.1.
C. Invest in an asset that will give a 30% return with probability 0.5 and a 20% return with probability 0.5.

The investor makes no allowance for discounting when making investment decisions. The investor must invest the whole $50,000 in a single option.

(ii) Determine which option the investor should choose to maximise their expected utility at the end of the year.

(iii) Comment on why the investor could not use U(w) to choose from the above options if their initial wealth was $65,000.

[(ii) Option C]

31. CM2A September 2022 Q1

An individual has the following utility function:

$$U(w) = \frac{(w^{\gamma} - 1)}{\gamma}, (w > 0),$$

where w is wealth in $000s. Their current wealth is $8,000 and their current utility is 2.1012.

(i) Show that γ = 0.01 to two decimal places.

(ii) Show that U(w) exhibits declining absolute risk aversion and constant relative risk aversion.

The individual has been offered a ticket to enter a lottery with a 1 in 10,000 chance to win $1m.

(iii) Calculate, to the nearest $, the maximum price, P, that the individual would pay for the ticket.

(iv) Discuss why this form of utility function with γ > 1 would be inconsistent with common utility theory.

Unit 1

PRACTICE QUESTION

32. CM2A September 2022 Q9

A pension fund has been offered two investment opportunities.

Asset A gives an annual return of 3B%, where B is a binomial random variable with parameters n = 4 and p = 0.4.

Asset B gives an annual return of 4P%, where P is a Poisson random variable with parameter μ = 2.

Calculate the following three measures of investment risk for each asset:

(a) Variance

(b) Semi-variance

(c) Shortfall probability versus a benchmark return of 4%.

[(a) A=8.64%%, B=32%%, (b) A=4.106%%, B =12.992%%, (c) A=0.4752, B= 0.13534]