$$\frac{d\, p_{ij}(t)}{dt} = \boxed{\sum_{k \in S.S.} p_{ik}(t) \cdot \mu_{kj}}$$

Small time prob. of transition

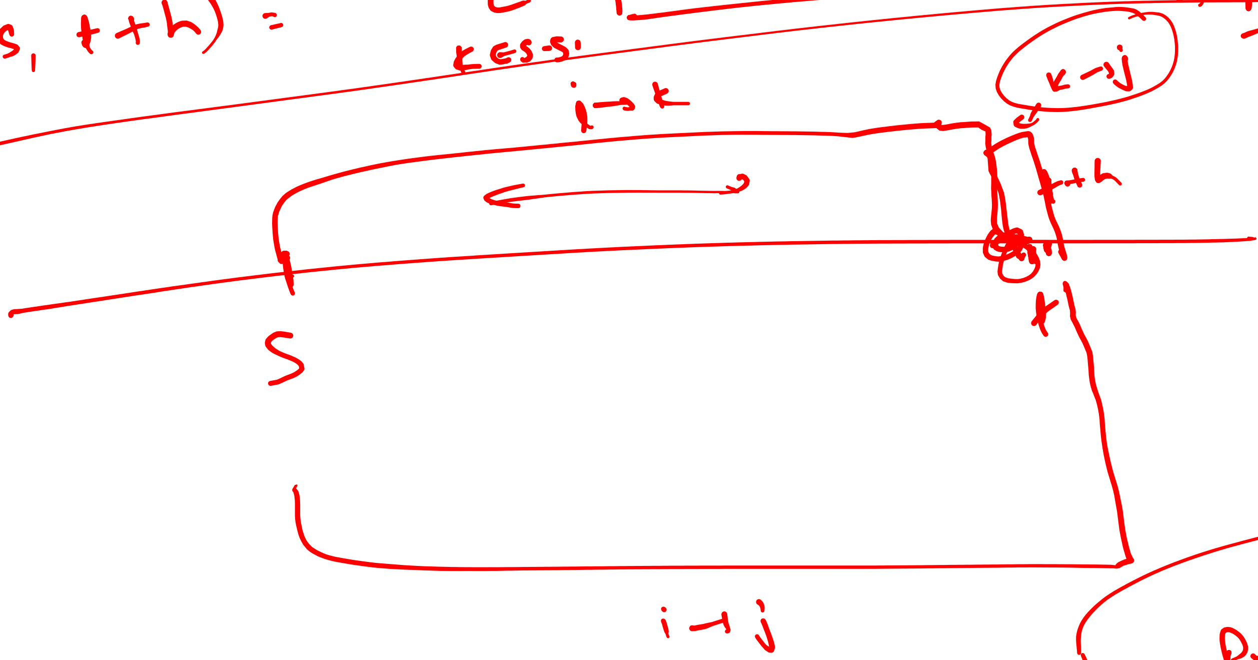
error term

$$\lim_{h \to 0} p_{ij}(h) = \begin{cases} h \cdot \mu_{ij} + o(h), & i \neq j \\ 1 + h\mu_{ii} + o(h), & i = j \end{cases}$$

$$\boxed{\lim_{h \to 0} p_{ij}(s, s+h) = \begin{cases} h \cdot \mu_{ij}(s) + o(h), & i \neq j \\ 1 + h \cdot \mu_{ii}(s) + o(h), & i = j \end{cases}}$$

Kolmogorov Forwd. Diff. eqn. (h is very small

$$p_{ij}(s, t+h) = \sum_{k \in S - S_1} p_{ik}(s,t) \cdot \cancel{\mu_{kj}(t)} \quad p_{kj}(t, t+h) \quad \mu_{kj}(t)$$



