



INSTITUTE OF ACTUARIAL
& QUANTITATIVE STUDIES

Subject:

SRM 1

Chapter:

UNIT 1

Category:

**Practice
Questions**

1. CT4 September 2010 Q9

A researcher is reviewing a study published in a medical journal into survival after a certain major operation. The journal only gives the following summary information:

- the study followed 16 patients from the point of surgery
- the patients were studied until the earliest of five years after the operation, the end of the study or the withdrawal of the patient from the study
- the Nelson-Aalen estimate, $S(t)$, of the survival function was as follows

Duration since operation t (years)	$S(t)$
$0 \leq t < 1$	1
$1 \leq t < 3$	0.9355
$3 \leq t < 4$	0.7122
$4 \leq t < 5$	0.6285

- Describe the types of censoring which are present in the study.
- Calculate the number of deaths which occurred, classified by duration since the operation.
- Calculate the number of patients who were censored.

Ans: (ii) 1 death at duration 1 year, 3 deaths at duration 3 years, 1 death at duration 4 years, (iii) 11 patients

2. CT4 April 2011 Q6

A study of the mortality of a certain species of insect reveals that for the first 30 days of life, the insects are subject to a constant force of mortality of 0.05. After 30 days, the force of mortality increases according to the formula:

$$\mu_{30+x} = 0.05 \exp(0.01x),$$

where x is the number of days after day 30.

- Calculate the probability that a newly born insect will survive for at least 10 days.
- Calculate the probability that an insect aged 10 days will survive for at least a further 30 days.
- Calculate the age in days by which 90 per cent of insects are expected to have died.

Ans: (i) 0.6065, (ii) 0.2174, (iii) 44.88 days

3. CT4 April 2012 Q7

Mr Bunn the baker made 12 pies to sell in his shop. He placed the pies in the shop at 9 a.m. During the rest of the day the following events took place.

<i>Time</i>	<i>Event</i>
10 a.m.	A boy bought two pies
11 a.m.	A man bought three pies
12 noon	Mr. Bunn accidentally sat on one pie and squashed it so it could not be sold
1 p.m.	A woman bought two pies
2 p.m.	A dog from across the street ran into Mr Bunn's shop and stole two pies
3 p.m.	A girl on the way home from school bought one pie.
5 p.m.	Mr Bunn closed for the day and the remaining pie was still in the shop

- (i) Estimate the time it takes Mr Bunn to sell 40% of the pies he makes, using the Nelson-Aalen estimator
- (ii) Comment on whether you think this estimate would be a good basis for Mr Bunn to plan his future production of pies.

Ans: (i) 4 hours

4. CT4 April 2013 Q7

The Shining Light Company has developed a new type of light bulb which it recently tested. 1,000 bulbs were switched on and observed until they failed, or until 500 hours had elapsed. For each bulb that failed, the duration in hours until failure was noted. Due to an earth tremor after 200 hours, 200 bulbs shattered and had to be removed from the test before failure.

The results showed that 10 bulbs failed after 50 hours, 20 bulbs failed after 100 hours, 50 bulbs failed after 250 hours, 300 bulbs failed after 400 hours, and 50 bulbs failed after 450 hours.

- (i) Calculate the Kaplan-Meier estimate of the survival function, $S(t)$, for the light bulbs in the test.
- (ii) Sketch the Kaplan-Meier estimate calculated in part (i).
- (iii) Estimate the probability that a bulb will not have failed after each of the following durations: 300 hours, 400 hours, and 600 hours. If it is not possible to obtain an estimate for any of the durations without additional assumptions, explain why

Ans: (iii) $S(300) = 0.9070$, $S(400) = 0.5291$, $S(600)$ cannot be estimated without additional assumptions as it lies outside the range of our data.

5. CT4 April 2013 Q2

In the context of a survival model:

- (i) Define right censoring, Type I censoring and Type II censoring
- (ii) Give an example of a practical situation in which censoring would be informative.

6. CT4 September 2013 Q6

- (i) Explain what is meant by censoring in the context of a mortality investigation.

A trial was conducted on the effectiveness of a new cream to treat a skin condition. 100 sufferers applied the cream daily for four weeks or until their symptoms disappeared if this happened sooner. Some of the sufferers left the trial before their symptoms disappeared.

- (ii) Describe two types of censoring that are present and state to whom they apply.

The following data were collected.

<i>Number of sufferers</i>	<i>Day symptoms disappeared</i>	<i>Number of sufferers</i>	<i>Day they left the trial</i>
2	6	3	2
1	7	1	10
1	10	3	13
2	14		

- (iii) Calculate the Nelson-Aalen estimate of the survival function for this trial.
- (iv) Sketch the survival function, labelling the axes.
- (v) Estimate the probability that a person using the cream will still have symptoms of the skin condition after two weeks.

