INSTITUTE OF ACTUARIAL & QUANTITATIVE STUDIES

Subject: PTSA

Chapter: Unit 3

Category: Practice questions

1. CT8 April 2011 Q3

A securities market has only three risky securities, A, B and C with the following annual return attributes:

	Asset A	*Asset B*	*Asset C*
Market capitalisation	£100bn	£150bn	£250bn
Annual expected return	4%	r_B	6%

Assume that:

- the assumptions of the Capital Asset Pricing Model hold
- the market price of risk is 10% per annum
- the risk free rate is 3.3% per annum
- the expected annual return on the market
- portfolio is 5.3% per annum.

(i) Calculate σ_M, the standard deviation of the annual return on the market portfolio. Quote any results that you use.

(ii) Calculate r_B, the expected annual return on asset *B*.

(iii) Calculate the covariance of the annual returns on each asset with the annual return on the market portfolio. State any further results that you use.

[(i) 20%, (ii) 5%, (iii) a= 0.014, b= 0.034, c= 0.054]

2. CT8 April 2012 Q2

In a market where the CAPM holds there are five risky assets with the following attributes per year.

Asset number	*1*	*2*	*3*	*4*	*5*
Expected return	6%	5%	8%	13%	11%
Market capitalisation (in \$)	2.6m	3.9m	5.2m		1.3m
Beta				1.5	

The risk-free rate is r = 1% p.a.

(i) Calculate the expected return on the market portfolio.

(ii) Deduce the market capitalisation of asset 4 and the betas of all the other assets.

(iii) Calculate the beta of a portfolio P which is equally weighted in the five assets and the risk-free asset.

(iv) Explain whether or not this portfolio P lies on the Capital Market Line.

[(i) 9%, (ii) 6.5m, $\beta_1 = \frac{5}{8}, \beta_2 = \frac{1}{2}, \beta_3 = \frac{7}{8}, \beta_4 = 1.5, \beta_5 = \frac{5}{4}$,(iii) 19/24]

3. CT8 September 2012 Q3

(i) State the three main assumptions of Modern Portfolio Theory.

Assume Modern Portfolio Theory holds true.

(ii) Write down equations for the expected return, E and variance, V of a portfolio of N securities, defining any notation used.

(iii) Describe how an efficient portfolio can be found.

4. CT8 September 2012 Q9

Consider a market where there are two risky assets A and B and a risk free asset. Both risky assets have the same market capitalisation.

Assume that all the assumptions of the CAPM hold.

(i) State the composition of the market portfolio.

(ii) Derive the expressions for the variance of the market portfolio and for the beta of each asset, in terms of the variance of each asset and of their covariance.

Assume now that the risk-free rate is r_f = 10%, the expected return of the market portfolio is r_M =18%, the variance of asset A is 4%, the variance of asset B is 2% and their covariance is 1%.

(iii) Derive the value for the expected return on asset A and asset B.

An investor wants an expected return of 20%.

(iv) Calculate the composition of the corresponding portfolio.

(v) Derive the corresponding standard deviation using the Capital Market Line.

[(i)0.5,0.5, (iii) a=0.2, b=0.16, (iv)$w_0 = -0.25, w_M = 1.25$, (v)17.6%]

5. CT8 April 2013 Q2

Consider a mean-variance portfolio model with two securities, S_A and S_B, where the expected return and the variance of return for S_B are twice the corresponding values for S_A. Suppose the correlation between the returns on the two securities is ρ.

(i)

a) Determine the values of ρ which allow the possibility of constructing a zero-risk portfolio, by calculating the variance of the return on a portfolio with weights x_A and x_B invested in the two assets.

b) Calculate the portfolio weights that lead to the most efficient zero-risk portfolio.

c) Calculate the expected return on the portfolio in part (i)(b) in terms of the expected return on S_A.

(ii) Calculate the maximum expected return for an investor:

(a) if portfolio weights are unlimited.

(b) if the investor can short sell at most one unit of either security and the total he has to invest is one unit.

(iii) Calculate the expected return on the minimum variance portfolio if the covariance between the two securities is 60% of the variance of S_A.