$P_{i+k}(s,t) \cdot \mu(t)$

$+ \; P_{in}(s,t) \cdot (0)$

$$= \underbrace{p_{ij}(s,t) \cdot p_{jj}(t,\, t+h)}_{k=j} + \sum_{k \neq j} \overbrace{p_{ik}(s,t) \cdot p_{kj}(t,t+h)}^{k \neq j}$$

$$p_{jj}(t,t+h) = 1 + h\,\boxed{\mu_{jj}(t)} + o(h)$$

$$p_{kj}(t,\, t+h) = h \cdot \mu_{kj}(t) + o(h)$$

$$p_{ij}(s,t+h) = p_{ij}(s,t)\left[1 + h\mu_{jj}(t) + o(h)\right] + \sum_{k \neq j} p_{ik}(s,t) \cdot \left[h \cdot \mu_{kj}(t) + o(h)\right]$$

$$p_{ij}(s,t+h) = p_{ij}(s,t) + h\cdot\mu_{jj}(t)\cdot p_{ij}(s,t) + o(h)\cdot p_{ij}(s,t) + \sum_{k\neq j} h\cdot\mu_{kj}(t)\cdot p_{ik}(s,t) + o(h)\cdot p_{ik}(s,t)$$

$$\frac{p_{ij}(s,t+h) - p_{ij}(s,t)}{h} = \mu_{jj}(t)\cdot p_{ij}(s,t) + \frac{o(h)}{h}\cdot p_{ij}(s,t) + \sum_{k\neq j}\mu_{kj}(t)\, p_{ik}(s,t) + \frac{o(h)}{h}\, p_{ik}(s,t)$$

$$\lim_{h\to 0}\frac{p_{ij}(s,t+h) - p_{ij}(s,t)}{h} = p_{ij}(s,t)\cdot\mu_{jj}(t) + \sum_{k\neq j} p_{ik}(s,t)\cdot\mu_{kj}(t)$$

$$\frac{d\,p_{ij}(s,t)}{dt} = \sum_{k\in S} p_{ik}(s,t)\,\mu_{kj}(t)$$

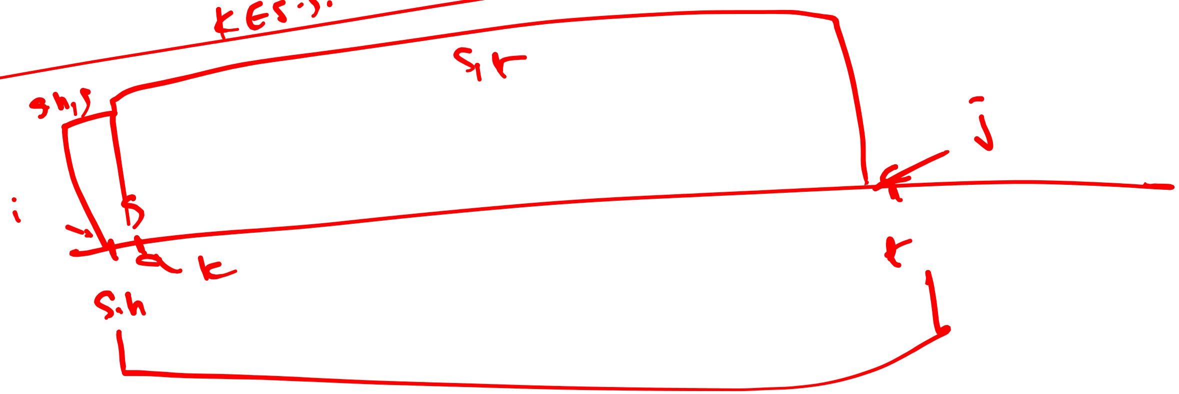
Derive KBE → Kolmogorov Backward Diffeqn

To prove:

$$\frac{d\, p_{ij}(s,t)}{ds} = -\sum_{k \in S.S.} \mu_{ik}(s)\, p_{kj}(s,t)$$

$$p_{ij}(s-h,t) = \sum_{k \in S.S.} p_{ik}(s-h,s) \cdot p_{kj}(s,t)$$

T.B.M. :)



$\cancel{P_{ij}(s-h, t) =}$ $\sum_{k \in E \setminus \{i\}}$

$P_{ij}(s-h, t) = \cancel{P_{ii}(s-h, s)} \cdot P_{ij}(s, t) + \sum_{k \neq i} P_{ik}(s-h, s) \cdot P_{kj}(s, t)$

$\cancel{+ o(h)}$

$P_{ij}(s-h, s) = \cancel{h \cdot \mu_{ij}(s)}$

$= \cancel{1} + h \cdot \mu$.

