Integration :

1. $$\frac{dy}{dx} = x$$

$$\therefore dy = x \cdot dx$$

$$\therefore \int dy = \int x \cdot dx$$

$$\therefore y = \frac{x^2}{2} + c$$

(2) $\frac{dy}{dx} = \frac{y}{x}$

$$\frac{1}{y} \cdot dy = \frac{1}{x} \cdot dx$$

$$\int 1/y \cdot dy = \int 1/x \cdot dx$$

$$\ln y = \ln x + C$$

$$= \ln x + \ln k$$

$$= \ln x \cdot k$$

$$\therefore \boxed{y = xk}$$

$$\boxed{C = \ln k}$$

③ Integrating factor:

$$\frac{dy}{dx} + f(x)\cdot y = g(x)$$

Integrating factor (IF) = $e^{\int f(x)\cdot dx}$

find answer:

$$y\cdot IF = \int IF\cdot g(x)\cdot dx$$

$$\left(\frac{dy}{dx} + f(x)\cdot y\right)\cdot e^{\int f(x)\,dx} = e^{\int f(x)\cdot dx}\cdot g(x)$$

$$\frac{d\,y\cdot e^{\int f(x)\cdot dx}}{dx} = e^{\int f(x)\,dx}\cdot g(x)$$

$$y\cdot e^{\int f(x)\cdot dx} = \int e^{\int f(x)\,dx}\cdot g(x)$$

$$y\cdot IF = \int IF\cdot g(x)\cdot dx$$

Ex : $\frac{dy}{dx} + 2y = e^x$

$\rightarrow f(x) = 2$

$g(x) = e^x$

$$IF = e^{\int 2 \cdot dx} = e^{2x}$$

$$y \cdot e^{2x} = \int e^{2x} \cdot e^{x} \cdot dx$$

$$y \cdot e^{2x} = \frac{e^{3x}}{3} + c$$

$$\boxed{\therefore y = \frac{e^x}{3} + \frac{c}{e^{2x}}}$$

Ex: $x \cdot \frac{dy}{dx} = 2x - (x+1) \cdot y$ [5 mins]

Find y in terms of x, given that $y(1) = 0$.

$$\int uv \cdot dx = u \int v \cdot dx - \int \left(\frac{du}{dx} \cdot \int v \cdot dx \right) \cdot dx$$

$$x \cdot \frac{dy}{dx} = 2x - (x+1) \cdot y$$

$$\frac{dy}{dx} = 2 - \left(\frac{x+1}{x} \right) \cdot y \qquad \therefore \frac{dy}{dx} + \underbrace{\left(\frac{x+1}{x} \right)}_{f(x)} \cdot y = \underbrace{2}_{g(x)}$$

$$IF = e^{\int g(x)\cdot dx} = \exp\left[\int\left(\frac{x+1}{x}\right)\cdot dx\right]$$

$$= \exp\left[\int 1\cdot dx + \int \frac{1}{x}\cdot dx\right]$$

$$= \exp[x + \ln x] = e^{x+\ln x}$$

$$= e^{x}\cdot e^{\ln x} = \underline{x\cdot e^{x}}$$

$$y\cdot IF = \int IF\cdot g(x)\cdot dx$$

$$y\cdot x\cdot e^{x} = \int x\cdot e^{x}\cdot 2\cdot dx = 2\int \underbrace{x\cdot e^{x}}_{\text{uv rule}}\cdot dx$$

I → Inverse

L → Log

A → Arithmetic

T → Trigonometric

E → Exponential

$$y \cdot x e^{x} = 2\left[x \int e^{x} \cdot dx - \int \left(\frac{dx}{dx} \cdot \int e^{x} \cdot dx\right) \cdot dx\right]$$

$$= 2\left[x \cdot e^{x} - \int 1 \cdot e^{x} \cdot dx\right] + c$$

$$= 2\left[\cancel{\frac{e^{x}}{x}}\, x \cdot e^{x} - e^{x}\right] + c$$

$$y \cdot x \cdot e^{x} = 2 e^{x}\left[x - 1\right] + c$$

$$\therefore\ y = \frac{2 e^{x}\left[x - 1\right]}{x \cdot e^{x}} + \frac{c}{x e^{x}}$$

$$\boxed{\therefore\ y = \frac{2(x - 1)}{x} + \frac{c}{x \cdot e^{x}}}$$

$$y(1) = 0$$

$$\therefore \frac{2(1-1)}{1} + \frac{c}{1 \cdot e^1} = 0$$

$$\therefore 0 + \frac{c}{e} = 0 \quad \therefore \frac{c}{e} = 0 \quad \boxed{\therefore c = 0}$$