[(i) a- ρ=-1, b - $x_A = \frac{\sqrt{2}}{\sqrt{2}+1}$, $x_B = \frac{1}{\sqrt{2}+1}$, c - $E_A.\frac{\sqrt{2}+2}{\sqrt{2}+1}$, (iii) $1.2222E_A$]

6. CT8 April 2013 Q4

In a market where the CAPM holds there are five assets with the following attributes.

Asset	*A*	*B*	*C*	*D*	*E*	*Probability of being in state*
Annual return in						
State 1	3 %	3 %	3 %	3 %	3 %	0.25
State 2	5	7	2	8	3	0.5

	%	%	%	%	%	
State 3	7 %	5 %	8 %	1 %	3 %	0.25
Market Capitalisation	10 m	20 m	40 m	30 m		

(i) Calculate the expected annual return on the market portfolio and σ*M*, the standard deviation of the annual return on the market portfolio.

(ii) Calculate the market price of risk under CAPM.

(iii) Calculate the beta of each asset.

(iv) Outline the limitations of the CAPM.

[(i) $E_M = 4.6\%, \sigma_M = 0.92466\%$, (ii) 173%, (iii) $\beta_A = 1.2865, \beta_B = 1.5205, \beta_C = 0.5848, \beta_D = 1.1111$]

7. CT8 September 2013 Q2

(i) Describe the single-index model of security returns, defining any terms used.

The single-index model is to be used in a particular market.

(ii) Determine the following results:

(a) the expected return on a security

(b) the variance of returns on a security; and

(c) the covariance of returns between two securities in the market, using the parameters described in part (i).

(iii) Show that investors can diversify away specific risk in this model by holding equal weights in an increasing number of securities.

(iv) State the potential impact of adding additional indices to the model:

(a) in terms of explaining historic data.

(b) in terms of forecasting security returns.

8. CT8 April 2014 Q2

(i) State the expression for the return on a security, *i*, in the single index model, defining all terms used.

(ii) Explain the difference between the single-index model and the Capital Asset Pricing Model.

Suppose the market has expected return 6% and standard deviation 10%. Two securities have expected returns 8% and 10%, and standard deviations 15% and 20%. The correlation between these two securities and the market is 0.25 and 0.4 respectively. Assume the single-index model described in (i) holds.

(iii) Calculate the constant parameters in the expression for the return of these two securities.

(iv) Explain how a multi-index model would be expected to perform relative to the single-index model in terms of fitting data and predicting future security price moves.

[(iii) $\alpha_1 = 5.75, \beta_1 = 0.375, \alpha_2 = 5.2, \beta_2 = 0.8$]

9. CT8 April 2014 Q6

(i) State the equation for the capital market line in the Capital Asset Pricing Model (CAPM), defining all the terms used.

In a market where the CAPM is assumed to hold, the expected annual return on the market portfolio is 12%, the variance is 4%% and the effective risk-free annual rate is 4%. An Agent wants an expected annual return of 18% on a portfolio worth £1,200,000.

(ii) Calculate the standard deviation of the return on the corresponding efficient portfolio.

(iii) Calculate the amount of money invested in each component of the Agent's portfolio.

[(ii) 3.5%, (iii) $w_M = 1.75, w_0 = -0.75$]

10. CT8 April 2014 Q9

Outline the evidence against normality assumptions in models of market returns.

11. CT8 September 2014 Q2

An investor wishes to allocate her capital between a service company S and a manufacturing company M. The investor believes that returns on shares in S have mean 10% and variance 16%% while returns on shares in M have mean 8% and variance 25%%. The correlation between the two companies is 0.3.

Assume the investor chooses their investments according to mean-variance portfolio theory.
(i) Explain which company's share she would prefer.

Assume the investor's preferences are described by a standard quadratic utility function.
(ii) State which assumption of the mean-variance portfolio theory can be relaxed.

(iii) Calculate the expected return and the standard deviation of a portfolio which is invested three quarters in S and one quarter in M.

(iv) Calculate the minimum variance portfolio.

A new study suggests that in the future, S will make more sales to M, when M is delivering strong profits.
(v) Describe the effect this will have on portfolios composed of M and S, including the minimum-variance portfolio.