$P_{ik}(s-h, s)$

$h \cdot \mu_{ik}(s) . + o(h)$

$P_{ik}(s-h, s) = h \cdot \mu_{ik}(s) + o(h)$

$P_{ii}(s-h, s) = 1 + h \cdot \mu_{ii}(s) + o(h)$

$$p_{ij}(s-h,t) = p_{ij}(s,t)\left[1+h\cdot\mu_{ii}(s)+o(h)\right] + \sum_{k\neq i} p_{kj}(s,t)\left[h\mu_{ik}(s)+o(h)\right]$$

$$= h\cdot\mu_{ii}(s)\cdot p_{ij}(s,t) + p_{ij}(s,t) + o(h)\cdot p_{ij}(s,t)$$

$$+ \sum_{k\neq i} p_{kj}(s,t)\cdot h\cdot\mu_{ik}(s) + o(h)\, p_{kj}(s,t)$$

$$\frac{p_{ij}(s-h,t)-p_{ij}(s,t)}{h} = \mu_{ii}(s)\, p_{ij}(s,t) + o(h)\ldots + \sum_{k\neq i}\mu_{ik}(s)\cdot p_{kj}(s,t) + o(h)\ldots$$

$$\lim_{h\to 0}\frac{p_{ij}(s-h,t)-p_{ij}(s,t)}{h} = \mu_{ii}(s)\,p_{ij}(s,t) + \sum_{k\neq i}\mu_{ik}(s)\cdot p_{kj}(s,t)$$

$$\lim_{h \to 0} \frac{p_{ij}(s,t) - p_{ij}(s-h,t)}{h} = -\mu_{ii}(s)\, p_{ij}(s,t) - \sum_{k \neq i} \mu_{ik}(s) \cdot p_{kj}(s,t)$$

$$= - \sum_{k \in S.S.} \mu_{ik}(s) \cdot p_{kj}(s,t)$$

$\underline{CKE}$: $\boxed{p_{\overline{ii}}(s, t+h)} = p_{\overline{ii}}(s,t) \cdot \boxed{p_{\overline{ii}}(t, t+h)}$

$= p_{\overline{ii}}(s,t) \cdot \left(1 + h \cdot \mu_{ii} + o(h)\right)$

Chapman Kolmogorov Eq^n

$$\boxed{p_{ij}(t) = \sum_{k \in S} p_{ik}(s)\, p_{kj}(t-s)}$$

$$p_{\overline{ii}}(s, t+h) = p_{\overline{ii}}(s,t) + h \cdot \mu_{ii}\, p_{\overline{ii}}(s,t) + o(h) \cdot p_{\overline{ii}}(s,t)$$

$$\lim_{h \to 0} \frac{p_{\overline{ii}}(s, t+h) - p_{\overline{ii}}(s,t)}{h} = \mu_{ii}\, p_{\overline{ii}}(s,t)$$

$$\therefore \frac{d\, p_{\overline{ii}}(s,t)}{dt} = p_{\overline{ii}}(s,t) \cdot \mu_{ii}(t)$$

$p_{ii}(s,t)$

$p_{ii}(t,t+h) \to 1 + h\cdot\mu_{ii}(t) + o(h)$

s $\qquad$ $t+h$

i $\qquad$ i

$p_{ii}(s,t+h)$

$$\frac{dp_{ii}(s,t)}{dt} = p_{ii}(s,t)\,\mu_{ii}(t)$$

Int. KBE :

$$p_{ij}(s,t) = \sum_{k\neq i}\int_s^t p_{ii}(s,s+w)\cdot \mu_{ik}(s+w)\cdot p_{kj}(s+w,t)\,dw$$

p_{kj}

$k \to j$

occ. prob.

