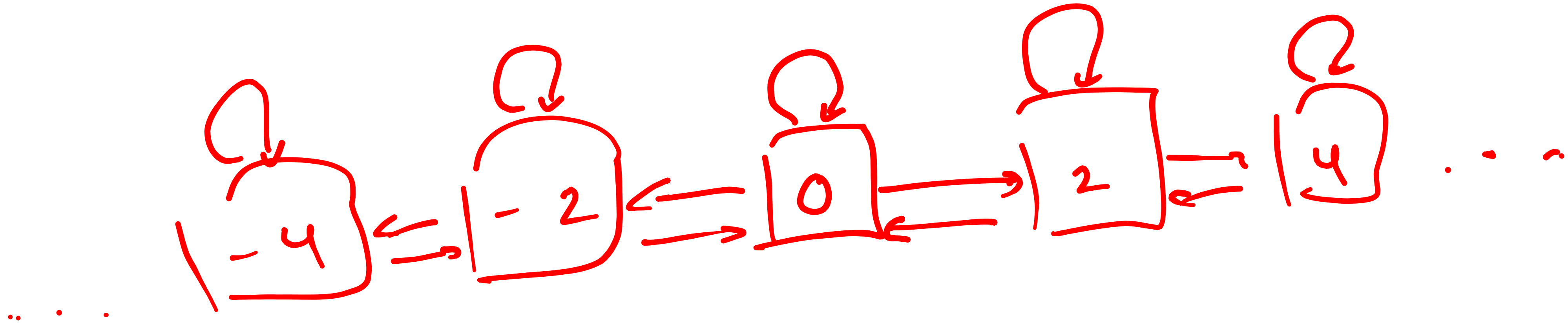
2 time step R.W. transition graph

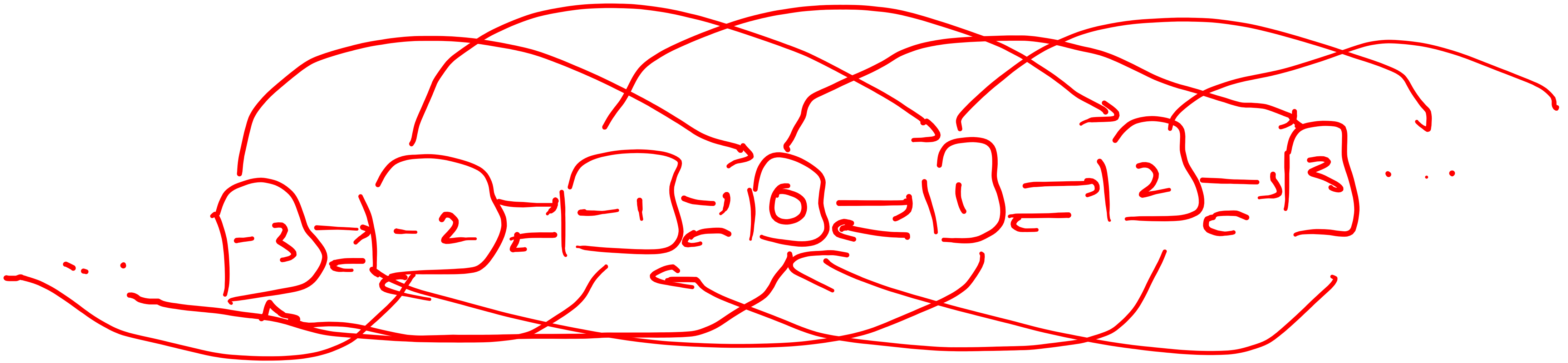
$$\sum_{i=1}^{2} Y_i = \begin{matrix} 2 \\ 0 \\ -2 \end{matrix}$$

1 time step	2 time steps
$+1$ or -1	$+1 +1 = +2$
	$+1 -1 = 0$
	$-1 -1 = 2$

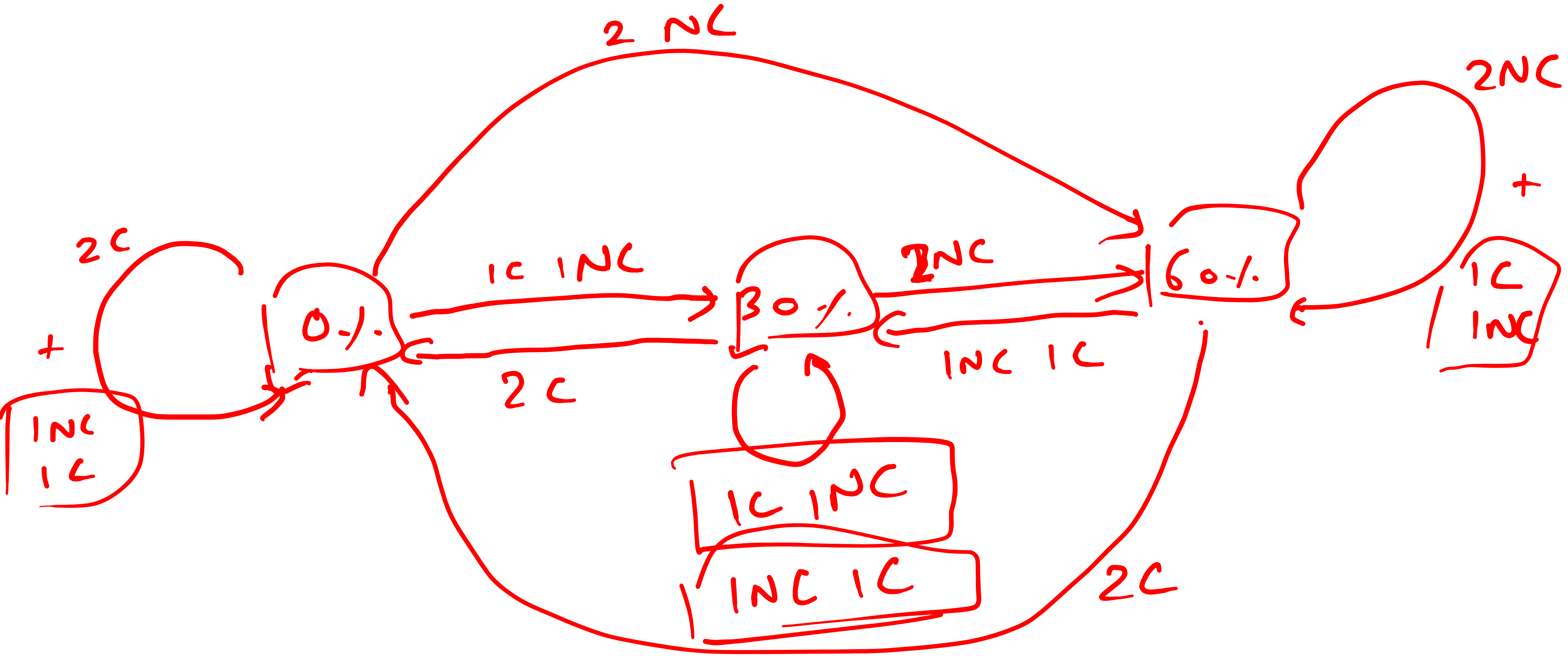


3 time steps R.W. transition graph

$$\sum_{i=1}^{3} Y_i = \begin{matrix} +3 \\ +1 \\ -1 \\ -3 \end{matrix} \qquad \begin{matrix} [1\ 1\ 1] \\ [1\ -1\ 1] \\ [-1\ 1\ -1] \\ [-1\ -1\ -1] \end{matrix}$$



