Lecture

INSTITUTE OF ACTUARIAL & QUANTITATIVE STUDIES

Class: TY BSc

Subject : Statistical and Risk Modelling-4

Chapter: Unit 4 Chapter 1

Chapter Name: Time Series Models Analysis and Forecasting- 2

(Non-stationary and Non-linear time series models)

Today’s Agenda

Non-stationary and Non-Linear Time series

1. Bilinear models
2. Threshold Autoregressive models
3. Random Coefficient Autoregressive models
4. Autoregressive models with conditional heteroscedasticity

1 Bilinear Models

The general class of bilinear models can be exemplified by its simplest representative, the random process X defined by the relation:

$$X_n - \alpha(X_{n-1} - \mu) = \mu + e_n + \beta e_{n-1} + b(X_{n-1} - \mu)e_{n-1}$$

Considered only as a function of X , this relation is linear; it is also linear when considered as a function of e only. Therefore, it is called ‘bilinear’.

The main qualitative difference between the bilinear model and models from the ARMA class is that many bilinear models exhibit ‘bursty’ behavior: when the process is far from its mean it tends to exhibit larger fluctuations. The difference between this model and an ARMA(1,1) process may be seen to lie in the last term on the right-hand side: when X_{n-1} is far from μ and e_{n-1} is far from 0 – events which are far from being independent – the final term assumes a much greater significance.

2 Threshold autoregressive models

A simple representative of the class of threshold autoregressive models is the random process X defined by the relation:

$$X_n = \mu + \begin{cases} \alpha_1(X_{n-1} - \mu) + e_n, & \text{if } X_{n-1} \le d \\ \alpha_2(X_{n-1} - \mu) + e_n, & \text{if } X_{n-1} > d \end{cases}$$

The distinctive feature of some models from the threshold autoregressive class is the limit cycle behavior. This makes the threshold autoregressive models suitable for the description of ‘cyclic’ phenomena.

3 Random coefficient autoregressive models

Another modification of the AR class of models is that of autoregressive models for which the coefficient is random. In other words:

$$\boldsymbol{X_t = \mu + \alpha_t(X_{t-1} - \mu) + e_t}$$

where $\{\alpha_1, \alpha_2, \dots, \alpha_n\}$ is a sequence of independent random variables.

Such a model could be used to represent the behavior of an investment fund, with $\mu = 0$ and $\alpha_t = 1 + i_t$ with i_t being the random rate of return.

The behavior of these processes can vary widely, depending on the distribution chosen for the α_t, but is in general more irregular than that of the corresponding $AR(1)$.

4 Autoregressive models with conditional heteroscedasticity

What is heteroscedasticity?

Financial assets often display the following behavior. After a large change in the asset price there follows a period of high volatility, which can be in either direction. Following small changes there tend to be further small changes. In other words, the variance of the process is dependent upon the size of the previous value. This is the property of conditional heteroscedasticity.

The words ‘homoscedastic’ and ‘heteroscedastic’ just mean having equal (i.e., constant) or different variances, respectively.

4 Autoregressive models with conditional heteroscedasticity

The class of autoregressive models with conditional heteroscedasticity of order p - the ARCH (p) - is defined by the relation:

$$X_t = \mu + e_t \sqrt{\alpha_0 + \sum_{k=1}^{p} \alpha_k \left(X_{t-k} - \mu\right)^2}$$

where e is a sequence of independent standard normal random variables. The simplest representative of the ARCH (p) class is the ARCH(1) model defined by the relation:

$$X_t = \mu + e_t \sqrt{\alpha_0 + \alpha_1 \left(X_{t-1} - \mu\right)^2}$$

If μ is zero, it can be shown that $\mathrm{cov}(X_t, X_s) = 0$ for $s \neq t$ confirming that X_t is white noise with uncorrelated but not independent components.

4 Autoregressive models with conditional heteroscedasticity

The ARCH models have been used for modelling financial time series. If Z_t is the price of an asset at the end of the t-th trading day, it is found that the ARCH model can be used to model $X_t = \ln(Z_t/Z_{t-1})$, interpreted as the daily return on day t.

The ARCH family of models captures the feature frequently observed in asset price data that a significant change in the price of an asset is often followed by a period of high volatility.

As may be seen from the $ARCH(1)$ model, a significant deviation of X_{t-1} from the mean μ gives rise to an increase in the conditional variance of X_t given X_{t-1}.