$$\boxed{\therefore y = \frac{2(x-1)}{x}}$$

$$
\begin{array}{c|cc}
 & 0 & 1 \\
\hline
0 & -0.01 & 0.01 \\
1 & 0.1 & -0.1
\end{array}
$$

$$\frac{d\,p_{01}(t)}{dt} = p_{00}(t)\cdot\mu_{01} + p_{01}(t)\cdot\mu_{11}$$

$$= p_{00}(t)\cdot 0.01 + (-0.1)\cdot p_{01}(t)$$

$$= \underline{p_{00}(t)}\cdot 0.01 - 0.1\, \underline{p_{01}(t)}$$

$$p_{00}(t) + p_{01}(t) = 1 \quad \therefore\ p_{00}(t) = 1 - p_{01}(t)$$

$$\frac{d\,p_{01}(t)}{dt} = \left(1 - p_{01}(t)\right)\cdot 0.01 - 0.1\, p_{01}(t)$$

$$= 0.01 - 0.01\, p_{01}(t) - 0.1\, p_{01}(t)$$

$$= \boxed{0.01 - 0.11\, p_{01}(t)}$$

$$\frac{dP_{10}(t)}{dt} = P_{10}(t)\cdot\mu_{00} + P_{11}(t)\cdot\mu_{10}$$

$$= P_{10}(t)\cdot(-0.01) + (1-P_{10}(t))\cdot 0.1$$

$$= -0.01\,P_{10}(t) + 0.1 - 0.1\,P_{10}(t)$$

$$\boxed{\frac{dP_{10}(t)}{dt} = 0.1 - 0.11\,P_{10}(t)}$$

$$\boxed{\frac{dP_{01}(t)}{dt}} = 0.01 - \underbrace{0.11\,P_{01}(t)}$$

$$\frac{dP_{01}(t)}{dt} + 0.11\,P_{01}(t) = 0.01$$

$\frac{dP_{01}(t)}{dt} \downarrow \frac{dy}{dx}$, $0.11 \downarrow f(x)$, $P_{01}(t) \downarrow y$, $0.01 \downarrow g(x)$

$$IF = e^{\int f(x)\,dx} = e^{\int 0.11\,dt} = e^{0.11t}$$

$$P_{01}(t) \cdot e^{0.11t} = \int e^{0.11t} \cdot 0.01 \cdot dt$$

$$P_{01}(t) \cdot e^{0.11t} = 0.01 \cdot \frac{e^{0.11t}}{0.11} + c$$

$$P_{01}(t) \cdot e^{0.11t} = \frac{1}{11} \cdot e^{0.11t} + c$$

$$\boxed{P_{01}(t) = 1/11 + c \cdot e^{-0.11t}}$$

$$P_{01}(0) = 0$$

$$\therefore 0 = 1/11 + c \cdot e^{-0.11(0)} \quad \therefore 0 = 1/11 + c$$

$$\therefore \boxed{c = -1/11}$$

$$\boxed{P_{01}(t) = \frac{1}{11} - \frac{1}{11} \cdot e^{-0.11t}}$$

$$P_{10}(t) = \frac{10}{11} - \frac{10}{11} e^{-0.11t}$$

$$\frac{dp_{10}(t)}{dt} = \overset{0.1}{\cancel{0.01}} - 0.11\, p_{10}(t)$$

$$\frac{dp_{10}(t)}{dt} + 0.11\, p_{10}(t) = \overset{0.1}{\cancel{0.01}}$$

$$IF = e^{\int 0.11\,dt} = e^{0.11t}$$

$$\therefore\ P_{10}(t)\cdot e^{0.11t} = \int e^{0.11t}\cdot \overset{0.1}{\cancel{0.01}}\,dt$$

