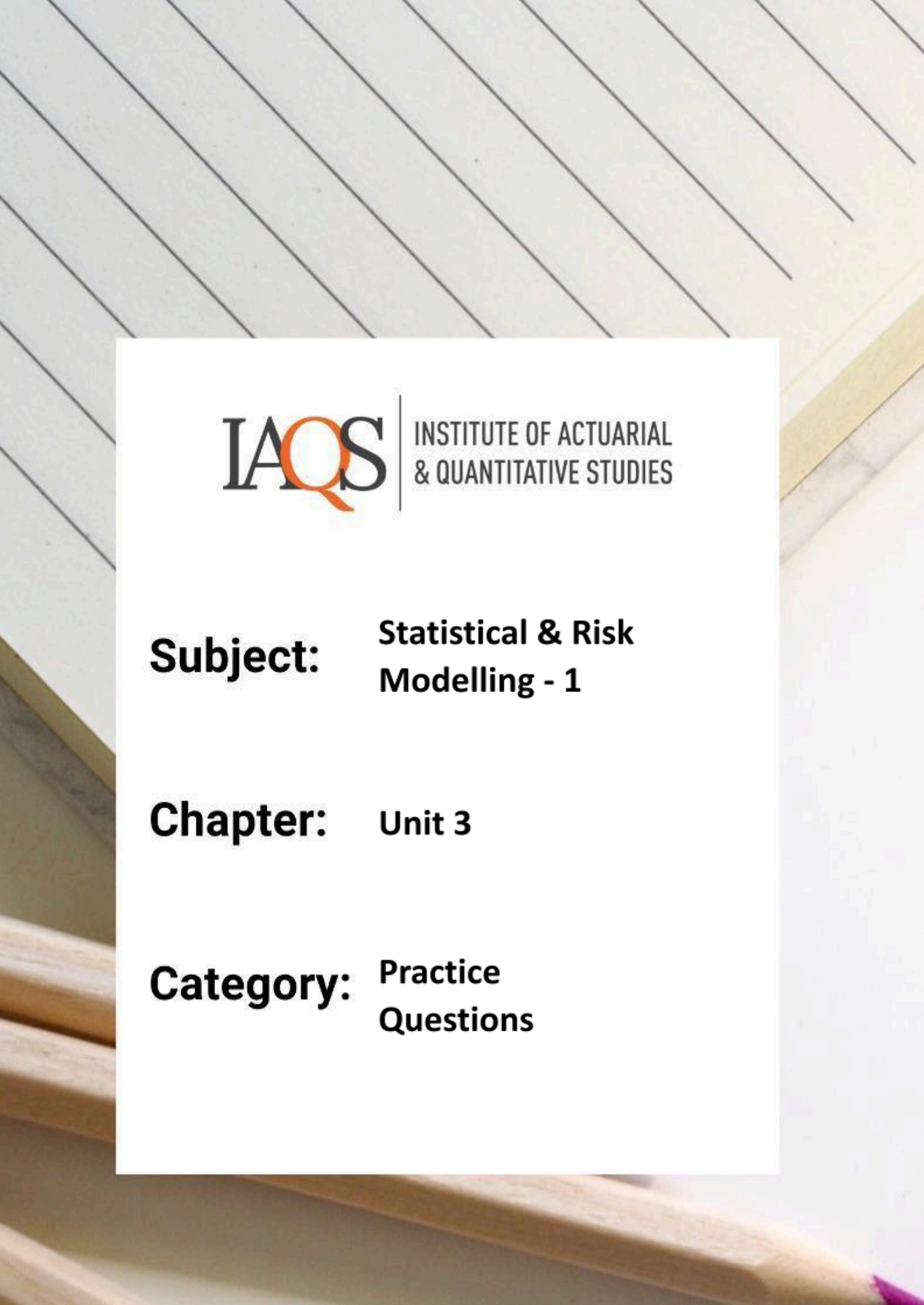

INSTITUTE OF ACTUARIAL & QUANTITATIVE STUDIES

**Subject:** Statistical & Risk Modelling - 1

**Chapter:** Unit 3

**Category:** Practice Questions

**Note: Older questions may have reference to graphical graduation method. However, after 2020, graphical graduation is not a part of the CS2 portion, and not a part of IAQS – SRM1 portion.**

**1. CT4 April 2010 Q12**

(i) State three different methods of graduating raw mortality data and for each method give an example of a situation when a method would be appropriate.