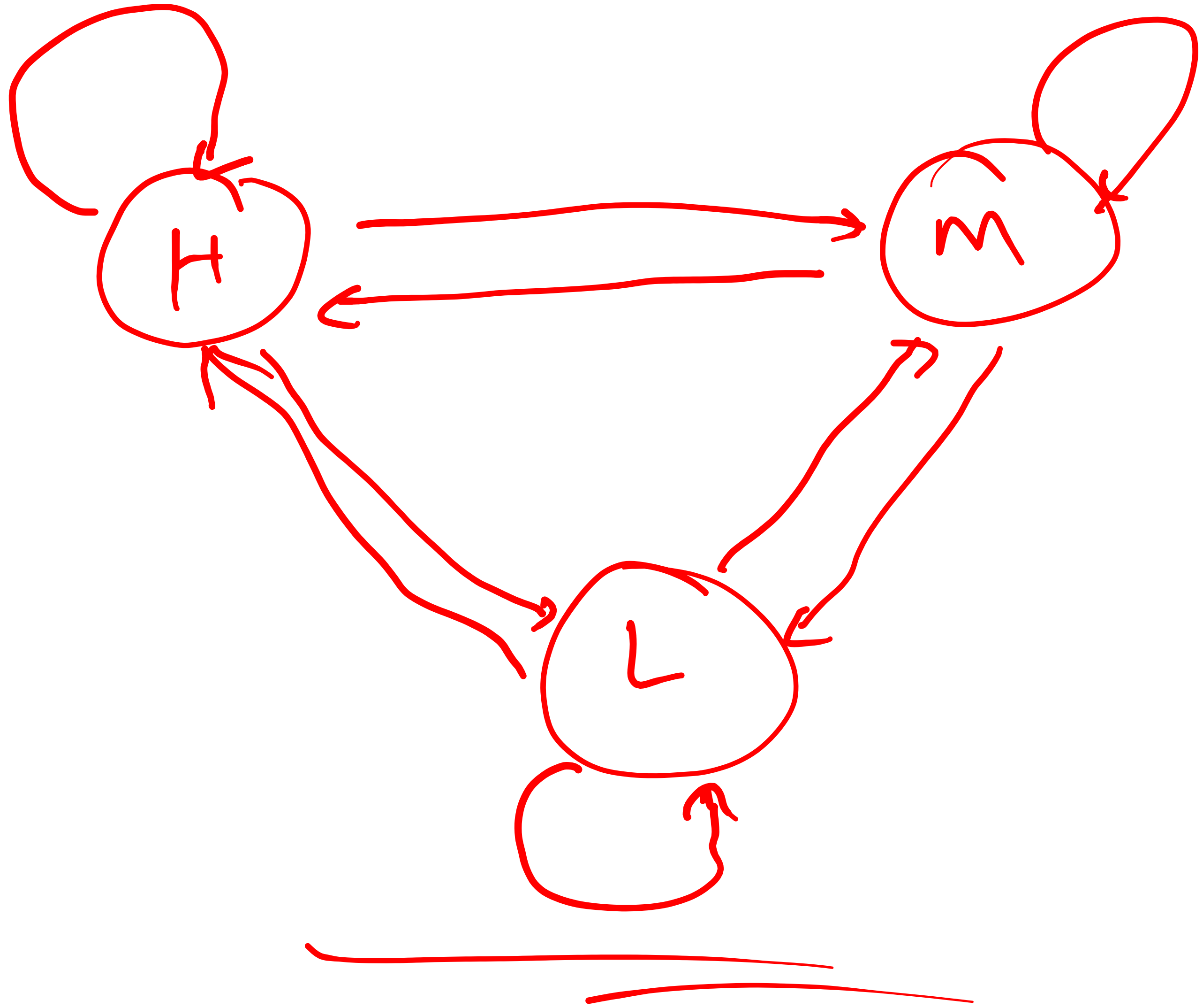
2 time step MCD



Q :

Sol :

1)



3) $\alpha = 0.2$

$$\begin{array}{llll} H & \xrightarrow[\text{steps}]{2 \text{ time}} & L & \rightarrow \underline{HM + ML} \quad \underline{HH + HL} \quad \underline{HL \; LL} \\ M & \rightarrow & L & . \\ L & \rightarrow & L & . \end{array}$$

$$P^{(1,3)}_{HL} = P_{HM} \cdot P_{ML} + P_{HH} \cdot P_{HL} + P_{HL} \cdot P_{LL}$$

$$= 0.2 \times 0.2 + 0.76 \times 0.04 + 0.04 \times 0.76$$

$$\underline{\underline{0.1008}}$$

$$P : \begin{array}{c|ccc} & H & M & L \\ \hline H & 0.76 & 0.2 & 0.04 \\ M & 0.2 & 0.6 & 0.2 \\ L & 0.04 & 0.2 & 0.76 \end{array}$$

Similarly, it can be done for x_1 as M & x_1 as L.

$\cancel{0 < \alpha < 1/2}$

ii) Range of α

$0 \le \alpha, \ \alpha^2, \ 1-2\alpha, \ 1-\alpha-\alpha^2 \le 1$

① $0 \le 1-2\alpha \le 1$

② $\boxed{0 \le 1-\alpha-\alpha^2} \le 1$

$$0 \le 1 - 2\alpha \rightarrow -2\alpha \geq -1$$

$$\therefore 2\alpha \le 1$$

$$\therefore \alpha \le 1/2$$

$$1 - 2\alpha \le 1 \rightarrow -2\alpha \le 0$$

$$\therefore \alpha \geq 0$$

Put $\alpha = 0$ in eqⁿ ③ →

$$1 - 0 - 0^2 = 1 \rightarrow \text{fine}$$

Put $\alpha = 1/2$ in eq^n (2) $\Rightarrow$

$1 - 1/2 - (1/2)^2 = 1 - 0.75 = 0.25 \rightarrow$ fine

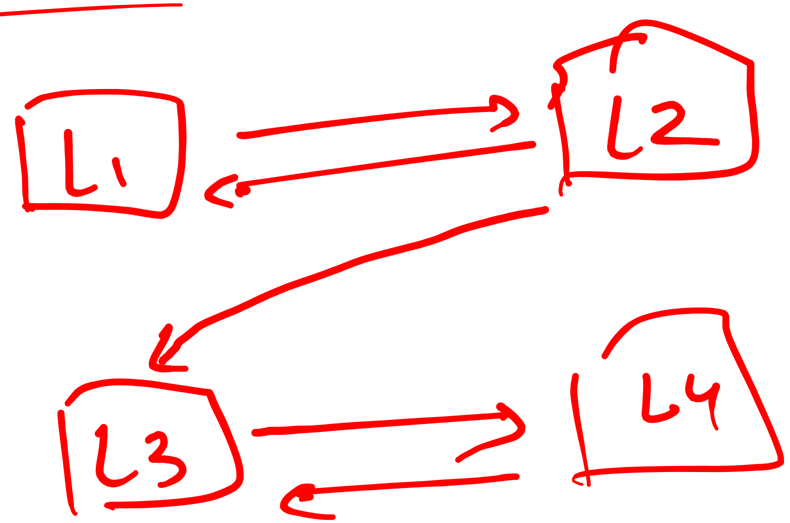
$\therefore \quad 0 \le \alpha \le 1/2$

If a Markov chain has an absorbing state, then it can never be an irreducible chain.

But vice-verse is not true.

