Matrices

) Solving of equation's

$$x + 2y + 3z = 10$$
$$2x + y - z = 2$$
$$5x - 2y - 3z = 2$$

$$Ax = B$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & -1 \\ 5 & -2 & -3 \end{bmatrix} X = \begin{bmatrix} 10 \\ 2 \\ 2 \end{bmatrix}$$

$$R_2 = R_2 - 2R_1$$

$$R_3 = R_3 - 5R_1$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -7 \\ 0 & -12 & -18 \end{bmatrix} X = \begin{bmatrix} 10 \\ -18 \\ -48 \end{bmatrix}$$

$$R_3 = R_3 - 4R_2$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -7 \\ 0 & 0 & 10 \end{bmatrix} X = \begin{bmatrix} 10 \\ -18 \\ 24 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -7 \\ 0 & 0 & 10 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ -18 \\ 24 \end{bmatrix}$$

$$\begin{bmatrix} x + 2y + 3z \\ -3y - 7z \\ 10z \end{bmatrix} = \begin{bmatrix} 10 \\ -18 \\ 24 \end{bmatrix}$$

$$10z = 24$$

$$\underline{z = 2.4}$$

$$-3y - 7z = -18$$

$$-3y - 16.8 = -18$$

$$-3y = -1.2$$

$$\underline{y = 0.4}$$

$$x + 2y + 3z = 10$$

$$x + 0.8 + 7.2 = 10 \qquad \therefore \underline{x = 2}$$

Q) find eigen values of $\begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

Consider $|A - \lambda I| = 0$

$$\therefore \left| \begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} \right| = 0$$

$$\left| \begin{bmatrix} \overset{+}{2-\lambda} & \overset{-}{-3} & 0 \\ 2 & -5-\lambda & 0 \\ 0 & 0 & 3-\lambda \end{bmatrix} \right| = 0$$

$$(2-\lambda) \begin{vmatrix} -(5+\lambda) & 0 \\ 0 & 3-\lambda \end{vmatrix} + 3 \begin{vmatrix} 2 & 0 \\ 0 & 3-\lambda \end{vmatrix} = 0$$

$$\therefore \quad (2-\lambda)\left[-(5+\lambda)(3-\lambda)\right] + 3(6-2\lambda) = 0$$

$$\therefore \quad -(2-\lambda)(5+\lambda)(3-\lambda) + 6(3-\lambda) = 0$$

$$\therefore \quad [3-\lambda]\left\{(\lambda-2)(\lambda+5) + 6\right\} = 0$$

$$\therefore \quad (3-\lambda)(\lambda^2 + 3\lambda - 4) = 0$$

$$(3-\lambda)(\lambda-1)(\lambda+4) = 0$$

$$\therefore \lambda = 3, \ 1, \ -4$$

Eigen Vectors

for $\lambda = 3$

$$(A - \lambda I)X = 0$$

$$\left(\begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix} - \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix} \right) \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{array}{c} \begin{bmatrix} -1 & -3 & 0 \\ 2 & -8 & 0 \\ 0 & 0 & 0 \end{bmatrix} \\ \begin{array}{ccc} x_1 & \quad x_2 & \quad x_3 \end{array} \end{array} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$R_2 = R_2 + 2R_1$$

$$\longrightarrow \begin{bmatrix} -1 & -3 & 0 \\ 0 & \boxed{-14} & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$R_2 = R_2 / -14$$

$$\begin{bmatrix} -1 & -3 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$R_1 = R_1 + 3R_2$$

$$\longrightarrow \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$x_1 = 0 \qquad x_2 = 0 \qquad x_3 = \text{Indeterminate} = \alpha$

for $\lambda = 3$, eigen vector $= \begin{bmatrix} 0 \\ 0 \\ \alpha \end{bmatrix}$

for $\lambda = 1$

$$\left(\begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\right)\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -3 & 0 \\ 2 & -6 & 0 \\ 0 & 0 & 2 \end{bmatrix} X = 0$$

$$\begin{bmatrix} 1 & -3 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x_1 - 3x_2 \\ 0 \\ 2x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$x_3 = 0$$

$$x_1 - 3x_2 = 0$$

$$x_1 = 3x_2$$

eigen vect $= \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3x_2 \\ x_2 \\ 0 \end{bmatrix}$

$$X = \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix} x_2$$

$\lambda = -4$

$$\left(\begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix} + \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix} \right) \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 6 & -3 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & 7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$R_1 = R_1 - 3R_2$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & 7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$\begin{bmatrix} 0 \\ 2x_1 - x_2 \\ 7x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$7x_3 = 0 \qquad x_3 = 0$$

$$2x_1 - x_2 = 0$$

$$x_2 = 2x_1 \quad , \quad x_1 = \tfrac{1}{2} x_2$$

$$\text{eigen vector} = \begin{bmatrix} x_1 \\ 2x_1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} x_1 \text{ or } \begin{bmatrix} \tfrac{1}{2} x_2 \\ x_2 \\ 0 \end{bmatrix} = \begin{bmatrix} \tfrac{1}{2} \\ 1 \\ 0 \end{bmatrix} x_2$$

8)

Marks	0 – 19	20 – 39	40 – 59	60 – 79	80 – 99
No of Candidates	41	62	65	50	17

Marks less than	19	39	59	79	99
Candidates	41	103	168	218	235

$$h = 20$$

$$U = \frac{x - x_0}{h} = \frac{70 - 19}{20} = \frac{51}{20}$$

x	y	Δy	$\Delta^2 y$	Δ^3	Δ^4
19	41				
		62			
39	103		3		
		65		−18	
59	168		−15		0
		50		−18	
79	218		−33		
		17			
99	235				

$$f(x) = 41 + \frac{51}{20}\,62 + \frac{\frac{51}{20}\left(\frac{51}{20} - 1\right)}{2!}\,3 + \frac{\frac{51}{20}\left(\frac{51}{20} - 1\right)\left(\frac{51}{20} - 2\right)}{3!}(-18)$$

$$= \underline{\underline{198.5}}$$