[(iii) E= 9.5%, S.D=3.57945%, (iv) 0.655173, 0.344828]

12. CT8 April 2015 Q8
(i) State the main assumptions of mean-variance portfolio theory.

There are only three assets available on a stock exchange:

- Asset 1, expected return 2%, standard deviation 4%
- Asset 2, expected return 4%, standard deviation 12%
- Asset 3, expected return 3%, standard deviation 8%

The correlation between the returns on assets 1 and 3 is 0.75. The return on asset 2 is uncorrelated with the returns on the other two assets.

An investor in this market wants to minimise the variance of his portfolio.

(ii) Determine the Lagrangian function that can be used to find the minimum variance portfolio for a given expected return.

Let x_i denote the weight of asset i (i = 1, 2, 3) in the minimum variance portfolio with an expected return of 4%.

(iii) Show, by taking partial derivatives of the Lagrangian function in part (ii), that:
$x_1 = -0.45$, $x_2 = 0.55$, $x_3 = 0.9$.

(iv) Comment on how the portfolio would change if short-selling was not allowed.

13. CT8 September 2015 Q3

(i) Define an "efficient portfolio" in the context of mean-variance portfolio theory.

(ii) State the assumptions required for the existence of efficient portfolios.

Suppose an investor invests his wealth in N securities, $i = 1, \ldots, N$, with x_i denoting the proportion of wealth invested in security i.

(iii) Write down a formula for the expected return on this portfolio.

(iv) Write down a formula for the variance of the return on this portfolio.

Now suppose the investor invests in only two securities, A and B.

(v) Derive the proportion x_A that should be invested in security A to minimise the portfolio variance.

14. CT8 April 2016 Q4

In a market where the assumptions of the Capital Asset Pricing Model (CAPM) hold, there are a risk-free asset and two risky assets with the following attributes:

		Rate of return (per annum)		
State	*Probability*	*Asset 1*	*Asset 2*	*Asset 3*
1	0.2	5.0%	15.0%	26.0%
2	0.3	5.0%	22.0%	15.0%
3	0.1	5.0%	10.0%	24.0%
4	0.4	5.0%	28.0%	7.0%
Market capitalisation			30,000	70,000

(i) Determine the composition of the market portfolio.

(ii) Determine the market price of risk.

(iii) Calculate the beta of each risky asset.

(iv) State the limitations of the CAPM.

[(i) 0.3,0.7, (ii) 3.35, (iii) $\beta_2 = 1.40, \beta_3 = 0.83$]

15. CT8 September 2016 Q2

(i) State the main assumptions of mean-variance portfolio theory.

Consider a mean-variance portfolio model with two securities, with respective returns S_A and S_B, where the expected return $E[S_B] = 0.25E[S_A]$ and the variance of return $V[S_B] = 0.25V[S_A]$.

Let the correlation between the returns on the two securities be ρ.

(ii) Determine, in terms of $E[S_A]$, the expected return on the minimum variance portfolio if:

(a) $\rho = 0$

(b) $\rho = 1$

(iii) (a) Calculate the variance of the return on the minimum variance portfolio for part (ii)(b).

(b) Comment on the risk in this portfolio.

[(ii) a= $0.4E_A$, b =$-0.5E_A$, (iii) a= 0]

16. CT8 September 2016 Q3

In a market where the assumptions of the Capital Asset Pricing Model hold, there are two risky assets with the following attributes:

Security	*A*	*B*
Expected return (p.a.)	20%	16%

(i) Determine the composition of the market portfolio with expected return 18% per annum.

(ii) Calculate the beta of each security under the assumption that the risk-free rate of interest is 10% per annum.

[(i) 0.5, 0.5, (ii) $\beta_A = 1.25, \beta_B = 0.75$]

17. CT8 April 2017 Q9

Let R_i denote the return on security i in a two-factor model.

(i) Write down the return equation for this two-factor model, defining all additional notation that you use.

(ii) Describe the three main categories of multifactor models.