→ Associate →

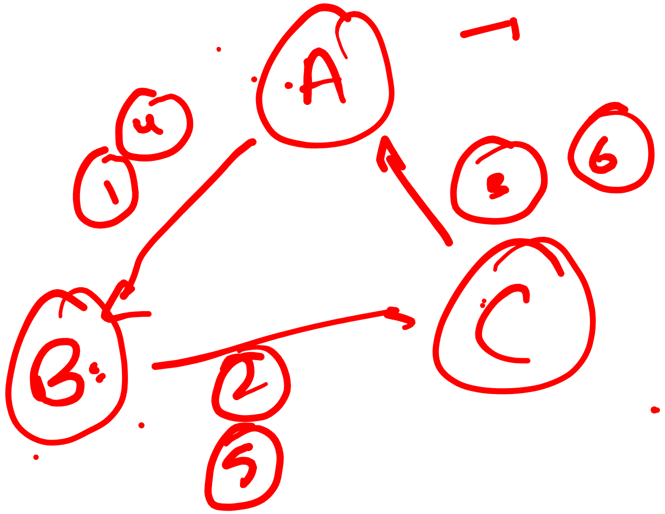
Manager →



Irreducible? No

Although no absorbing state.

Periodicity → Return to state 'i' possible in no. of steps that are multiple of 'd'.



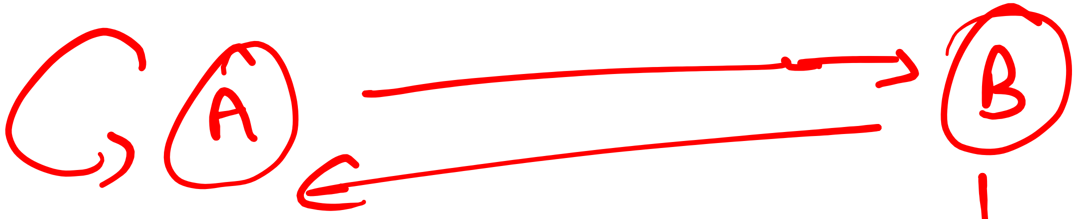
→ Periodicity of 'A'? → 3, 6, 9, 12, ...

↑

Multiple of 3

State A is periodic with multiple of 3.

State B & C → 3



Periodicity?

1, 2, 3, 4, ...

LCM = 1

Period of 1.

Aperiodic state

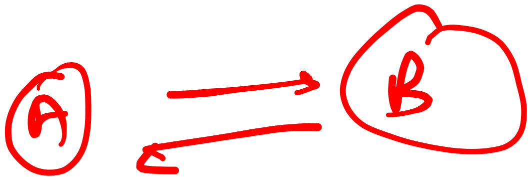
Periodicity?

2, 3, 4, 5, ...

LCM = 1

Period of 1

Aperiodic



2, 4, 6, ...

2, 4, 6, ...

LCM = 2

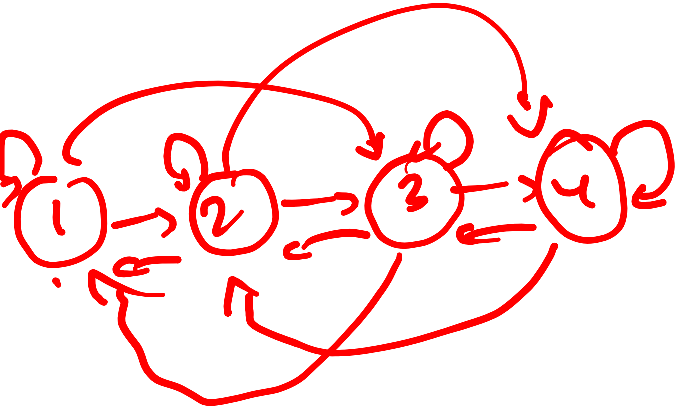
Periodicity of both = 2

→ If a chain is irreducible, and one of the state is aperiodic, then

All the states are aperiodic.

Sol^n to bakery Q:

	1	2	3	4
1	0.75	0.15	0.1	–
2	0.15	0.6	0.15	0.1
3	0.1	0.15	0.6	0.15
4	–	0.1	0.15	0.75



↓

Irreducible & aperiodic ✓

$$P_{21}^{(2)} = P_{221} + P_{211} + P_{231}$$

$$= 0.6 \times 0.15 + 0.15 \times 0.75 + 0.15 \times 0.1$$

$$= \underline{0.2175}$$

	1	2	3	4
1	0.75	0.15	0.1	–
2	0.15	0.6	0.15	0.1
3	0.1	0.15	0.6	0.1
4	–	0.1	0.15	0.75

$$P_{22}^{(2)} = P_{222} + P_{212} + P_{232} + P_{242} = \underline{0.415}$$

$$P_{23}^{(2)} = P_{233} + P_{243} + P_{223} + P_{213} = \underline{0.21}$$

$$P_{24}^{(2)} = \underline{0.1575}$$

Assumpt^n that 25% chance of going up when you are at 3 & 25% " " going down " " " at 2.

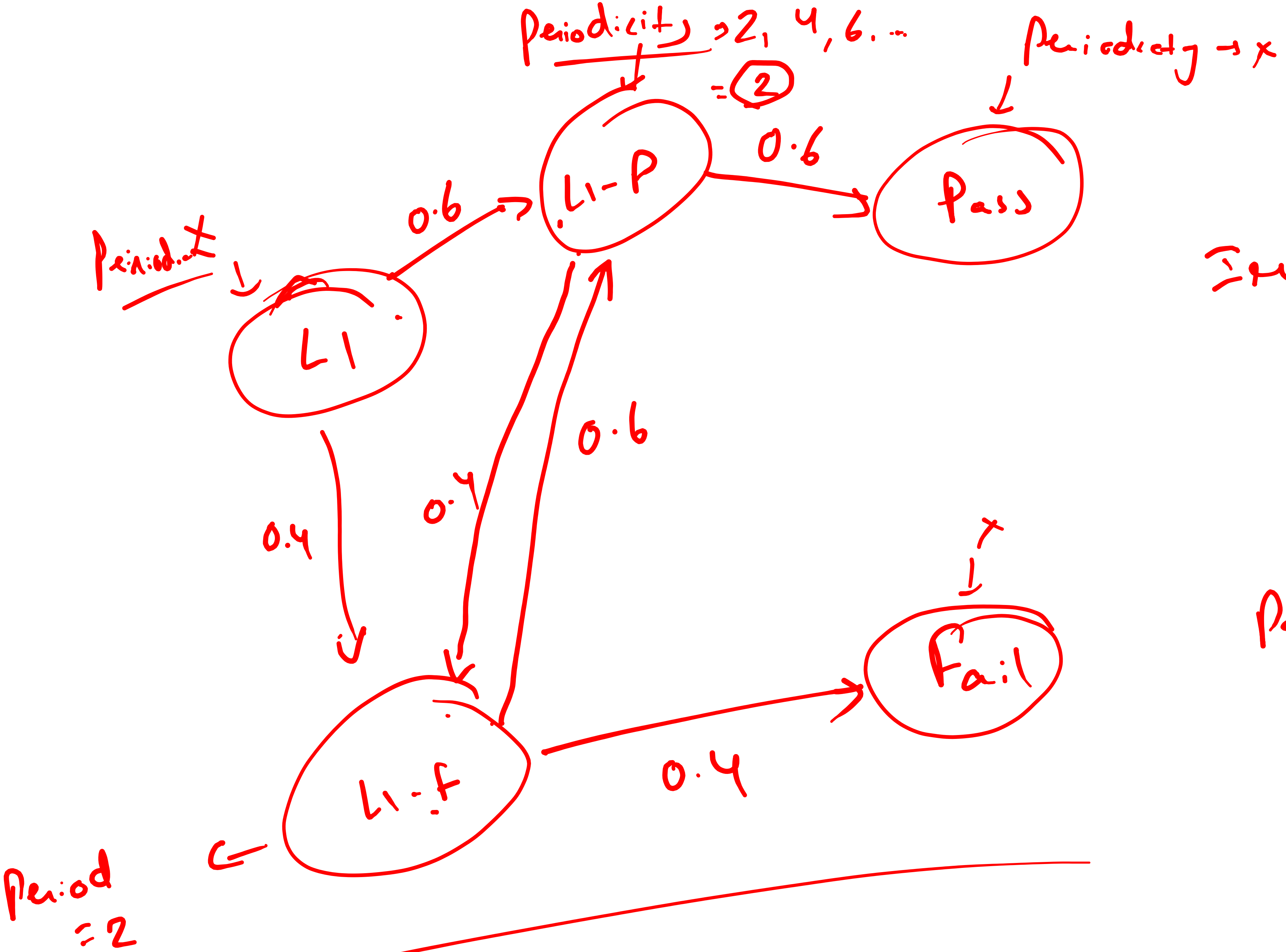
H.W.