$i \to k$

1st jump

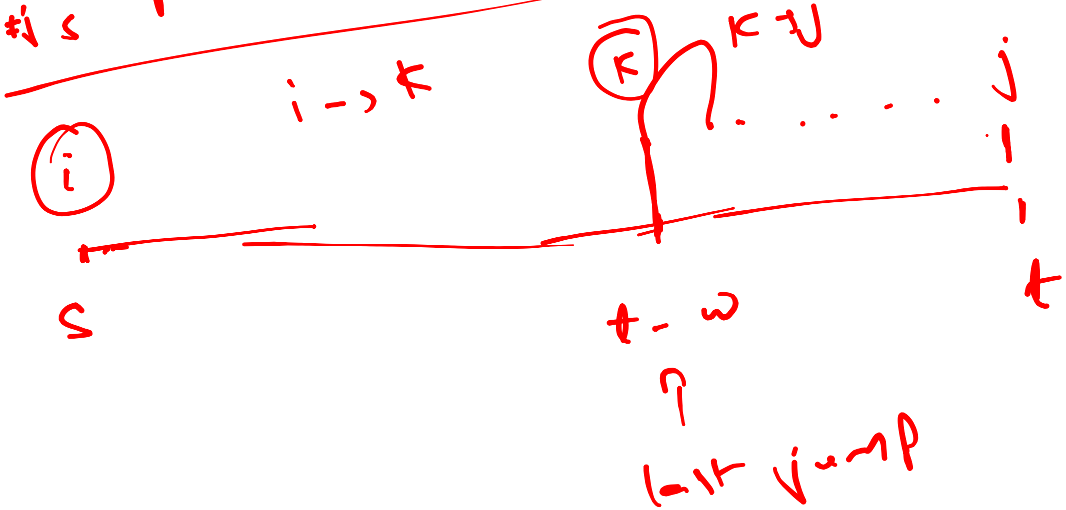
w

↓

Residual holding time for state i.

Int. KFE:

$$P_{ij}(s,t) = \sum_{k \neq j} \int_s^t P_{ik}(s, t-w) \cdot P_{kj}(t-w) \cdot \boxed{P_{jj}(t-w, t)}$$



① Prob. that child remains a good child till time 't' :

or. prob. → P_{ii} $\boxed{P_{ii}(s,t) = \exp\left[-\int_s^t \lambda(k)\, dk\right]}$

$$P_{GG}(0,t) = \exp\left[-\int_0^t 0.2 + 0.04s \,.\, ds\right]$$

$$= \exp\left[-\left(0.2s + 0.02s^2\right)_0^t\right]$$

$$= \exp\left[-0.2t - 0.02t^2\right]$$

0 → 3

9am → 12pm

$$= \exp\left[-0.2\times 3 - 0.02\times 3^2\right]$$

$=$

③ ~~Proof~~ Time 't' by when there is 50% chance that student has been naughty at some point.

$$P(X_s = N \mid 0 < s < t) = 1 - P_{\bar{G}G}(0,t)$$

$$0.5 = 1 - P_{\bar{G}G}(0,t)$$

$$\therefore P_{\bar{G}G}(0,t) = 0.5$$

$$\exp[-0.2t - 0.02t^2] = 0.5$$

$$-0.2t - 0.02t^2 = \ln 0.5$$

$\therefore \quad 10t + t^2 - 34.6574 = 0$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-10 \pm \sqrt{10^2 + 4 \times 34.6574 \times 1}}{2 \times 1}$$

$= \cancel{2.72}$ or $-$

$= 2.72$ or $\cancel{-6.41}$

$\downarrow$

2 hrs 43 mins

⑤ P_{ij} eq^n for $\cancel{P_{jj}(0,t)}$ $P_{GG}(0,t)$

$$\frac{d\,P_{ij}(s,t)}{dt} = \sum_{k\in S.S.} P_{ik}(s,t)\cdot \mu_{kj}(t)$$

$$\frac{d\,P_{GG}(0,t)}{dt} = \sum_{k\in S.S.} P_{Gk}(0,t)\cdot \mu_{kG}(t)$$

$$= P_{GG}(0,t)\cdot \mu_{GG}(t) + P_{GN}(0,t)\cdot \mu_{NG}(t)$$

$$= \underline{P_{GG}(0,t)}\,[\cancel{-0.2-0.0}\,0.0$$
$$[-0.2-0.04t) + P_{GN}(0,t)\,(0.4 - 0.04t)$$

$$P_{GG}(0,t) + P_{GN}(0,t) = 1$$

$$\therefore P_{GN}(0,t) = 1 - P_{GG}(0,t)$$

$$\frac{dP_{GG}(0,t)}{dt} = -P_{GG}(0,t)\left[0.2 + 0.04t\right] + \left[1 - P_{GG}(0,t)\right]\left[0.4 - 0.04t\right]$$

$$= 0.4 - 0.2P_{GG}(0,t) - 0.4P_{GG}(0,t) - 0.04t \cancel{- 0.08t\,P_{GG}(0,t)}$$

$$= \underline{0.4 - 0.6P_{GG}(0,t)} \cancel{- 0.08t\,P_{GG}(0,t)} \underline{- 0.04t}$$