$$= \frac{\cancel{0.01}\ 0.1}{0.11}\cdot e^{0.11t} + C$$

$$\therefore\ P_{10}(t)\cdot \cancel{e} = \frac{10}{11} + C\cdot e^{-0.11t}$$

$$P_{10}(0) = 0$$

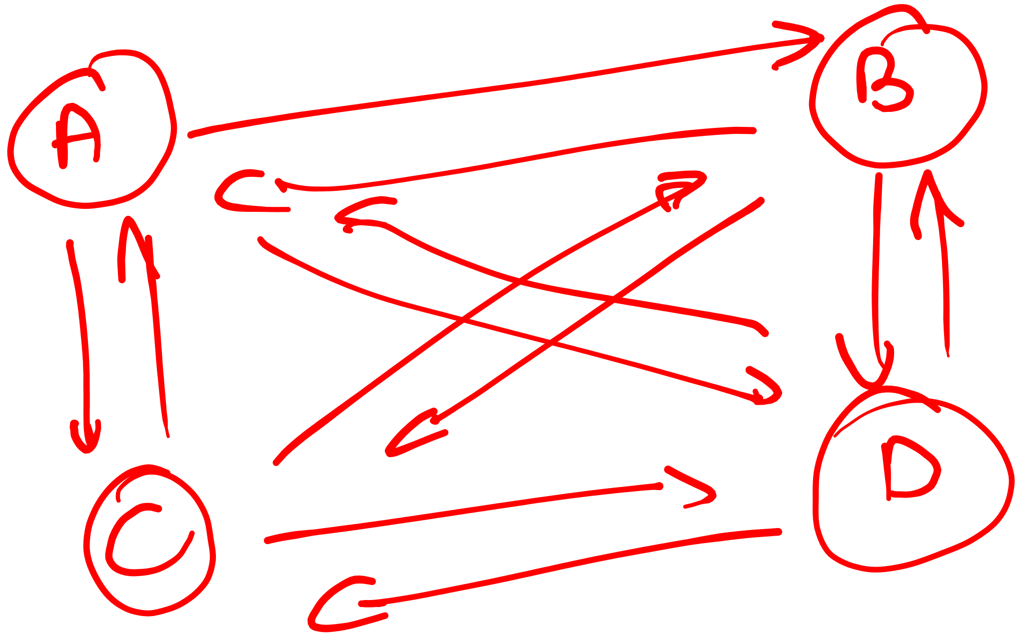
$$\therefore\ 0 = \frac{10}{11} + C\cdot e^{-0.11(0)}$$

$$\therefore\ C = -\frac{10}{11}$$

$$P_{10}(t) = \frac{10}{11} - \frac{10}{11}\cdot e^{-0.11t}$$

Exp. time to reach state 'k' from 'i'

Ex :



→ All transitions are possible

Let's say time taken to reach state 'D' from state 'i' is m_i

$A \longrightarrow D = \underline{m_A}$

$B \to D = m_B$

$C \longrightarrow D \to \underline{m_C}$

$D \longrightarrow D = 0$

$$m_A = \underline{\text{Exp. time in state } A} + \text{Go to state } B \times m_B + \text{Go to state } C \times m_C$$

$$\sum_{i \neq j} \mu_{ij} = \lambda_i$$

Prob. of going to state B whenever I leave state A

$$+ \underline{\text{Go to state } D} \times 0$$

$$= \frac{1}{\lambda_A} + \boxed{\frac{\mu_{AB}}{\sum_{A \neq i} \mu_{Ai}}} \cdot m_B + \frac{\mu_{AC}}{\sum_{A \neq i} \mu_{Ai}} \cdot m_C$$

$$\therefore m_A = \frac{1}{\lambda_A} + \frac{\mu_{AB}}{\lambda_A} \cdot m_B + \frac{\mu_{AC}}{\lambda_A} \cdot m_C$$

$$m_B = \frac{1}{\lambda_B} + \frac{\mu_{BA}}{\lambda_B} \cdot m_A + \frac{\mu_{BC}}{\lambda_B} \cdot m_C$$

↓

cross-dependency in the times

↓

recurssive maths

Exp. time to reach state 'D' from state 'H'.

$$m_H = \frac{1}{\lambda_H} + \frac{\mu_{HS}}{\lambda_H} \cdot m_S$$

$$= \frac{1}{0.07} + \frac{0.02}{0.07} \cdot m_S$$

$$m_S = \frac{1}{\lambda_S} + \frac{\mu_{SH}}{\lambda_S} \cdot m_H + \frac{\mu_{ST}}{\lambda_S} \cdot m_T$$

$$= \frac{1}{1.2} + \frac{1}{1.2} \cdot m_H + \frac{0.15}{1.2} \cdot m_T$$

$$m_T = \frac{1}{\lambda_T} = \frac{1}{0.4} = 2.5$$

$$\therefore m_S = \frac{1}{1.2} + \frac{1}{1.2} m_H + \frac{0.15}{1.2} \cdot (2.5)$$