$$\underline{P_{0,0}^{(3)}} = P_{0,0} \cdot P_{0,0} P_{0,0} + \underline{P_{0,30}} \cdot \underline{P_{30,0}} \cdot \underline{P_{0,0}}$$

$$+ \underline{P_{0,0}} \cdot \underline{P_{0,30}} \cdot \underline{P_{30,0}}$$

$$= p^3 + (1-p) \cdot p^2 + p^2 (1-p)$$

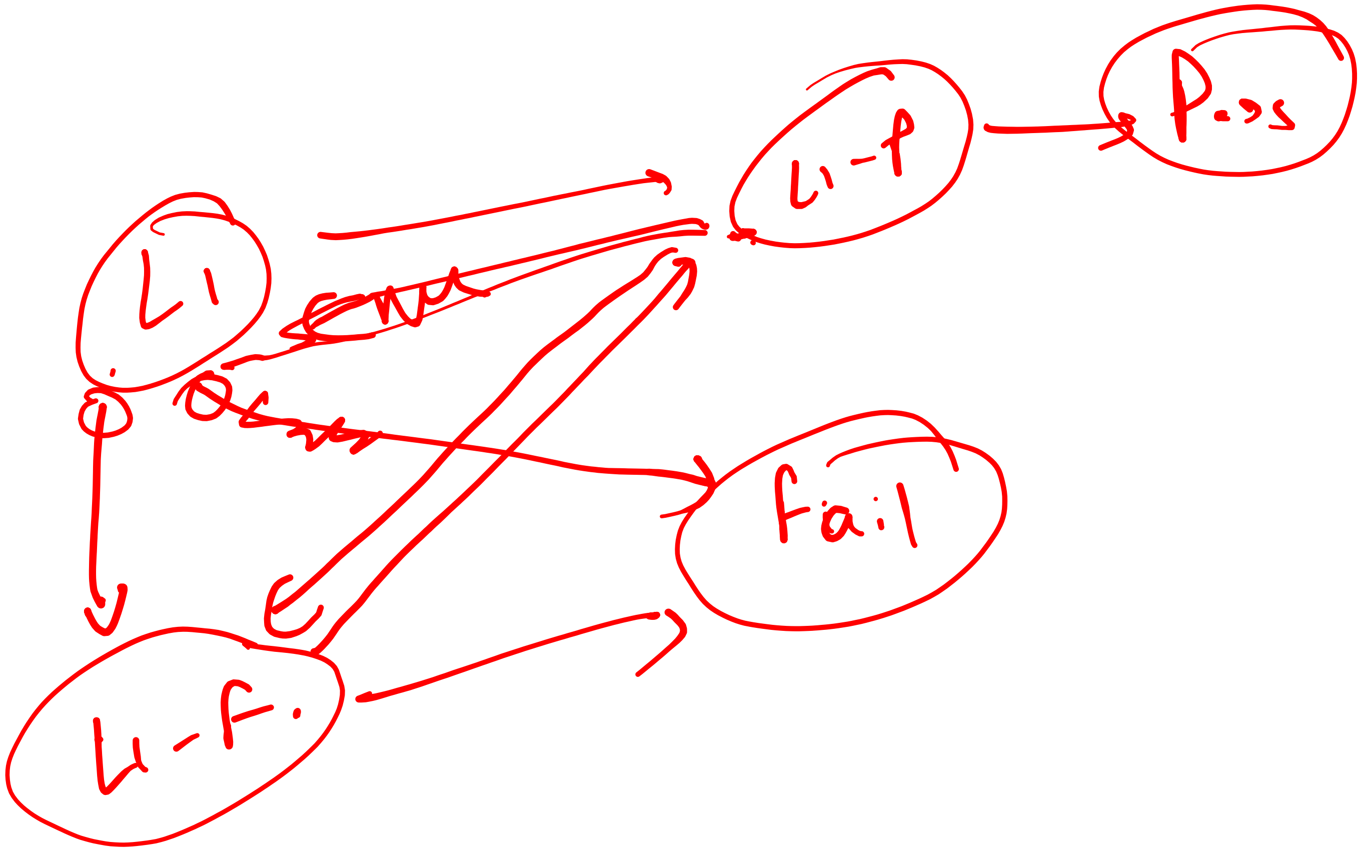
$$= \underline{p^2 (2-p)}$$



Irreducible?

No

Pass & Fail are absorbing states


L1
L1-P
Pass
Fail
L1-F.

	L1	L1F	L1P	P	F
L1	–	0.4	0.6	–	–
L1F	–	–	0.6	–	0.4
L1P	–	0.4	–	0.6	–
P	–	–	–	1	–
F	–	–	–	–	1

No. of attempts post which you won't give any exam with at least 90% chance.

(L1) → Pass → Pass → No Exam

→ Pass → Fail → Exam

→ Fail → Pass → Exam

→ Fail → Fail → No Exam

$P[\text{Exam after 2 attempts}] = P(\text{pass}) \cdot P(\text{fail}) + P(\text{fail}) \cdot P(\text{pass}) = 0.6 \times 0.4 + 0.4 \times 0.6 = 0.48$

$P[E \text{ after 2 attempts}] = 0.48$

$0.48^4 = 0.05$ 8 attempts

$0.48^3 = 0.11$

$P[E \text{ after 4 attempts}] = 0.48^2 = 0.24\ldots$

$\vdots$

$P[E \text{ after } 2N \text{ attempts}] = 0.48^N$

$1 - 0.48^N \geq 0.9$

$P[\text{NE after } 2N \text{ attempts}] \geq 0.9$

$1 - P[E \text{ after } 2N \text{ attempts}] \geq 0.9$

$P[E \text{ after } 2N \text{ attempts}] \leq 0.1$

$0.48^N \leq 0.1$

$N \cdot \log(0.48) \leq \log(0.1)$

$N \geq 3.14$

$P[E \text{ after } 2N] \leq 0.1$

$0.48^N \leq 0.1$

$\log(0.48^N) \leq \log(0.1)$

$N \cdot \log(0.48) \leq \log(0.1)$

$N \geq \dfrac{\log(0.1)}{\log(0.48)}$

$N \geq 3.1378$

$\rightarrow N \geq 4$

$\therefore 2N = 8$ attempts

$$P(\text{pass}) = P(\text{pass-pass}) + P(\text{Exam}) \cdot P(\text{pass-pass})$$

$$+$$

$$= \underline{0.6 \times 0.6} + \underline{0.48} \times 0.6^2$$

$$+ 0.48^2 \times \underline{0.6}^2$$

$$+ \underline{0.48}^3 \times 0.6^2$$

$$+ \ldots$$

$$= 0.6^2 + 0.48 \times 0.6^2 + 0.48^2 \times 0.6^2 + \ldots$$

$\rightarrow$ <u>Infinite GP series.</u>

$$a = 0.6^2, \quad r = 0.48$$

$$= \frac{a}{1-r}$$

$$= \frac{0.36}{1-0.48} = 0.6923$$

36% → Pass $\quad \frac{36}{36+16} = \boxed{69.23\%}$

16% → Fail $\quad \boxed{30.77\%}$

$$\pi^0 = (100\% \quad 0\% \quad 0\%)$$

$$P = \begin{bmatrix} 0.25 & 0.75 & - \\ 0.25 & - & 0.75 \\ - & 0.25 & 0.75 \end{bmatrix}$$

$$\underline{\pi^1 = \pi^0 \cdot P} = (100\% \quad 0 \quad 0) \begin{bmatrix} \text{Matrix} \end{bmatrix}$$

$$= \begin{bmatrix} 100 \times 0.25 & 100 \times 0.75 & - \end{bmatrix}$$

$$= \underline{\begin{bmatrix} 25\% & 75\% & - \end{bmatrix}}$$

$\pi^1 = (25 \quad 75 \quad 0)$

$\pi^2 = (25 \quad 18.75 \quad 56.25)$

⋮

after some time

↓

Stability

↓

long term stationary dist^n

MCD

$$P = \begin{bmatrix} 0.2 & 0.8 & - \\ 0.2 & - & 0.8 \\ - & 0.2 & 0.8 \end{bmatrix}$$