A life insurance company last period its whole of life contract 30 years ago using a standard mortality table. The company wishes to establish whether recent mortality experience in the portfolio of business is in line with the pricing basis. These are the data:

| *Recent Experience* | | | *Extract from the standard table used for pricing the product* | |
|---|---|---|---|---|
| *Age last birthday* | *Exposed to Risk during 2009* | *Deaths during 2009* | *x* | *Number of survivors to age x* |
| 50 | 2,381 | 16 | 50 | 32,669 |
| 51 | 3,177 | 21 | 51 | 32,513 |
| 52 | 3,460 | 22 | 52 | 32,338 |
| 53 | 1,955 | 15 | 53 | 32,143 |
| 54 | 3,122 | 24 | 54 | 31,926 |
| 55 | 3,485 | 29 | 55 | 31,685 |
| 56 | 2,781 | 26 | 56 | 31,417 |
| 57 | 3,150 | 31 | 57 | 31,121 |
| 58 | 3,651 | 39 | 58 | 30,795 |
| 59 | 3,991 | 48 | 59 | 30,435 |
| | | | 60 | 30,039 |

(ii) Test the goodness of fit to these data with the pricing basis and comment on your results.

(iii) (a) State with reason one further test which you would deem appropriate to perform on these data.

(b) carry out the test.

**2. CT4 April 2011 Q5**

(i) Explain why mortality experiences would need to be graduated.

**UNIT 3**

**PRACTICE QUESTIONS**

An actuary was conducted investigations into the mortality of the following classes of lives:

(a) The female members of a medium-sized pension scheme

(b) The male population of a large industrial country

(c) The population of a particular species of reptile in the zoological collections of the southern hemisphere.

The actuary wishes to graduate the crude rates.

(ii) State an appropriate method of graduation for each of three classes of lives and, for each class, briefly explain your choice.

**3. CT4 April 2011 Q11**

An historian has investigated the force of mortality from tuberculosis in a particular town in a developed country in the 1860s using a sample of records from a cemetery.

He wishes to test whether the underlying mortality from tuberculosis in the town is the same as the national force of mortality from this cause of death, as reported in death registration data. The data are shown in the table below.

| *Age-group* | *Deaths in sample* | *Central exposed to risk in sample* | *National force of mortality* |
|---|---|---|---|
| 5–14 | 13 | 3,685 | 0.0051 |
| 15–24 | 47 | 2,540 | 0.0199 |
| 25–34 | 52 | 1,938 | 0.0309 |
| 35–44 | 50 | 1,687 | 0.0316 |
| 45–54 | 33 | 1,386 | 0.0286 |
| 55–64 | 23 | 1,018 | 0.0230 |
| 65–74 | 13 | 663 | 0.0202 |
| 75–84 | 3 | 260 | 0.0070 |

(i) Carry out an overall test of the null hypothesis that the underlying mortality from tuberculosis in the town is the same as the national force of mortality and state your conclusion.

(ii) (a) Identify two differences between the experience of the sample and the national experience which the test you performed in (i) might not detect.

(b) Carry out a test for each of the differences in (ii)(a).

**UNIT 3**

**PRACTICE QUESTIONS**

(iii) Comment on the results from all the tests carried out in (i) and (ii).

**4. CT4 October 2011 Question 10**

(i) Describe three shortcomings of the $\chi^2$ test for comparing crude estimates of mortality with a standard table and why they may occur.

The following table gives an extract of data from a mortality investigation conducted in the rural highlands of a developed country. The raw data have been graduated by reference to a standard mortality table of assured lives.

| *Age* $x$ | *Expected deaths* | *Observed deaths* | $z_x$ | $z_x^2$ |
|---|---|---|---|---|
| 60 | 36.15 | 35 | –0.191 | 0.037 |
| 61 | 28.92 | 24 | –0.915 | 0.837 |
| 62 | 31.34 | 27 | –0.775 | 0.601 |
| 63 | 38.01 | 35 | –0.488 | 0.238 |
| 64 | 26.88 | 32 | 0.988 | 0.975 |
| 65 | 37.59 | 36 | –0.259 | 0.067 |
| 66 | 33.85 | 34 | 0.026 | 0.001 |
| 67 | 26.66 | 32 | 1.034 | 1.070 |
| 68 | 22.37 | 26 | 0.767 | 0.589 |
| 69 | 18.69 | 33 | 3.310 | 10.956 |
| 70 | 18.24 | 22 | 0.880 | 0.775 |



(ii) For each of the three shortcomings you described in (i):
(a) Name a test that would detect that shortcoming.
(b) Carry out the test on the data above.