$$m_S = \frac{5}{6} + \frac{5}{6} m_H + \frac{5}{16}$$

$$\therefore \boxed{m_S = \frac{55}{48} + \frac{5}{6} m_H} \quad -(1)$$

$$m_s = \frac{55}{48} + \frac{5}{6}\left[\frac{100}{7} + \frac{2}{7}\cdot m_s\right]$$

$$m_s = \frac{55}{48} + \frac{500}{42} + \frac{10}{42} m_s$$

$$\therefore \frac{32}{42} m_s = \frac{55}{48} + \frac{500}{42}$$

$$\therefore m_s = \left(\frac{55}{48} + \frac{500}{42}\right)\frac{42}{32}$$

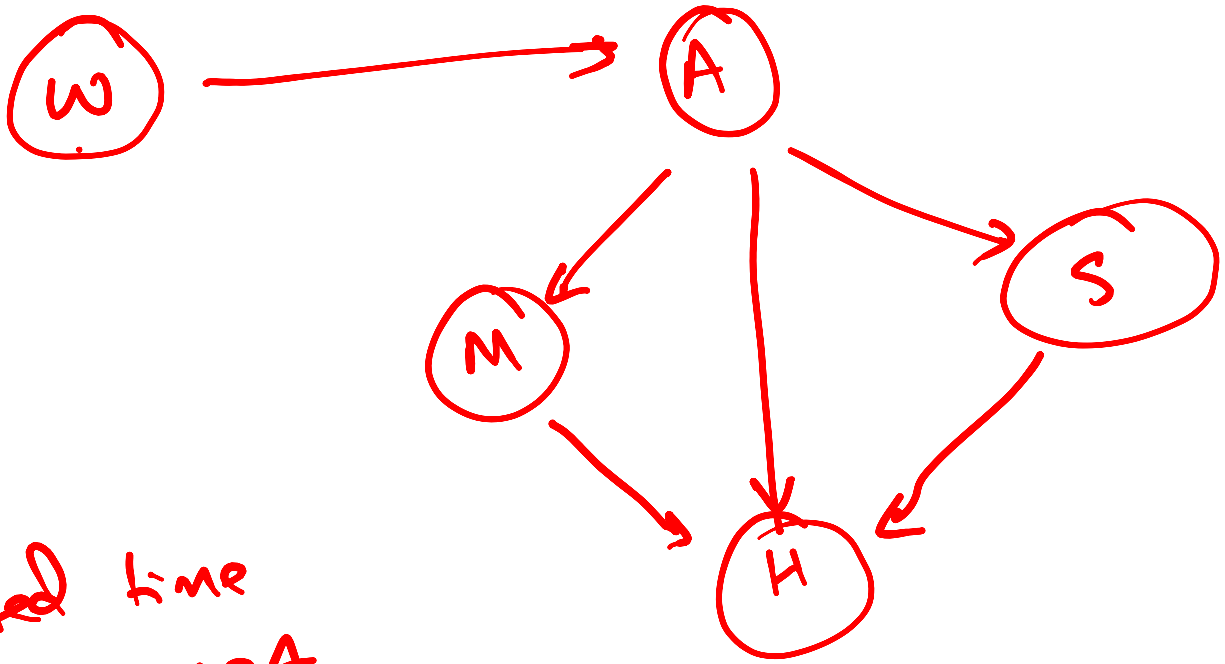
$$= 4385/256 = 17.1289$$

$$m_{lt} = \frac{100}{7} + \frac{2}{7} \times 17.1289$$

$$= 19.1797$$

Generator Matrix:

Transition graph:



$$
\begin{array}{c|ccccc}
 & W & A & M & S & H \\
\hline
W & -1/15 & 1/15 & - & - & - \\
A & - & \frac{-1}{20} & \frac{3}{400} & \frac{1}{400} & \frac{1}{25} \\
M & - & - & \frac{-1}{30} & - & \frac{1}{30} \\
S & - & - & - & \frac{-1}{180} & \frac{1}{180} \\
H & - & - & - & - & -
\end{array}
$$