Ans: (v) The survival probability at $t = 14$ is 0.93777.

7. CT4 April 2014 Q8

- (i) Describe what is meant by censoring in the context of a mortality investigation.
- (ii) Explain what right-censoring, left-censoring and interval censoring are, giving an example of each

A toy manufacturer is testing the lifetime of its new electric children's toy. 500 are set going at 9 a.m. one morning on test rigs plugged into the electricity supply and are run until 5 p.m. the next day or until they fail, whichever comes first.

Unfortunately, the cleaner unplugged a test rig on which 17 toys were still working at 7 p.m. on the first evening in order to plug his floor polisher in.

Then, as he left work three hours later, he took three of the still working toys for his children to play with.

Of the other 480 toys it was found that 12 failed after four hours, 25 failed after 11 hours and a further 8 failed after 31 hours.

- (iii) Explain which forms of censoring are present in this investigation
- (iv) Calculate the Nelson-Aalen estimate of the survival function.
- (v) Sketch a graph of the Nelson-Aalen estimate of the survival function, labelling the axes.
- (vi) Comment on the length of time for which a new toy has a 60% probability of surviving.

8. CT4 September 2016 Q6

Brian worked in a large open-plan office with a communal kitchen in which the workers made coffee. Each worker supplied his or her own coffee cup. For several years Brian was annoyed by his coffee cups being taken away by colleagues and never returned to the kitchen, so he decided to do an experiment. He brought into the kitchen 20 cups which were distinguishable from the other cups in the kitchen. At the end of each day for 15 days he counted the number of his 20 cups which remained.

The results were as follows:

Day	Number of Cups	Day	Number of Cups
1	20	9	15
2	19	10	15
3	18	11	15
4	18	12	15
5	17	13	13
6	17	14	12
7	17	15	10
8	16		

Brian noted that:

- the cup that “disappeared” during day 2 was taken home by Brian to be used by his mother.
- the two cups that “disappeared” during day 13 were accidentally broken b

Brian when doing his daily check.

Let $h(x)$ be the hazard that each of Brian's cups is taken by colleagues during day x and not returned and let $S(x)$ be the corresponding survival function.

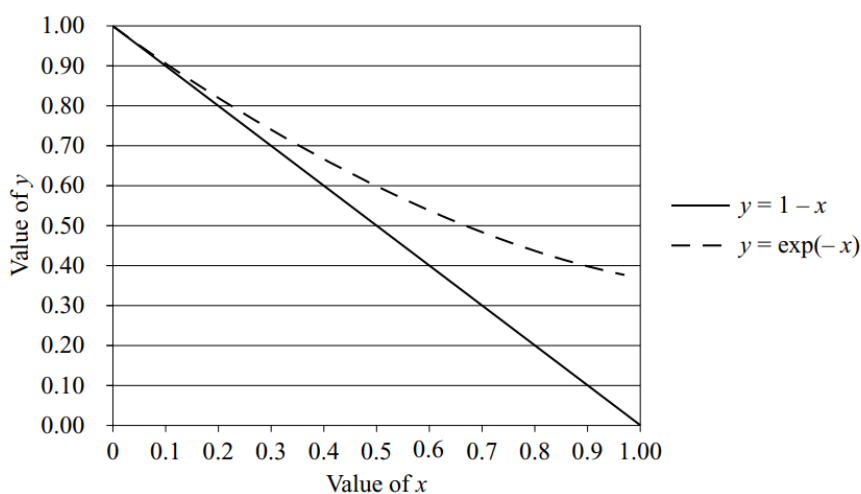
- (i) Determine an estimate of $S(x)$ for Brian's cups using the Nelson-Aalen estimator.
- (ii) Sketch a chart for your estimated $S(x)$.

9. CT4 September 2017 Q10

- (i) Write down the formulae for the Kaplan-Meier estimator $\hat{S}(t)$ and Nelson-Aalen estimator $\hat{S}(t)$ of survival in the presence of a stated hazard, defining all terms used.

The following graph shows the functions:

- $y = 1 - x$ and
- $y = e^{-x}$ over the range $0 \leq x \leq 1$.



- (ii) Demonstrate that the Nelson-Aalen estimator is never lower than the Kaplan-Meier estimator.

A trial is conducted amongst 20 patients who have suffered from eczema but are in remission (that is, they are clear of the condition). The trial is to assess whether continuing with periodic doses of a certain steroid cream in remission reduces the rate at which eczema recurs. Patients are invited to tests every 3 months for a period of up to 5 years from when first declared to be in remission.

- (iii) Describe THREE types of censoring present in the investigation.