18. CT8 April 2017 Q10

In a market in which the Capital Asset Pricing Model (CAPM) holds, there are two securities with the following attributes (expressed per annum):

	security	*A*	*B*
$E(r_i)$		0.196	0.164
$\text{Cov}(r_i, r_j)$	A	0.05	0.01
	B	0.01	0.03

(i) Determine the composition of the market portfolio with expected return 18% per annum.

(ii) Calculate the beta of each security, under the assumption that the risk-free rate of interest is 10% per annum.

(iii) State the limitations of the CAPM.

[(i) 0.5,0.5, (ii) $\beta_A = 1.2, \beta_B = 0.8$]

19. CT8 September 2017 Q2

(i) Define in the context of mean-variance portfolio theory:

(a) an inefficient portfolio

(b) an efficient portfolio

(ii) State the two assumptions about investor behaviour that are needed for the existence of efficient portfolios.

An investment universe includes two assets, A and B, with expected return on asset i of r_i and variance v_i as set out below:

Asset i	*Expected return* r_i	*Variance of return* v_i
A	$r_A = 0.05$	$v_A = 0.16$
B	$r_B = 0.07$	$v_B = 0.25$

The correlation of returns is $c_{AB} = -0.2$.

In an efficient portfolio, let a be the proportion which is held in asset A.

(iii) Express the portfolio variance V in terms of a quadratic function in a, showing your workings.

Let R be the expected return on the portfolio.

(iv) Express the portfolio variance V in terms of a quadratic function in R, using your result from part (iii) and showing your workings. [Your expression should not include a.]

The expression in part (iv) represents the efficient frontier.

An investor uses a utility function that gives rise to an indifference curve $V = 16R - 200R^2$.

(v) Determine the two portfolios on the efficient frontier that also lie on the investor's indifference curve.

(vi) Comment on the implications for part (v) if short selling is not allowed in the market.

[(iii) $V = 0.49a^2-0.58a+0.25$, (iv) $V = 1225R^2-142.5R+4.2225$, (v) 0.1497, 1.2889]

20. CT8 April 2018 Q11

Consider a market in which the Capital Asset Pricing Model (CAPM) holds.

(i) List the assumptions, additional to those used in modern portfolio theory, of the CAPM.

(ii) Prove that the market portfolio has unit beta.

In the same market as above, there are two assets with the following attributes.

Rate of return (per annum)					*Variance/Covariance Matrix*		
State	*Probability*	*Asset 1*	*Asset 2*			*Asset 1*	*Asset 2*
1	0.2	5.00%	11.00%		Asset 1	0.00068	0.00102
2	0.3	10.00%	15.00%		Asset 2	0.00102	0.00181
3	0.1	8.00%	12.00%				
4	0.4	4.00%	5.00%				
Market capitalisation		40,000	60,000				

(iii) Calculate the beta of each security.

(iv) Determine the value of the risk-free rate of interest which is consistent with the results obtained in part (iii), under the assumption that the CAPM holds.

[(iii) $\beta_1 = 0.70915, \beta_2 = 1.1939$, (iv) 0.012797]

21. CT8 September 2018 Q11

Consider a market in which the Capital Asset Pricing Model (CAPM) holds.

(i) Write down the equation of the Security Market Line, defining all the notation you use.

In this market, the risk-free rate of interest is 9.44% per annum. There are only two risky assets in the market with the following attributes.

Rate of return (per annum)					*Variance/Covariance Matrix*		
State	*Probability*	*Asset 1*	*Asset 2*			*Asset 1*	*Asset 2*
1	0.2	10.00%	11.00%		Asset 1	0.00142	0.00379
2	0.3	15.00%	30.00%		Asset 2	0.00379	0.01146
3	0.1	18.00%	25.00%				
4	0.4	20.00%	40.00%				

(ii) Determine the weight of each asset in the market portfolio to be consistent with $\beta_1 = 0.46$, $\beta_2 = 1.36$.

(iii) Calculate the Market Price of Risk.

[(ii) x_1=0.4, x_2=0.6, (iii) 1.897]

22. CM2A April 2019 Q5

(i) Define an efficient portfolio in the context of portfolio theory.

(ii) State the two assumptions required in portfolio theory for the existence of an efficient portfolio.