$\frac{1}{\lambda_A} \to$ expected time in state A

$\lambda_A = \sum_{A \neq j} \lambda_{Aj}$ $\quad$ ~~$\lambda_A = 20$ m.~~ $\quad$ $\lambda_A = \frac{1}{20}$

$$\mu_{AS} = \lambda_A \times \text{Prob}(S) = \frac{1}{20} \times 5\% = \frac{1}{400}$$

$$\mu_{AM} = \lambda_A \times \text{Prob}(M) = \frac{1}{20} \times 15\% = \frac{3}{400}$$

$$\mu_{AH} = \lambda_A \times \text{Prob}(H) = \frac{1}{20} \times 80\% = \frac{1}{25}$$

$$\boxed{\underline{\text{Prob}\,[A} \to H] = \frac{\mu_{AH}}{\lambda_A}}$$

$$\therefore\ \mu_{AH} = \underline{\text{Prob.} \times \lambda_A}$$

$$= 80\% \times \frac{1}{20} = \frac{1}{25}$$

KFD :

$$\frac{dP_{\omega M}}{dt} = \sum_{k \in S_3} P_{\omega k}(t) \cdot \mu_{kM}$$

$$= P_{\omega A}(t) \cdot \mu_{AM} + P_{\omega M}(t) \cdot \mu_{MM}$$

$$= \frac{3}{400} P_{\omega A}(t) - \frac{1}{30} P_{\omega M}(t)$$

$$\frac{dP_{WA}}{dt} = \sum_{k \in S.S.} P_{Wk}(t) \cdot \mu_{kA}$$

$$= P_{WW}(t) \cdot \mu_{WA} + P_{WA}(t) \cdot \mu_{AA}$$

$$\frac{dP_{WA}(t)}{dt} = \frac{1}{15} \cdot P_{WW}(t) + \left(-\frac{1}{20}\right) \cdot P_{WA}(t)$$

Verify $\underline{P_{WA}(t) = 4e^{-t/20} - 4e^{-t/15}}$

$$\frac{dP_{WA}(t)}{dt} = \frac{1}{15}\underline{P_{WW}(t)} - \frac{1}{20}P_{WA}(t)$$

$$\underline{P_{WW}(t)} = P[T = \omega \mid \omega] = e^{-p_{WA}t} \leftarrow \text{[Similar to survival prob.]}$$

$$= e^{-t/15}$$

L.H.S. $= \dfrac{dP_{WA}(t)}{dt}$

R.H.S. $= \dfrac{1}{15}e^{-t/15} - \dfrac{1}{20}P_{WA}(t)$

$$L.H.S. = \frac{d\left[4e^{-t/20} - 4e^{-t/15}\right]}{dt}$$

$$= \frac{-1}{20} \cdot 4e^{-t/20} + \frac{1}{15} \cdot 4e^{-t/15}$$

$$= -\frac{1}{5} e^{-t/20} + \frac{4}{15} e^{-t/15}$$

$$R.H.S. = \frac{1}{15} e^{-t/15} - \frac{1}{20}\left[4e^{-t/20} - 4e^{-t/15}\right]$$

$$= \frac{1}{15} e^{-t/15} - \frac{1}{5} e^{-t/20} + \frac{1}{5} e^{-t/15}$$

$$= \frac{4}{15} e^{-t/15} - \frac{1}{5} e^{-t/20}$$

$\therefore$ L.H.S. = R.H.S.

Try solving using I.F.

$$\frac{d P_{WA}(t)}{dt} = \frac{1}{15} P_{WW}(t) - \frac{1}{20} P_{WA}(t)$$

$$P_{WW}(t) + P_{WA}(t) = 1$$

$$\therefore P_{WW}(t) = 1 - P_{WA}(t)$$

$$\frac{d\,P_{wA}(t)}{dt} = \frac{1}{15}\left[1 - P_{wA}(t)\right] - \frac{1}{20} P_{wA}(t)$$

$$\frac{dy}{dx} + f(x)\cdot y = g(x)$$

$$\frac{dP_{wA}(t)}{dt} = \frac{1}{15} - \frac{1}{15} P_{wA}(t) - \frac{1}{20} P_{wA}(t)$$

$$= \frac{1}{15} - \frac{7}{60} P_{wA}(t)$$

$$\therefore \frac{dP_{WA}(t)}{dt} + \frac{7}{60} P_{WA}(t) = \frac{1}{15}$$

$$\therefore f(x) = 7/60 \qquad g(x) = 1/15$$

$$I.F. = \exp\left[\int 7/60 . dt\right] = e^{7/60\, t}$$

$$\therefore P_{WA}(t) . e^{7/60\, t} = \int 1/15 . e^{7/60\, t} . dt$$

$$= \frac{1}{15} . \frac{60}{7} e^{7/60\, t} + c$$

$$= \frac{4}{7} e^{7/60\, t} + c$$

$$P_{WA}(t) = \frac{4}{7} + c \cdot e^{-7/60\, t}$$

Now, $P_{WA}(0) = 0$

$$\therefore \quad 0 = \frac{4}{7} + c \cdot e^{-7/60(0)}$$

$$\therefore \quad c = -4/7$$

$$\therefore P_{WA}(t) = \frac{4}{7} - \frac{4}{7} \cdot e^{-7/60\, t}$$

This does not lead to the answer.