The data for the trial are subdivided into a group who continued to receive the steroid cream, and a control group who did not receive the steroid cream. The data for the patients in the trial showing the quarterly test at which eczema recurred, or censoring occurred, are as follows (an * indicates a patient who was censored):

For group receiving steroid cream: 3, 5, 6*, 7*, 10, 10, 12*, 14*, 18, 19*
 For control group: 6, 8, 8, 10*, 11*, 12*, 14, 15*, 18, 18

(iv) Calculate the Kaplan-Meier estimates of the survival function for remaining clear of eczema for:

- (a) the group who continued to receive the steroid cream; and
- (b) the control group.

(v) (a) Recommend, without performing any calculations, a method of establishing whether the hazard of eczema returning is statistically lower for those continuing to receive the steroid cream.

(b) Comment on the chance of being able to conclude from the trial data that continuing to receive the steroid cream reduces the risk of recurrence of eczema.

10. CT4 September 2018 Q7

(i) Define the following types of censoring in the context of a mortality investigation:

- random censoring;
- right censoring;
- informative censoring.

Anwen received a bunch of 17 fresh red roses on the evening of her birthday from her boyfriend. She arranged them in a vase and placed them on the table in the garden for all to admire. She needed to do a project for school so decided to use them to conduct an experiment as to how long roses live before they start to wilt. She checked them very often, and noted down the date when any was showing signs of wilting, and immediately removed the wilting rose from the vase.

The following shows what she discovered.

Day 2. Very disappointing, already two roses wilting.

Day 3. A neighbour passed with his goat which took a nibble at the bunch, so three damaged, but otherwise fresh, roses had to be removed.

Day 5. One more wilting.

Day 7. Three more wilting.

Day 8. The boy down the road stole a fresh rose to give to his sweetheart.

Day 9. Another one is wilting and it is hard to make the remaining ones look good in the vase, so the project is terminated.

(ii) For each of the three types of censoring listed in part (i):

(a) State which roses (if any) experience that censoring.

(b) Explain why those roses (if any) experience that censoring.

11. CS2A September 2020 Q5

Zebras in a large African wildlife park have recently become susceptible to a particular disease called Zebra rabies. Zebras often die as a direct result of contracting this disease. An investigation to monitor deaths due to this disease was carried out between 1 January 2018 and 1 January 2019.

A researcher was interested in the rate at which zebras die once contracting this disease and decided to monitor the health of each zebra on the first day of each month. All zebras in the wildlife park were tagged to ensure that they were identifiable. 14 zebras were diagnosed with rabies during 2018. The data recorded on these 14 zebras are set out below:

<i>Reference tag</i>	<i>Date of diagnosis</i>	<i>Date of death</i>	<i>Reason for death</i>
1	1 Jan 2018	1 Jun 2018	Rabies
3	1 Jan 2018	1 Dec 2018	Rabies
4	1 Apr 2018	1 Jul 2018	Killed by lion
7	1 Apr 2018	1 Jun 2018	Rabies
8	1 Apr 2018	1 Dec 2018	Rabies
10	1 Jul 2018	1 Sep 2018	Rabies
11	1 Aug 2018	1 Oct 2018	Rabies
12	1 Aug 2018	1 Nov 2018	Rabies
19	1 Sep 2018	1 Oct 2018	Rabies
20	1 Oct 2018	1 Nov 2018	Rabies

Two zebras (reference tags 9 and 21) escaped from the wildlife park on 1 December 2018 having been diagnosed with rabies on 1 July 2018 and 1 November 2018, respectively.

In addition, the following two zebras that contracted the disease were still alive at the end of the investigation:

<i>Reference tag</i>	<i>Date of diagnosis</i>
13	1 Aug 2018
25	1 Dec 2018

- (i) Discuss whether the different types of censoring present in the above investigation are likely to be informative.
- (ii) Determine the Kaplan–Meier estimate of the survival function, where the decrement of interest is death due to rabies.

12. CS2A April 2021 Q7

An Actuarial Analyst is investigating the forces of mortality for males aged 65 and over. The Analyst has studied a group of 100 male lives, all of whom were exactly 65 years old at the beginning of the study, over a 10-year period and has estimated the following forces of mortality based on the lives observed in the study

<i>Duration from age 65 (years)</i>	<i>Force of mortality (p.a.)</i>
0–1	0.040
1–8	0.005
8+	0.080

- (i) Determine, to six decimal places, the probability that a 65-year-old male is alive at age 75, using the estimated forces of mortality in the table above.

A colleague has suggested that the Analyst use the following formula to model the forces of mortality at age x :

$$\mu_x = 0.0020291 + 0.0001000 \times 1.0793496^x$$

- (ii) Verify that the probability that a 65-year-old male is alive at age 75 using the suggested model matches the result calculated in part (i) to six decimal places.

13. CS2A September 2021 Q5

A study was undertaken into survival rates following major heart surgery. Patients who underwent this surgery were monitored from the date of surgery until whichever of the following events occurred first:

- they died
- they left the hospital where the surgery was carried out, or
- a period of 30 days had elapsed.