(iii) Comment on your result from (ii).

**5. CT4 April 2012 Question 8**

The mortality experience of a large company pension scheme is to be tested to see if the experience of males aged 65-72 years is constant with a standard table. The results were collected by the firm conducting the analysis on a computer spread sheet, with positive and negative standardized deviations being distinguished only by being in a different coloured font.

Unfortunately, the results have been supplied to the company in the form of a printout produced on a black-and-white printer from which it is not possible to tell the signs of the deviations.

The values of the standardized deviations shown are as follows:

0.052, 0.967, 2.528, 0.328, 1.234, 0.250, 1.023, 0.756

(i) Suggest two tests which could be conducted from the information given.

(ii) Carry out the tests you suggested in your answer to part (i).

**6. CT4 September 2012 Question 8**

(i) Describe a situation when graduation of raw mortality data using a parametric formula might be appropriate and explain why.

(ii) (a) state another method of graduation.

(b) suggest the situation in which its use may be appropriate.

A large insurance company has graduated the mortality experience of part of its business. The original data and the graduated rates are as follows.

(iii) Test this graduation for overall goodness of fit.

(iv) Discuss whether it may be necessary to test for smoothness.

(v) Test the data for individual outliers.



**7. CT4 April 2013 Question 9**

A life office compared the mortality of its policyholders in the age range 30 to 60 years inclusive with a set of mortality rates prepared by the continuous mortality investigation (CMI). The mortality of the life office policyholders was higher than the CMI rates at ages 30-35, 38-41, 41-50 and 54-59 years inclusive, and lower than the CMI rates at all other ages in the age range.

(i) Perform two tests of the null hypothesis that the underlying mortality of the life office policyholders is represented by the CMI rates.

(ii) Comment on your results from part (i).

(iii) Explain the problem which duplicate policies cause in the context of the CMI mortality investigation.

**8. CT4 September 2013 Q9**

(i) (a) State three different methods of graduating crude mortality data. (b) Give, for each method, one advantage and one disadvantage.

An insurance company has graduated the experience of one block of its life business against a standard table, the following is an extract of the data.

(ii) Carry out a test for overall goodness of fit.

(iii) Carry out two other statistical tests to check the validity of the graduation.

(iv) Discuss, with reference to the tests you have performed, whether it would be reasonable for the company to use the graduated rates to price life insurance policies.

**9. CT4 April 2014 Q9**

(i) (a) State three features which are desirable when a graduation is performed.

(b) Explain why they are desirable.

The actuary to a large pension scheme has attempted to graduate the scheme's recent mortality experience with reference to a table used for similar sized schemes in a different industry. He has calculated the standardised deviations between the crude, and he graduated rates, zx, at each age and has sent you a printout of the figures over a small range of ages.

Unfortunately, the dot matrix printer on which he printed the results was very old and the dots which would form the minus sign in front of numbers no longer function, so you cannot tell which of the standardised deviations is positive and which negative. Below is the data which you have.

| *Age* | *Standardised deviation* |
|---|---|
| 60 | 2.40 |
| 61 | 0.08 |
| 62 | 0.80 |
| 63 | 0.76 |
| 64 | 1.04 |
| 65 | 0.77 |
| 66 | 1.30 |
| 67 | 1.76 |
| 68 | 0.28 |
| 69 | 0.68 |
| 70 | 0.93 |

(ii) (a) Carry out an overall goodness-of-fit test on the data.

(b) Comment on your result.

(iii) (a) List four defects of a graduation which the test you have carried out would fail to detect.

(b) Suggest, for each of the defects, a test which could be used to detect it.

**UNIT 3**

**PRACTICE QUESTIONS**

(iv) Carry out one of the tests suggested in part (iii)(b).

**10. CT4 September 2014 Q9**

A life insurance company is developing a new class of annuity business. It has conducted a study of mortality among lives it believes represents this new business. It wishes to graduate the data so that they are suitable for use in financial calculations. It decides to use a standard table as a basis for graduation and the function:

$$\overset{\circ}{\mu}_x = \mu^s_x + 0.01$$

where $\overset{\circ}{\mu}_x$ are the graduated rates and $\mu^s_x$ are the rates from the standard table.