Find long term stationary dist^2

$$\pi = \pi \cdot P$$

$$(\pi_1 \quad \pi_2 \quad \pi_3) = (\pi_1 \quad \pi_2 \quad \pi_3)\begin{bmatrix} \text{matrix} \end{bmatrix}$$

$$\pi_1 = 0.2\pi_1 + 0.2\pi_2 \quad — ①$$

$$\pi_2 = 0.8\pi_1 + 0.2\pi_3 \quad — ②$$

$$\pi_3 = 0.8\pi_2 + 0.8\pi_3 \quad — ③$$

$$\pi_1 + \pi_2 + \pi_3 = 1 \quad — ④$$

$\boxed{\begin{matrix} 1/21 \\ 4/21 \\ 16/21 \end{matrix}}$

From ①,

$$\pi_1 - 0.2\pi_1 = 0.2\pi_2$$

$$\therefore 0.8\pi_1 = 0.2\pi_2 \quad \therefore \pi_1 = 1/4\,\pi_2$$

From ③,

$$\pi_3 - 0.8\pi_3 = 0.8\pi_2 \quad \therefore 0.2\pi_3 = 0.8\pi_2 \quad \therefore \pi_3 = 4\pi_2$$

$$\pi_1 + \pi_2 + \pi_3 = 1$$

$$\pi_1 = 1/4\, \pi_2$$

$$1/4\, \pi_2 + \pi_2 + 4\pi_2 = 1$$

$$= 1/4 \times 4/21$$

$$= 1/21$$

$$\pi_2 (1/4 + 1 + 4) = 1$$

$$\pi_2 (21/4) = 1 \qquad \therefore \underline{\pi_2 = 4/21}$$

$$\pi_3 = 4\pi_2 = 4 \times 4/21 = \underline{16/21}$$

Stationary dist^n = $(1/21 \quad 4/21 \quad 16/21)$

Q: Sol^n:

$$\pi_1 = 1/4\,\pi_1 + 1/4\,\pi_2 + 1/4\,\pi_4$$

$$\pi_2 = 3/4\,\pi_1 + 1/4\,\pi_3$$

$$\boxed{\pi_3 = 3/4\,\pi_2}$$

$$\pi_4 = 1/4\,\pi_5$$

$$\pi_5 = 3/4\,(\pi_3 + \pi_4 + \pi_5)$$

$$\pi_1 + \pi_2 + \pi_3 + \pi_4 + \pi_5 = 1$$

$$\pi_2 = \frac{3}{4}\pi_1 + \frac{1}{4}\left(\frac{3}{4}\pi_2\right)$$

$$\therefore \frac{13}{16}\pi_2 = \frac{3}{4}\pi_1 \quad \therefore \boxed{\pi_1 = \cancel{\frac{12}{32}}\ \frac{13}{12}\pi_2}$$

$$\pi_1 = \frac{1}{4}\pi_1 + \frac{1}{4}\pi_2 + \frac{1}{4}\pi_4$$

$$\frac{3}{4}\pi_1 - \frac{1}{4}\pi_2 = \frac{1}{4}\pi_4$$

$$\frac{3}{4}\left(\frac{13}{12}\pi_2\right) - \frac{1}{4}\pi_2 = \frac{1}{4}\pi_4 \quad \therefore \frac{1}{4}\pi_4 = \frac{27}{48}\pi_2$$

$$\therefore \pi_4 = \frac{27}{12}\pi_2 = \underline{\frac{9}{4}\pi_2}$$

$\pi_4 = \frac{1}{4}\pi_5 \quad \therefore \pi_5 = 4\pi_4 = 4 \times \frac{9}{4}\pi_2$

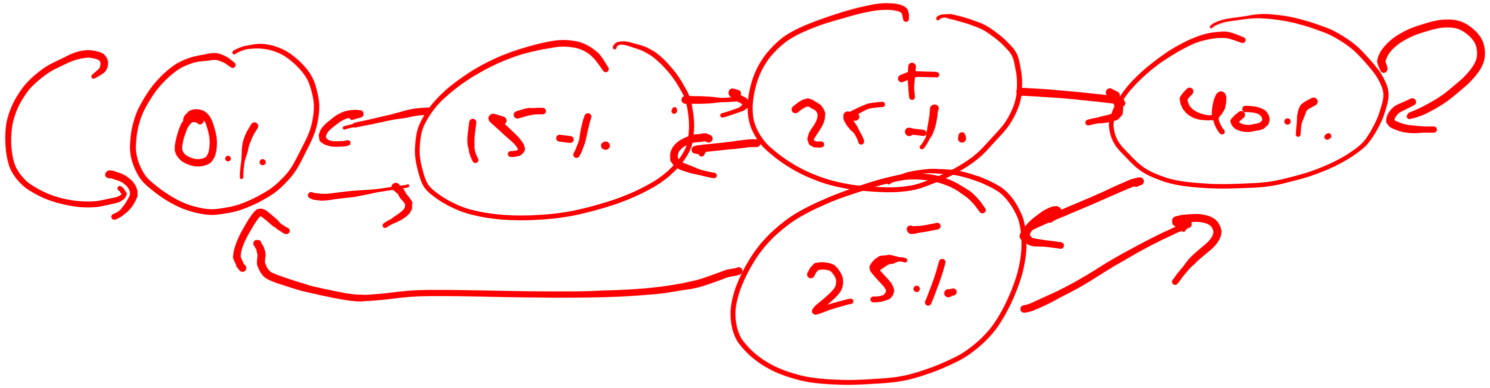
$\cancel{\pi_5 = 100\pi_2} \qquad \pi_5 = 9\pi_2$