- (i) State, with reasons, two forms of censoring that are present in this study and one form of censoring that is not present.

The Analyst collating the results calculated the Nelson–Aalen estimate of the survival function, $S(t)$, as follows:

t (days)	$S(t)$
$0 \leq t < 5$	1.0000
$5 \leq t < 17$	0.9001
$17 \leq t < 25$	0.8456
$25 \leq t$	0.7157

(ii) State, using the Nelson–Aalen estimate, the probability of survival for 20 days after the surgery.

The Analyst also wishes to calculate the Kaplan–Meier estimate of the survival function.

(iii) Determine the Kaplan–Meier estimate of the survival function.

14. CS2A April 2023 Q7

A mountain rescue service is looking to introduce a new training programme for volunteers who wish to join the service. Each Saturday for 10 weeks trainee rescuers are asked to join a mountain climb. Only those who successfully complete the climb are invited back the following week.

The rescue service will recruit those trainees who successfully complete a certain number of Saturday climbs. To decide on how many successful weeks should be required for a new recruit, the rescue service conducts a trial with 20 volunteers. The table below shows how many of these volunteers fail to complete the climb each week and the number who are eligible but do not arrive for the beginning of each climb.

<i>Week</i>	<i>Eligible but do not arrive</i>	<i>Arrive but fail to complete the climb</i>
1	0	1
2	0	2
3	1	2
4	0	0
5	0	1
6	4	1
7	0	2
8	0	1
9	0	2
10	0	1

(i) Explain why the Kaplan–Meier estimate is a suitable way to evaluate this training programme.

The rescue service would like to recruit 30% of the volunteers who start the programme.

(ii) Calculate the number of successful weeks the service should require trainees to complete using the Kaplan–Meier estimate.

(iii) Discuss what concerns the rescue service should have about using this study to set the recruitment criteria for all future volunteers.

Ans: (ii) Weeks required is 8

15. CS2A September 2023 Q1

A leap year is a calendar year which has 366 days including 29 February. This occurs once every 4 years and the most recent leap year was 2020. A group of people who were born on 29 February in different leap years meet for dinner once every 4 years when it is a leap year to celebrate their unusual birthdays. The number of people at the dinner on 29 February 2020 was recorded by their year of birth as follows:

Year of birth	1948	1952	1956
Number attending	3	8	18

One of the group decides to estimate the survival of group members using Gompertz Law with the force of mortality at age x given by:

$$\mu_x = 0.0045 (1.0004)^x$$

(i) Comment on the choice of formula for the force of mortality.

(ii) Calculate the expected number of people that will be at the next dinner stating any assumptions you make.

A new member, born on 29 February 1960, asks to join the group.

(iii) Calculate the expected cost of dinners for this member up to and including 29 February 2040 if each dinner costs £60 per person and ignoring interest and inflation.

Ans: (ii) 28.47 people. (iii) £283.91

16. CS2A September 2023 Q7

A survival study followed twelve patients, for a maximum of 10 days each, following a major surgical operation. From previous similar studies, around one-third of patients survived 10 days. The condition of all patients (i.e. whether a patient was alive or dead) was monitored daily. The results are set out below where $S(t)$ is the Kaplan–Meier estimate of the survival function:

Time since operation (days)	$S(t)$
$0 \leq t < 2$	1
$2 \leq t < 4$	0.9
$4 \leq t < 5$	0.7
$5 \leq t < 7$	0.56
$7 \leq t < 10$	0.373

(i) Calculate the number of deaths and the number of patients who were censored, stating the times of all deaths and censoring events.

An expert analyst has voiced concerns about the accuracy of the data, thinking there is likely to have been an error.

(ii) Briefly explain the likely source of the expert's concerns.

(iii) State ways in which this study could be improved.

Ans:

Time (days)	deaths	Censoring events
<2	0	2
2	1	0
4	2	2
5	1	1 or
6	0	1
7	1	0

17. CS2A September 2024 Q2

The mortality of the population of a particular country has been studied. A recent investigation suggests that for individuals aged 10 exact and over, the force of mortality is constant, μ . Further, the study suggests that 25% of those born survived to exact age 20 and 20% of those born survived to exact age 25 years.

(i) Calculate:

- the constant force of mortality, μ .
- the survival probability, ${}_{10}p_0$.

SRM UNIT 1

PRACTICE QUESTIONS

A new study suggests that mortality at older ages follows Makeham's Law and that:

$$\mu_{70} = 0.026742$$

$$\mu_{75} = 0.149824$$

$$\mu_{80} = 0.358927.$$

(ii) Determine the probability that a life age 72 exact will survive 15 years.

(iii) Comment on the validity of the assumptions made about forces of mortality in parts (i) and (ii) above.

Ans: (i) $\mu = 0.044629$, $10p_0 = 0.390625$, (ii) 0.003100



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