



INSTITUTE OF ACTUARIAL
& QUANTITATIVE STUDIES

Subject:

SRM 1

Chapter:

UNIT 4

Category:

Practice Questions

1. CS2A A2022 Q1

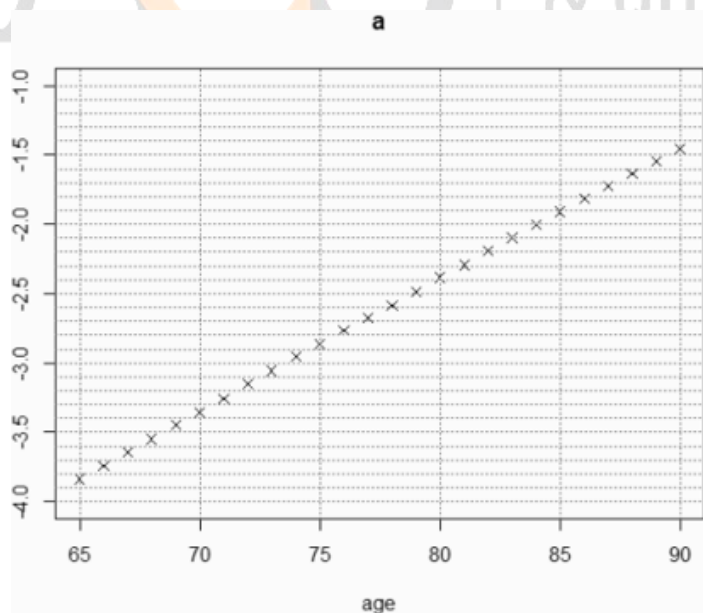
(i) Write down the two-factor Lee–Carter model, clearly defining each of the terms you use. [3]

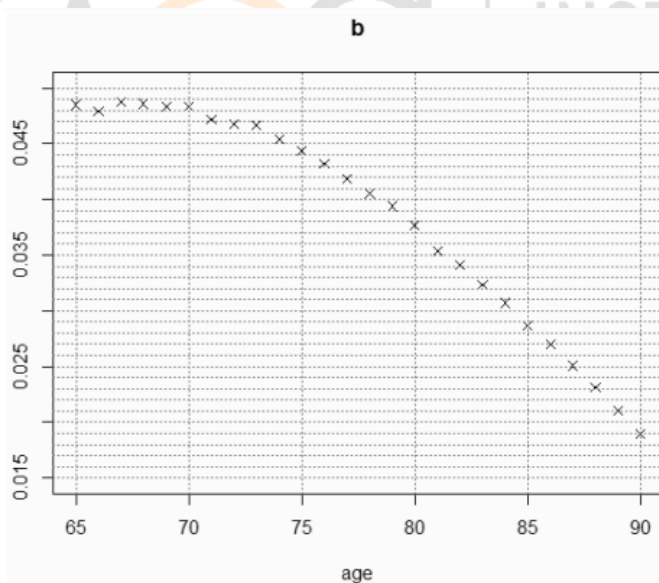
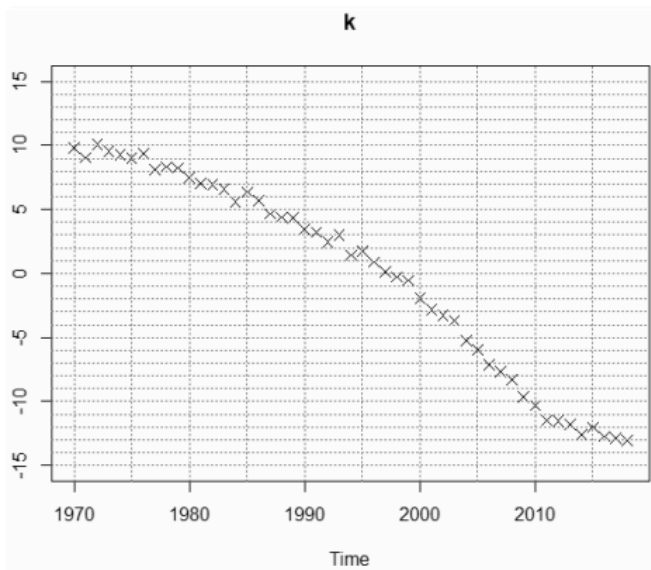
A statistician is using the two-factor Lee–Carter model to project future mortality rates and has fitted the model to a set of mortality data. The statistician has observed that the fitted forces of mortality do not vary regularly with each calendar year but vary more regularly with age. Therefore, the statistician suggests that, before projecting future mortality rates, the k parameters in the Lee–Carter model should be smoothed using penalised regression splines, but the a and b parameters should be kept as separate, unsmoothed parameters for each age.