(iii) Explain what it means if a portfolio is not on the efficient frontier.

Consider a market with just two securities, S_A and S_B, with expected return E_A and E_B, variance V_A and V_B and covariance C_{AB}.

(iv) Derive a formula for the amount, x_A, that should be invested in S_A to minimise the portfolio variance.

23. CM2A September 2019 Q7

A market that satisfies the assumptions of the Capital Asset Pricing Model (CAPM) comprises n assets.

Let the random return on asset i be denoted by R_i, the expected return by r_i, and the corresponding returns for the market portfolio by R_M and r_M. Let π_i be the proportion of asset i in the market portfolio.

(i) (a) Define β_i algebraically in this market.

(b) Write down the relationship between these expected returns in this market, including a definition of any additional notation that you use.

(c) Show that $\Sigma\pi_i \beta_i = 1$.

Consider a market where n = 4, that is, there are four assets in the market with the following attributes:

Asset	*1*	*2*	*3*	*4*	*Risk-free Asset*
Expected return (per annum)	14%	r_2	r_3	r_4	3%
Market capitalisation	£4m	£2m	£2m	£2m	

The variance-covariance matrix (in %%) of annual returns on the four assets is as follows:

Asset	*1*	*2*	*3*	*4*
1	4	1	1	1
2	1	3	1	1
3	1	1	2	1
4	1	1	1	x^2

for some $x^2 > 0$. The variance of the return on the market portfolio is 8/5 %%.

(ii) (a) Calculate the proportions of each asset in the market portfolio.

(b) Calculate β_1, β_2, β_3 and β_4.

(c) Calculate r_M, r_2, r_3 and r_4. [

[(ii) a= 2/5,1/5, 1/5,1/5, b = 11/8, 7/8, 6/8, 5/8, c = 11%, 10%, 9%, 8%]

24. CM2A April 2021 Q9

In a particular market, there are two assets, A and B, for which the annualised rates of return have the following characteristics:

Asset	*Expected value (%)*	*Standard deviation (%)*
A	10	20
B	5	0

An investor is considering investing a proportion of their wealth x_A in asset A and x_B in asset B.

(i) State the formula for the market price of risk in this market.

(ii) Show that the efficient frontier for this investor is a straight line passing through the points (0, 0.05) and (0.1, 0.075) in (Standard Deviation, Expected Return) space.

A third asset, C, becomes available in the market. It has an annualised expected return of 6% and an annualised standard deviation of 10%. It is uncorrelated with assets A and B.

(iii) Show that the new efficient frontier using A, B and C, passes through the point (0.1, 0.0769).
[Hint: the market price of risk for a portfolio involving only assets A and C is maximised when $x_A = 5/9$ and $x_C = 4/9$.]

25. CM2A September 2021 Q5

Consider the following assets in a world where the Capital Asset Pricing Model (CAPM) holds. There are three risky assets and one risk-free asset. No other assets exist in the market.

Asset	*Expected return (% p.a.)*	*Total value of assets in market ($m)*	*Beta*
Risky asset A	5	10	β
Risky asset B	10	50	1
Risky asset C	x	20	2
Risk-free asset	3	40	n/a

(i) Calculate the expected return on the market portfolio.

(ii) Calculate x.

(iii) Calculate β.

(iv) Discuss the limitations of the CAPM.

[(i) 12.25% or 10%, (ii) 21.5% or 17%, (iii) 0.216 or 0.286]

26. CM2A September 2022 Q8

Consider the following assets in a world where the capital asset pricing model holds. These are the only risky assets in the market.

Asset	*Expected return (% p.a.)*	*Total value of assets in market ($m)*	*Beta*
Risky asset A	3.5	20	1.5
Risky asset B	2.2	30	0.2
Risky asset C	4.4	10	2.4

(i) Calculate:
(a) the risk-free rate of interest.
(b) the expected return on the market portfolio.

The standard deviation of the return on the market portfolio is 10%.
(ii) Calculate the market price of risk.

The risk-free rate of interest now increases to 3% p.a.
(iii) Explain why one or more of the figures in the table must change.

[(i) a = 2% , b = 3%, (ii) 0.1]