$$\left(\frac{13}{12} + 1 + \frac{3}{4} + \frac{9}{4} + 9\right)\pi_2 = 1$$

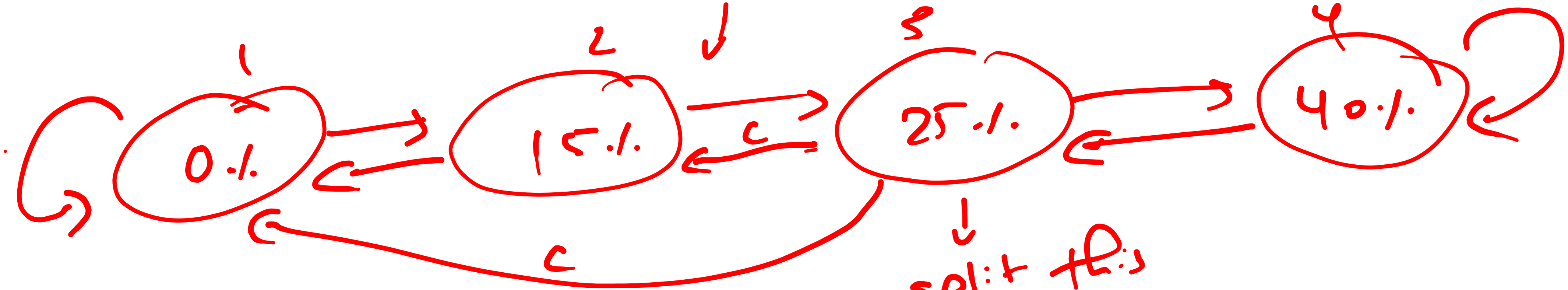
$$\frac{169}{12}\pi_2 = 1$$

$$\therefore \boxed{\pi_2 = \frac{12}{169}}$$

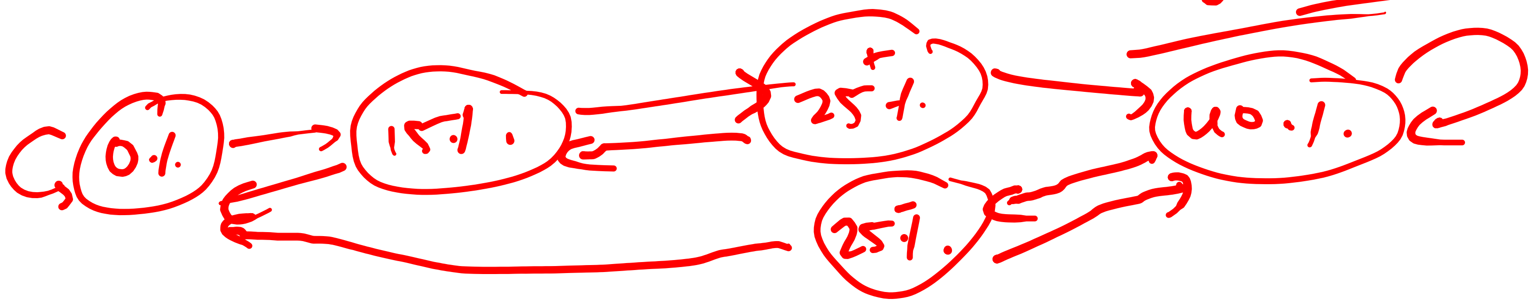
$$\pi = \left(1/13 \quad 12/169 \quad 9/169 \quad 27/169 \quad 108/169\right)$$