(ii) Discuss the statistician’s suggestion by considering each of the three parameters of the Lee–Carter model, k , a and b , in turn. [3] [Total 6]

2. CS2A A2023 Q3

The figure below shows three vectors ax , bx and kt , which were obtained by applying the two-factor Lee–Carter model to mortality data derived from the period 1970 to 2018 for the age range 65 to 90 years.





- (i) Discuss what each of these three vectors suggest about the characteristics of the mortality rates in the data. [4]
- (ii) Suggest a socio-economic reason for your conclusion about vector b in part (i) above. [1]
- (iii) Calculate $m_{70, 2018}$ using the output from the model (reading the values from the plots in the figure). [1]

On analysing the past data over a larger age range (from 20 to 90 years) it was determined that there was an increase in rates for 20–30 year olds in the 1970s due to an illness that almost exclusively affected the younger population. The higher mortality rates only lasted until 1980 when a cure was found and introduced to the population.

(v) Explain how this cohort effect may manifest itself in the output from the two-factor Lee–Carter model, fitted to ages 20–90 and calendar years 1970–2018, in terms of a , b and k . [5]

[Total 11]

3. IAQS PAST PAPER

For the senior citizen population in a country, a statistician has decided to use the two-factor Lee–Carter model to project future mortality rates and has fitted the model to a set of mortality data.

i) Write down the two-factor Lee–Carter model, clearly defining each of the terms you use. Also list out the constraints that are normally imposed, in order for the model to be uniquely specified. (3)

ii) The parameters a_x and b_x of the model for ages between 60 to 80 (inclusive) are expressed using linear functions as below.

$$a_x = 0.105x - 10.95$$

$$b_x = -0.004x + 0.48$$

Further, it is assumed that the factor k_t decreases linearly from 2.75 at time 0 to -1.25 at time 40.

Estimate the central mortality rate for ages 60 and 70 and at times 0, 10, 20, 30 and 40. (6)

iii) From the results in ii above, comment on the trend in central mortality rate with time for ages 60 and 70. (2)

iv) Describe the advantages and disadvantages of Lee-Carter Model. (4)

Total [15]

Solution:

i)

The two-factor Lee-Carter model may be written:

$$\ln m_{x,t} = a_x + b_x \cdot k_t + \varepsilon_{x,t}$$

where:

$m_{x,t}$ is the central mortality rate at age x in year t

[0.5]

a_x describes the general shape of mortality at age x or a_x is the mean of the time-averaged logarithms of the central mortality rate at age x .

[0.5]

b_x measures the change in the rates in response to an underlying time trend in the level of mortality of k_t .

[0.5]

k_t reflects the effect of the time trend on mortality at time t , and

[0.5]

$\varepsilon_{x,t}$ are independently distributed normal random variables with means of zero and some variance to be estimated.

[0.5]

The usual constraints imposed in the Lee-Carter Model are that $\sum b_x x = 1$

and

$$\sum k_t t = 0$$

[0.5]

[3]

ii)

Ignoring error term $\epsilon_{x,t}$, the mortality rate at age x in projection year t is:

$$m_{x,t} = \exp(ax + b_x * kt)$$

[0.5]

The time trend factor kt is assumed to decrease linearly from 2.75 at time 0 to -1.25 at time 40. Hence, can be expressed as

$$kt = 2.75 - 0.1 * t$$

[0.5]

Further, parameters a_x and b_x of the model for ages between 60 to 80 (inclusive) are expressed using linear functions as below.

$$a_x = 0.105x - 10.95$$

&

$$b_x = -0.004x + 0.48$$

x	a_x	b_x
60	-4.65	0.24
70	-3.6	0.2

[1]

From the above,

$$\begin{aligned} m_{60,t} &= \exp(a_{60} + b_{60} * kt) \\ &= \exp(-4.65 + 0.24 * (2.75 - 0.1t)) \\ &= \exp(-4.65 + 0.66 - 0.024t) \\ &= \exp(-3.99 - 0.024t) \end{aligned}$$

[1]

$$m_{60,0} = \exp(-3.99) = 0.0185$$

$$m_{60,10} = \exp(-3.99 - 0.24) = \exp(-4.23) = 0.014552$$

$$m_{60,20} = \exp(-3.99 - 0.48) = \exp(-4.47) = 0.011447$$

$$\begin{aligned}
 m_{60,30} &= \exp(-3.99 - 0.72) = \exp(-4.71) = 0.009005 \\
 m_{60,40} &= \exp(-3.99 - 0.96) = \exp(-4.95) = 0.007083 \\
 [1]
 \end{aligned}$$