The table below gives some results from the graduation

| *Age x* | *Crude rates* $\hat{\mu}_x$ | *Graduated rates* $\overset{\circ}{\mu}_x$ | *Exposed to risk* |
|---|---|---|---|
| 70 | 0.0167 | 0.022661 | 1,200 |
| 71 | 0.0209 | 0.024783 | 1,194 |
| 72 | 0.0236 | 0.027204 | 973 |
| 73 | 0.0324 | 0.029956 | 956 |
| 74 | 0.0362 | 0.033072 | 912 |
| 75 | 0.0402 | 0.036587 | 845 |
| 76 | 0.0561 | 0.040357 | 820 |
| 77 | 0.0623 | 0.044962 | 369 |
| 78 | 0.0552 | 0.049899 | 489 |
| 79 | 0.0640 | 0.055390 | 500 |

(i) Carry out an overall test of the goodness-of-fit of this graduation to the crude rates.

(ii) List three defects of a graduation which the test you conducted in (i) may not detect.

(iii) Perform, for each of two of the defects listed in (ii), an additional test which can detect the defect.

(iv) Comment on the results of the tests carried out in parts (i) and (iii).

**11. CT4 September 2015 Q11**

(i) Describe why an insurance company might want to compare the results of a mortality investigation with previous experience.

A large life insurance company has undertaken an investigation of the mortality of its policyholders. Currently it assumes that mortality at age x, μx, is equal to a standard table. The company wishes to use the results from the investigation to see whether the

standard table is still appropriate. Below are shown some data from the investigation.

| *Age x* | *Number of policies in force* | *Actual death claims* | *Expected death claims from standard table* |
|---|---|---|---|
| 70 | 1,000 | 13 | 23.74 |
| 71 | 1,200 | 28 | 31.80 |
| 72 | 1,100 | 31 | 32.50 |
| 73 | 1,100 | 34 | 36.20 |
| 74 | 1,000 | 39 | 36.63 |
| 75 | 1,000 | 41 | 40.73 |
| 76 | 950 | 41 | 42.99 |
| 77 | 900 | 40 | 45.20 |
| 78 | 850 | 46 | 47.34 |
| 79 | 800 | 48 | 49.35 |

(ii) Perform an overall test of the hypothesis that the underlying mortality of the company's policyholders is, over this range of ages, represented by the standard table.

(iii) Evaluate the suitability of the standard table for use in the company's financial modelling by performing two additional tests for different possible inconsistencies between the actual death rates and those represented by the standard table.

The company discovers that at age 70 years, one individual owns 25 of the policies in the investigation, the remaining policies each being owned by different individuals.

(iv) Assess the impact of this on the variance of the number of claims at age 70 years.

**12. CT4 April 2016 Q8**

(i) List THREE different methods of graduating crude mortality rates.

(ii) State the advantages of each method listed in part (i).

A life insurance company has graduated its own mortality experience for term assurance business over the past 15 years against a standard table using the following equation:

$$q_x = 0.94q_x^s - 0.0001$$

where $q_x^s$ is the mortality rate from the standard table.

The following is an extract from the data.

**UNIT 3**

**PRACTICE QUESTIONS**

| *Age x* | *Exposed to Risk* | *Deaths* | *Graduated Rate* |
|---|---|---|---|
| **40** | 24,584 | 14 | 0.000625 |
| **41** | 32,587 | 32 | 0.000683 |
| **42** | 15,784 | 16 | 0.000748 |
| **43** | 21,336 | 22 | 0.000823 |
| **44** | 25,874 | 24 | 0.000908 |
| **45** | 21,544 | 22 | 0.001005 |
| **46** | 23,967 | 25 | 0.001114 |
| **47** | 25,811 | 30 | 0.001239 |
| **48** | 26,911 | 28 | 0.001378 |
| **49** | 28,445 | 38 | 0.001536 |
| **50** | 30,205 | 45 | 0.001713 |

(iii) Carry out a test for overall goodness of fit of the data, using a 95% significance level.

## 13. CT4 September 2016 Q12

A large life insurance company is conducting an investigation into the mortality of its policyholders to see if this has changed since the previous investigation ten years ago. Below is a sample of the results:

| | *Current investigation* | | *Previous investigation* |
|---|---|---|---|
| *Age* | *Exposed to risk* | *Observed deaths* | *mortality rates* |
| 55 | 5,842 | 150 | 0.0267 |
| 56 | 5,630 | 132 | 0.0278 |
| 57 | 4,281 | 126 | 0.0301 |
| 58 | 3,955 | 98 | 0.0325 |
| 59 | 3,879 | 142 | 0.0356 |
| 60 | 3,550 | 149 | 0.0387 |
| 61 | 4,006 | 162 | 0.0396 |
| 62 | 4,150 | 173 | 0.0410 |
| 63 | 3,520 | 158 | 0.0433 |
| 64 | 3,057 | 150 | 0.0458 |
| 65 | 3,666 | 200 | 0.0490 |

(i) Explain how many degrees of freedom would be used to conduct a chi-squared test for goodness of fit on these data.