Not a Markov chain



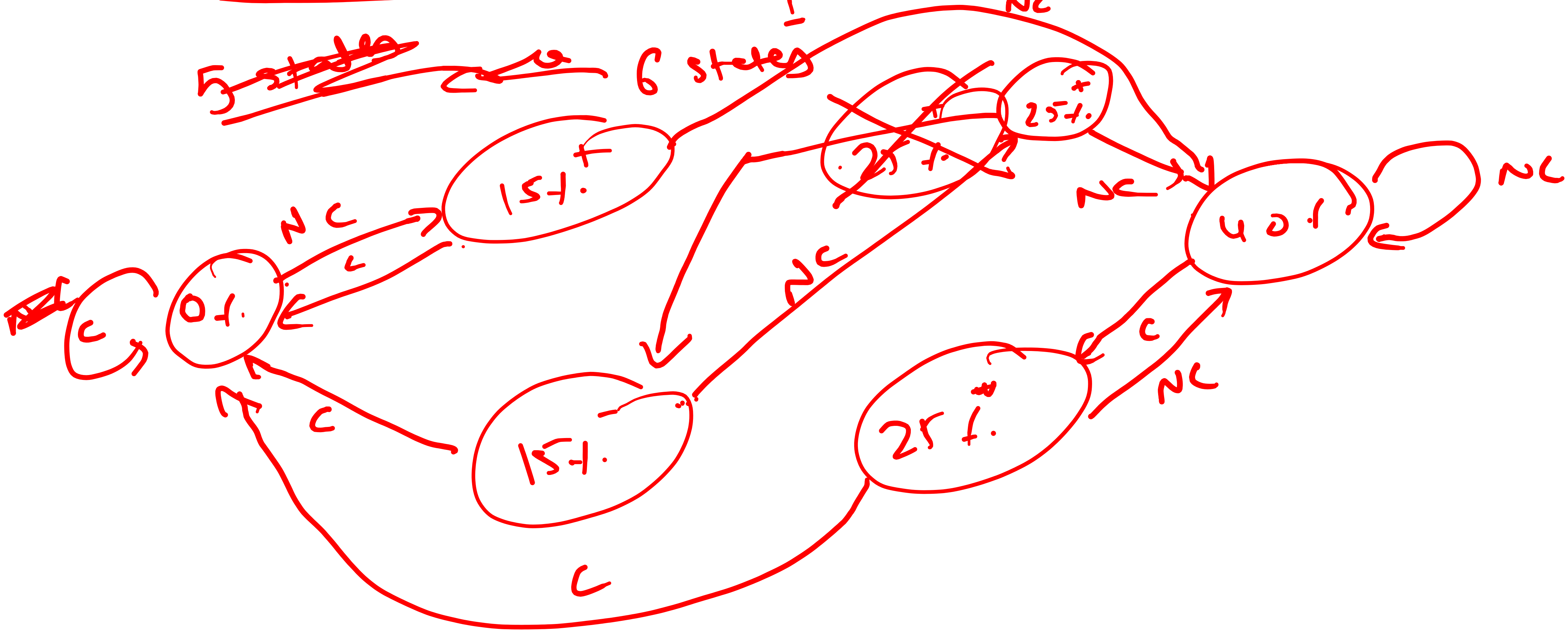
split this state

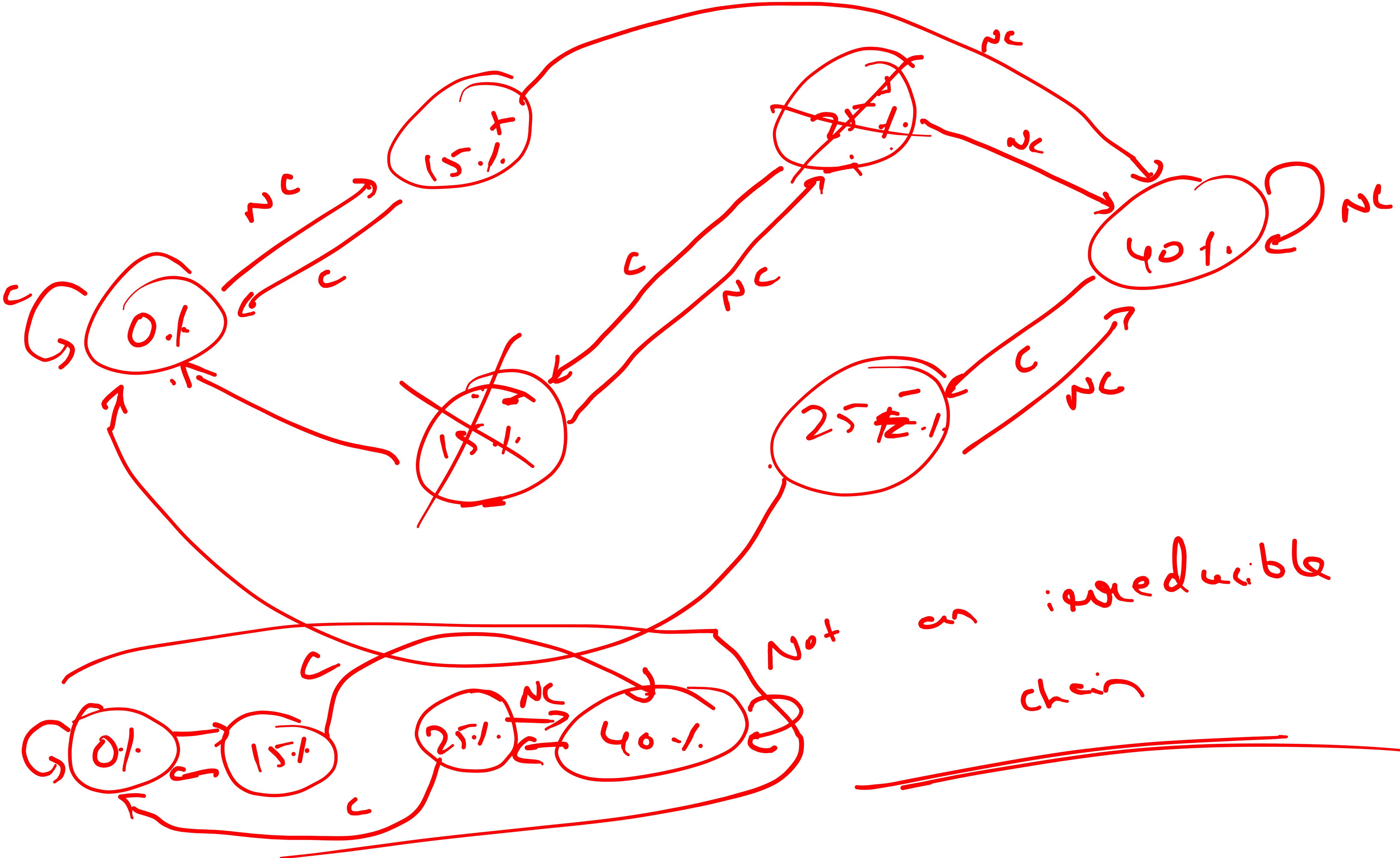


If I do not claim for 2 consecutive years,

I go up 2 levels.

~~5 states~~ 6 states





Not an irreducible chain

$$\pi = \pi \begin{array}{c} \begin{array}{ccccc} 0 & 1 & 2^{+} & 2^{-} & 3 \end{array} \\ \begin{bmatrix} p & 1-p & - & - & - \\ p & - & 1-p & - & - \\ - & p & - & - & 1-p \\ p & - & - & - & 1-p \\ - & - & - & p & 1-p \end{bmatrix} \end{array}$$

$$\pi_1 = p(\pi_1 + \pi_2 + \pi_4)$$

$$\pi_4 = p \cdot \pi_5$$

$$\pi_2 = (1-p)\pi_1 + p\pi_3 .$$

$$\pi_5 = (1-p)(\pi_3 + \pi_4 + \pi_5)$$

$$\pi_3 = (1-p)\pi_2$$

Also, $\pi_5 = 9\pi_2$

To solve for p:

$\leftarrow \pi_5 = 9\pi_2$

$$\pi_5 = (1-p)[\pi_3 + \pi_4 + \pi_5]$$

$$\pi_5 = (1-p)[(1-p)\pi_2 + p\pi_5 + \pi_5]$$

$$\pi_5 = (1-p)[(1-p)\pi_2 + (1+p)\pi_5]$$

$$\pi_5 = (1-p)^2\pi_2 + (1-p^2)\pi_5$$

$$\pi_5[1 - 1 + p^2] \cancel{\pi_5} = (1-p)^2\pi_2$$

$$\therefore \boxed{p^2\pi_5} = (1-p)^2\pi_2 \quad \therefore \pi_5 = \left(\frac{1-p}{p}\right)^2\pi_2$$

$\therefore \left(\frac{1-p}{p}\right)^2 = 9 \quad \therefore \quad \frac{1-p}{p} = 3$

$0.75\pi_1 = \frac{1}{4}\pi_2 + \frac{9}{4}\pi_2$

$\therefore \boxed{p = 1/4}$

$\pi_1 = \frac{1}{4}\pi_1 + \frac{1}{4}\pi_2 + \pi_4$

$\pi_1 = \frac{1}{4}(\pi_1 + \pi_2 + \pi_4)$

$\pi_2 = \frac{3}{4}\pi_1 + \frac{1}{4}\pi_3$

$\pi_4 = \frac{1}{4}\pi_5$

$\pi_5 = \frac{3}{4}(\pi_3 + \pi_4 + \pi_5)$

$\pi_3 = \frac{3}{4}\pi_2$

$\pi_5 = 9\pi_2$

$\pi_3 = \frac{3}{4}\pi_2$

$\therefore \pi_4 = \frac{9}{4}\pi_2$

$$\pi_2 = \tfrac{3}{4}\pi_1 + \tfrac{1}{4}\left(\tfrac{3}{4}\pi_2\right)$$

$$\therefore \pi_2 = \tfrac{3}{4}\pi_1 + \tfrac{3}{16}\pi_2 \quad \therefore \tfrac{3}{4}\pi_1 = \tfrac{13}{16}\pi_2$$

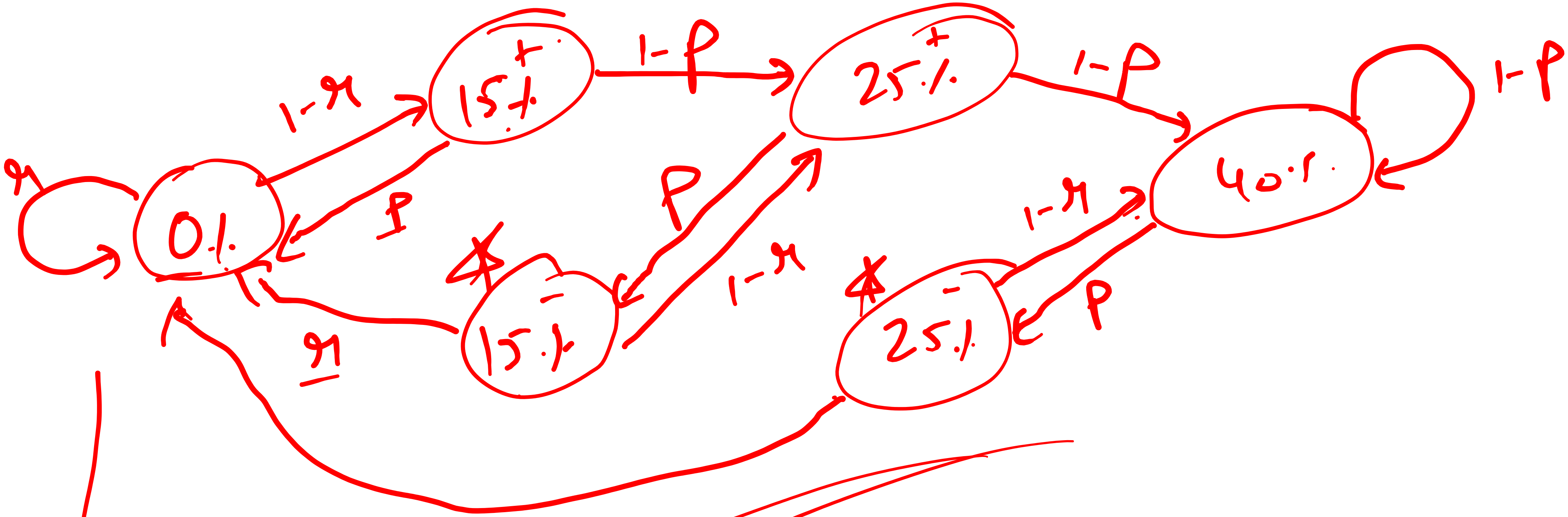
$$\therefore \pi_1 = \tfrac{13}{12}\pi_2$$

$$\left(\tfrac{13}{12} + 1 + \tfrac{3}{4} + \tfrac{9}{4} + 9\right)\pi_2 = 1$$

$$\therefore \boxed{\pi_2 = \cancel{16}\ \tfrac{12}{169}}$$

Prop. at 25% level $= \pi_3 + \pi_4 = \left(\tfrac{3}{4} + \tfrac{9}{4}\right) \times \tfrac{12}{169}$

$= 36/169 = 0.213$



Assume that initial prob of claim = r.