Similarly,

$$\begin{aligned}
 m_{70,t} &= \exp(a_{70} + b_{70} \cdot kt) \\
 &= \exp(-3.6 + 0.2 \cdot (2.75 - 0.1t)) \\
 &= \exp(-3.6 + 0.55 - 0.02t) \\
 &= \exp(-3.05 - 0.02t) \\
 [1]
 \end{aligned}$$

$$\begin{aligned}
 m_{70,0} &= \exp(-3.05) = 0.047359 \\
 m_{70,10} &= \exp(-3.05 - 0.2) = \exp(-3.25) = 0.038774 \\
 m_{70,20} &= \exp(-3.05 - 0.4) = \exp(-3.45) = 0.031746 \\
 m_{70,30} &= \exp(-3.05 - 0.6) = \exp(-3.65) = 0.025991 \\
 m_{70,40} &= \exp(-3.05 - 0.8) = \exp(-3.85) = 0.02128 \\
 [1]
 \end{aligned}$$

[6]

Alternatively, Ignoring error term $e_{x,t}$, the mortality rate at age x in projection year t is:

$$\begin{aligned}
 m_{x,t} &= \exp(ax + bx \cdot kt) \\
 [0.5]
 \end{aligned}$$

The time trend factor kt is assumed to decrease linearly from 2.75 at time 0 to -1.25 at time 40.

Hence, can be expressed as

$$\begin{aligned}
 kt &= 2.75 - 0.1 \cdot t \\
 [0.5]
 \end{aligned}$$

Further, parameters a_x and b_x of the model for ages between 60 to 80 (inclusive) are expressed using linear functions as below.

$$a_x = 0.105x - 10.95$$

&

$$b_x = -0.004x + 0.48$$

$$\begin{aligned}
 m_{x,t} &= \exp(0.105x - 10.95 + (-0.004x + 0.48) \cdot (2.75 - 0.1 \cdot t)) \\
 &= \exp(0.105x - 10.95 - 0.004 \cdot 2.75x + 0.004 \cdot 0.1 \cdot x \cdot t + 0.48 \cdot 2.75 - 0.048 \cdot t) \\
 &= \exp(0.105x - 10.95 - 0.011x + 1.32 - 0.048 \cdot t + 0.0004 \cdot x \cdot t) \\
 &= \exp(0.094x - 9.63 - 0.048 \cdot t + 0.0004 \cdot x \cdot t) \\
 [2]
 \end{aligned}$$

$$\begin{aligned}
 m_{60,0} &= \exp(-3.99) = 0.0185 \\
 m_{60,10} &= \exp(-4.23) = 0.014552 \\
 m_{60,20} &= \exp(-4.47) = 0.011447
 \end{aligned}$$

$$m_{60,30} = \exp(-4.71) = 0.009005$$

$$m_{60,40} = \exp(-4.95) = 0.007083$$

[1.5]

$$m_{70,0} = \exp(-3.05) = 0.047359$$

$$m_{70,10} = \exp(-3.25) = 0.038774$$

$$m_{70,20} = \exp(-3.45) = 0.031746$$

$$m_{70,30} = \exp(-3.65) = 0.025991$$

$$m_{70,40} = \exp(-3.85) = 0.02128$$

[1.5]

[6]

iii) For every year that we project into the future, the mortality rate $m_{60,t}$ is multiplied by a factor of $e^{-0.024}$ (or 0.9763), i.e. it decreases by approximately 2.4% p.a.

[1]

For $m_{70,t}$, we multiply by a factor of $e^{-0.02}$ (or 0.9802) for each year that we project into the future and hence the values of $m_{70,t}$ decrease by approximately 2% p.a.

The percentage reduction in $m_{70,t}$ is smaller than that for $m_{60,t}$ since $b_{70} < b_{60}$.

[1]

[2]

iv) Future estimates of mortality at different ages are heavily dependent on the original estimates of the parameters a_x and b_x , which are assumed to remain constant into the future. These parameters are estimated from past data, and will incorporate any roughness contained in the data. In particular, they may be distorted by past period events which might affect different ages to different degrees.

[1]

- If the estimated b_x values show variability from age to age, it is possible for the forecast age-specific mortality rates to 'cross over' (such that, for example, projected rates may increase with age at one duration, but decrease with age at the next). [0.5]
- There is a tendency for Lee-Carter forecasts to become increasingly rough over time. [0.5]
- The model assumes that the underlying rates of mortality change are constant over time across all ages, when there is empirical evidence that this is not so. [0.5]
- The Lee-Carter model does not include a cohort term, whereas there is evidence from some countries that certain cohorts exhibit higher mortality improvements than others. [0.5]

- Unless observed rates are used for the forecasting, it can produce ‘jump-off’ effects (ie an implausible jump between the most recent observed mortality rate and the forecast for the first future period). [0.5]

[4]



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