(ii) Carry out a chi-squared test on these data.

(iii) Perform a test to determine whether the shape of the mortality rates has changed over the age range.

(iv) Comment on your results to parts (ii) and (iii).

(v) Suggest reasons why the mortality experience may have changed over the past ten years.

## 14. CT4 April 2017 Q10

The government statistical service in a country with a population of 10 million has estimated mortality rates among males in that country aged 20 to 99 years inclusive.

It wishes to create a new standard mortality table.

(i) Describe why the crude mortality rates should be graduated during the production of this standard mortality table.

(ii) Describe a suitable method of graduation for these mortality rates.

(iii) Explain the limitations of the method described in your answer to part (ii) in this situation.

The government performs the graduation and compares the crude and graduated rates.

Below are some of the results of the comparison:

| *Value of individual standardised deviation at age x, $z_x$* | *Number of ages* |
|---|---|
| $z_x < -3$ | 0 |
| $-2 > z_x \geq -3$ | 7 |
| $-1 > z_x \geq -2$ | 16 |
| $0 > z_x \geq -1$ | 26 |
| $1 > z_x \geq 0$ | 16 |
| $2 > z_x \geq 1$ | 10 |
| $3 > z_x \geq 2$ | 2 |
| $z_x \geq 3$ | 3 |

(iv) Assess the quality of the graduated rates for use as a new standard mortality table by applying TWO statistical tests to the above information. The two tests should each examine a different aspect of the graduation.

(v) Comment on the implications of your results in part (iv) for the government using the new standard mortality table for economic and financial planning purposes.

**15. CT4 September 2017 Q9**

(i) (a) List TWO different methods of graduating crude mortality data.

(b) State, for each method, TWO advantages and ONE disadvantage.

A large pension scheme is examining its most recent experience and has graduated its data over a range of ages using μx = 0.0005 + 0.00005(1.1^x). The table below gives some of the data

| *Age* | *Exposed to Risk* | *Observed Deaths* | *Graduated Rates* |
|---|---|---|---|
| 60 | 7,966 | 127 | 0.015724 |
| 61 | 7,728 | 139 | 0.017246 |
| 62 | 7,870 | 162 | 0.018921 |
| 63 | 7,622 | 167 | 0.020763 |
| 64 | 7,097 | 205 | 0.022790 |
| 65 | 7,208 | 179 | 0.025019 |
| 66 | 6,833 | 185 | 0.027470 |
| 67 | 6,474 | 212 | 0.030167 |
| 68 | 6,208 | 209 | 0.033134 |
| 69 | 5,914 | 195 | 0.036398 |

(ii) Perform an overall goodness of fit test on the data.

(iii) (a) State THREE possible defects of the graduation which the test you performed in (ii) would fail to detect.

(b) Suggest, for each defect in part (a), an alternative test which would detect each defect.

(iv) Carry out TWO of the tests you mentioned in part (iii), clearly stating your conclusions in relation to the relevant defect.

**16. CT4 September 2018 Q9**

(i) Describe why a mortality experience would need to be graduated.

(ii) Describe how smoothness is achieved when using the following graduation methods:

(a) parametric formula.

(b) with reference to a standard table.

An insurance company conducts an investigation into the mortality rates of policyholders who choose to retire at a relatively young age.

The following table shows data from the investigation, together with graduated rates $\overset{o}{q}_x$ which were fitted with reference to standard table rates, $\overset{s}{q}_x$ using a link function $\overset{o}{q}_x = \overset{s}{q}_x$ + constant.

| *Age x* | *Exposed to risk* | *Deaths* | $\overset{o}{q}_x$ |
|---|---|---|---|
| 55 | 1,550 | 15 | 0.00673 |
| 56 | 2,100 | 18 | 0.00689 |
| 57 | 2,300 | 15 | 0.00709 |
| 58 | 2,450 | 21 | 0.00736 |
| 59 | 2,700 | 18 | 0.00770 |
| 60 | 3,250 | 29 | 0.00820 |
| 61 | 3,100 | 25 | 0.00891 |
| 62 | 3,450 | 30 | 0.00978 |
| 63 | 3,600 | 45 | 0.01084 |
| 64 | 3,750 | 41 | 0.01210 |

(iii) Test the goodness-of-fit of the graduated rates using a chi-square test.