① Prob of claim (if NC last yr) = p [0.15]

② Prob of claim (if claim last yr) = q [0.25]

	0	15^+	15^-	25^+	25^-	40
0	r	$1-r$	–	–	–	–
15^+	p	–	–	$1-p$	–	–
15^-	r	–	–	$1-r$	–	–
25^+	–	–	p	–	–	$1-p$
25^-	r	–	–	–	–	$1-r$
40	–	–	–	–	p	$1-p$

Q.2 :

Per policy:

Total Premium = Total Claims

$P(X=x) = \frac{e^{-\lambda}.\lambda^{x}}{x!}$

Total Claims :

No. of claims $\sim Poi(0.35)$, Cost = 2500

Claim		Prob.		Cost
1 →		$e^{-0.35}.0.35$	×	2500
2 →		$\frac{e^{-0.35}.0.35^{2}}{2}$	+	5000
3 →		$\frac{e^{-0.35}.0.35^{3}}{6}$	+	7500
>3		0.00047	×	7500

873.3

$$P(X>3) = 1 - \left[P(X=0) + P(X=1) + P(X=2) + P(X=3)\right]$$

$$= 1 - \left[e^{-0.35} + e^{-0.35} \cdot 0.35 + \frac{e^{-0.35} \cdot 0.35^2}{2} + \frac{e^{-0.35} \times 0.35^3}{6}\right]$$

$$= \underline{0.00047}$$

Note: Would have been the avg. cost of claims w/o max. 3 claims →

$$0.35 \times 2500 = \underline{875}$$

Total Prem. collected =

	0%	15%	30%	40%
Base Premium	P	$0.85P$	$0.7P$	$0.6P$
	$\times$ 4.4%	+ $\times$ 10.5%	$\times$ 25.1%	$\times$ 60%

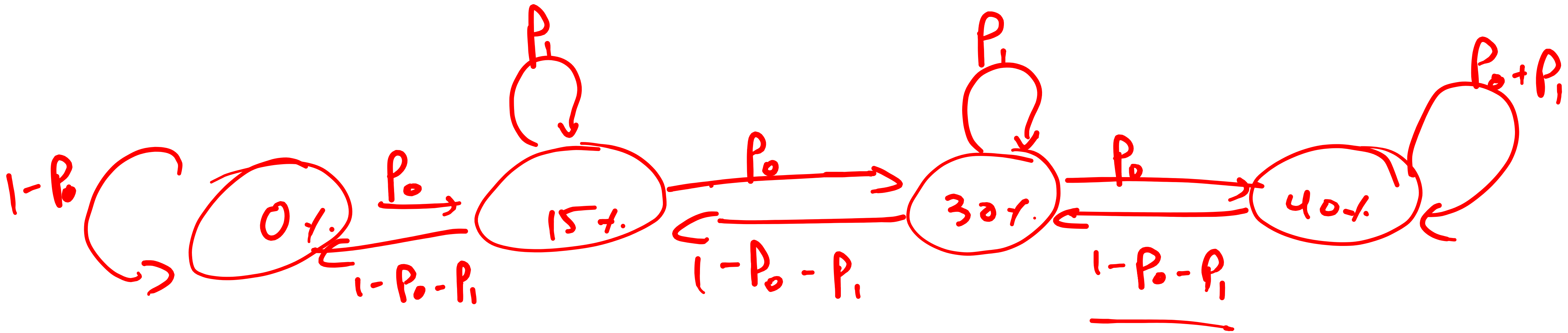
$$\boxed{0.66895P}$$

$$0.66895P = 873.3$$

$$\therefore \boxed{P = 1306.12}$$

$\therefore$ 40% disc. level $= 0.6 \times P$

$= 783.67$



P_0 = Prob. of no claim

P_1 = " " 1 claim

P_{1+} = " " more than 1 claim

Poi(0.35)

$$\boxed{P_{1+} = 1 - P_0 - P_1}$$

$P(x=0) = 0.7049$

$P(x=1) = 0.2466$

$P(x>1) = 0.0487$

	0%	15%	30%	40%
0%	0.2953	0.7049	–	–
15%	0.0487	0.2466	0.7047	–
30%	–	0.0487	0.2466	0.7047
40%	–	–	0.0487	0.9513

Q3 :

i) Transit^{n} prob.

$$\begin{array}{c|ccc|c} & C & R & D & \\ \hline C & 3/7 & 2/7 & 2/7 & 7 \\ R & 2/6 & 1/6 & 3/6 & 6 \\ D & 3/6 & 2/6 & 1/6 & 6 \\ \hline & & & & \boxed{19} \end{array}$$

ii) Prob. that at least 2 of 3 → RCB wins

last match → C

$$\frac{2}{7} \times \frac{1}{6} \times \frac{1}{6}$$

① ~~R R R~~ →

② R C R

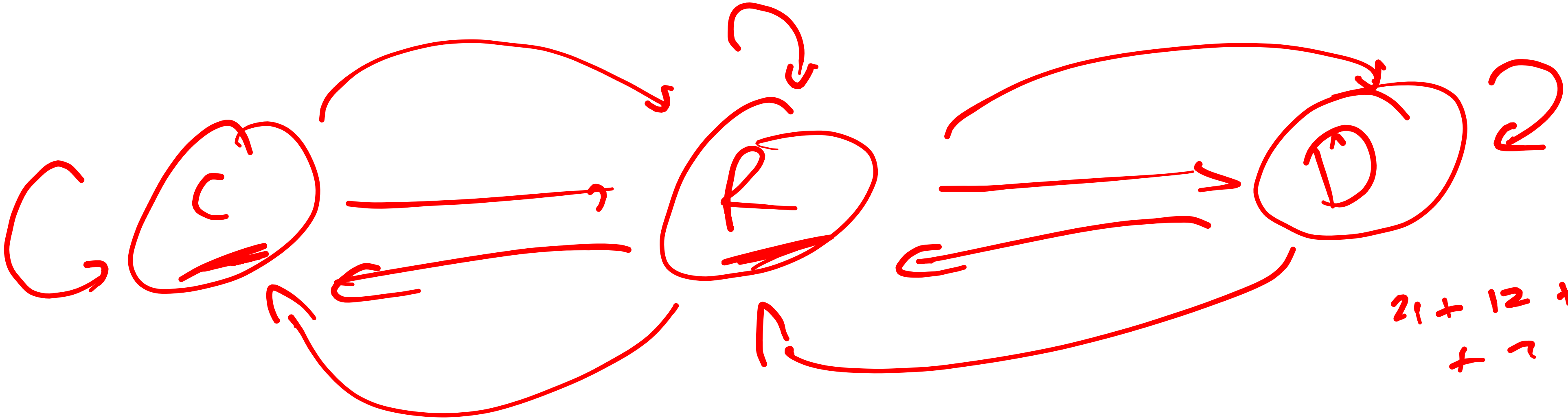
③ R D R

④ ~~R R~~ C

⑤ ~~R R~~ D

⑥ C R R

⑦ D R R



$21 + 12 + 21 + 9 + 7 = 70/441 = \underline{10/63}$

1. $RRX \to 2/7 \times 1/6$ — $1/21$
2. $+\ RCR \to 2/7 \times 1/3 \times 2/7$ — $4/147$
3. $+\ RDR \to 2/7 \times 1/2 \times 1/3$ — $1/21$
4. $+\ CRR \to 3/7 \times 2/7 \times 1/6$ — $1/49$
5. $+\ DRR \to 2/7 \times 1/3 \times 1/6$ — $1/63$

$= \underline{10/63}$