we have to take $\underline{P_{WW}(t) = e^{-t/15}}$

$$\frac{d\,P_{wM}(t)}{dt} = \frac{3}{400}\,P_{wA}(t) - \frac{1}{30}\,P_{wM}(t)$$

$$\frac{d\,P_{wM}(t)}{dt} = \frac{3}{400}\left[4e^{-t/20} - 4e^{-t/5}\right] - \frac{1}{30}\,P_{wM}(t)$$

$$\frac{dy}{dx} + f(x)\cdot y = g(x)$$

Use I.F. to solve

$$\frac{d\,P_{wm}(t)}{dt} + \frac{1}{30} P_{wm}(t) = \frac{3}{400}\left[4e^{-t/20} - 4e^{-t/15}\right]$$

$$= \frac{3}{100}\left[e^{-t/20} - e^{-t/15}\right]$$

$$f(x) = 1/30 \quad , \quad g(x) = \frac{3}{100}\left[e^{-t/20} - e^{-t/15}\right]$$

$$I.F. = \exp\left[\int 1/30\, dt\right] = e^{1/30\, t}$$

$$P_{wm}(t) \cdot e^{1/30\, t} = \int e^{1/30\, t} \cdot \frac{3}{100}\left[e^{-t/20} - e^{-t/15}\right] dt$$

$$= \frac{3}{100} \int \left[e^{-t/20 + t/30} - e^{-t/15 + t/30} \right] . dt$$

$$= \frac{3}{100} \int \left(e^{-t/60} - e^{-t/30} \right) . dt$$

$$= \frac{3}{100} \left[(-60) . e^{-t/60} + 30 e^{-t/30} \right] + c$$

$$P_{wm}(t) . e^{1/30 t} = -\frac{9}{5} e^{-t/60} + \frac{9}{10} e^{-t/30} + c$$

$$P_{wm}(t) = -\frac{9}{5} e^{-t/60 - t/30} + \frac{9}{10} e^{-t/30 - t/30} + c . e^{-t/30}$$

$$P_{WM}(t) = -\frac{9}{5} e^{-t/20} + \frac{9}{10} e^{-t/15} + c \cdot e^{-t/30}$$

$$\boxed{P_{ii}(0) = 1}$$

$$P_{WM}(0) = 0$$

$$\therefore 0 = -\frac{9}{5} e^{0} + \frac{9}{10} e^{0} + c \cdot e^{0}$$

$$\therefore 0 = -\frac{9}{5} + \frac{9}{10} + c \quad \therefore \boxed{c = \frac{9}{10}}$$

$$P_{WM}(t) = -\frac{9}{5} e^{-t/20} + \frac{9}{10} e^{-t/15} + \frac{9}{10} e^{-t/30}$$

$T_A,\ T_m,\ T_S$:

$T_M = 30$, $T_S = 180$

$T_A = 0.8 \times 0 + 0.15 \times T_M + 0.05 \times T_S + 20$

$= 0.15 \times 30 + 0.05 \times 180 + 20$

$= 4.5 + 9 + 20$

$= 33.5$ mins

$$T_{WD} = 15 + T_A$$

$$= 15 + 33.5$$

$$= \underline{\underline{48.5}}$$

Solⁿ-

i) Assumptions of Markov jump model-

→ Process has the Markov property.

→ Transition intensities are constant.

→ Transition rates are independent of each other.

→ Small time transition prob. ie, $p_{ij}(h)$ can be defined as $h.\mu_{ij} + o(h)$.

iv)

Dist^2 of n_{ij}/n_i

Binomial dist^2

$$X \sim Bin(n, p)$$

Total no. (n)

Prob. of success (p)

Prob. of success = $\boxed{p_{ij} = \frac{N_{ij}}{n_i}}$

Total no. = n_i

Ex. $p_{AB} = \frac{N_{AB}}{N_{AB} + N_{AC}}$

$= \frac{12/7}{3} = \frac{4}{7}$