## 17. CS2A April 2019 Q11

A life insurance company is investigating the mortality of its policyholders over the past year. It wishes to compare the current mortality rates with those obtained from a similar investigation ten years ago. The following is an extract of the data:

| ***Current investigation*** | | | ***Previous investigation*** |
|---|---|---|---|
| *Age x last birthday* | *Exposed to risk* | *Observed deaths* | *Mortality rate* |
| 50 | 5,368 | 25 | 0.00479 |
| 51 | 4,986 | 26 | 0.00538 |
| 52 | 4,832 | 30 | 0.00603 |
| 53 | 5,298 | 37 | 0.00675 |
| 54 | 5,741 | 45 | 0.00756 |
| 55 | 4,866 | 46 | 0.00844 |
| 56 | 4,901 | 52 | 0.00942 |
| 57 | 5,003 | 63 | 0.01050 |
| 58 | 3,952 | 45 | 0.01169 |
| 59 | 2,786 | 45 | 0.01299 |

(i) Sketch a graph, showing clearly both the current and the previous mortality rates.

(ii) Carry out a goodness-of-fit test on the data.

(iii) Carry out the following additional statistical tests:

- Signs test.
- Grouping of Signs test.

(iv) Comment on your answers to part (iii) in the light of your sketch in part (i).

**18. CS2A September 2019 Q6**

The Eternal Life insurance company insures the lives of community leaders. Its pricing policy assumes that the mortality of its policyholders reflects the national mortality levels. Recently, however, it has been losing business to rivals, so it has commissioned an investigation into the mortality of its policyholders. Some results from the investigation are shown below

| *Age x nearest birthday* | *Person-years at risk,* $E^c_x$ | *Number of deaths,* $d_x$ | *National mortality rate* |
|---|---|---|---|
| 60 | 750 | 10 | 0.01323 |
| 61 | 740 | 11 | 0.01483 |
| 62 | 710 | 12 | 0.01664 |
| 63 | 700 | 15 | 0.01870 |
| 64 | 680 | 12 | 0.02101 |
| 65 | 400 | 5 | 0.02348 |
| 66 | 390 | 5 | 0.02610 |
| 67 | 380 | 6 | 0.02893 |
| 68 | 360 | 8 | 0.03192 |
| 69 | 350 | 8 | 0.03505 |



(i) Carry out a general goodness-of-fit test of the hypothesis that the mortality of the company's policyholders is the same as the national mortality rate.

(ii) Comment on your results in part (i).

(iii) Perform one other test designed to explore further the hypothesis that the mortality of the company's policyholders is the same as the national mortality rate.

(iv) Suggest explanations for the results you have obtained in parts (i) and (iii).

**19. CS2A September 2020 Q5**

An investigation was undertaken into the mortality of policyholders for a large life insurance company. The crude mortality rates were graduated using a formula of the form: μx = exp(ax + bx^2).

An extract of the results is set out below. All data have been collated between 1 January 2018 and 1 January 2019 on an 'age last birthday' basis:

| *Age* | *Exposed-to-risk (years)* | *Observed deaths* | *Graduated rates* |
|---|---|---|---|
| 50 | 23,308 | 70 | 0.00368 |
| 51 | 19,316 | 58 | 0.00379 |
| 52 | 16,914 | 54 | 0.00391 |
| 53 | 21,082 | 90 | 0.00402 |
| 54 | 14,820 | 70 | 0.00415 |
| 55 | 24,084 | 96 | 0.00428 |
| 56 | 28,076 | 114 | 0.00441 |
| 57 | 22,958 | 86 | 0.00455 |
| 58 | 24,960 | 102 | 0.00469 |
| 59 | 21,134 | 86 | 0.00485 |
| 60 | 18,374 | 94 | 0.00500 |



(i) Perform the signs test to explore the hypothesis that the graduated mortality rates are the true rates underlying the observed data from the life insurance company.

(ii) State the main limitation of the signs test in assessing the suitability of the graduated rates.

(iii) Perform a chi-square goodness-of-fit test to explore further the hypothesis that the graduated mortality rates are the true rates underlying the observed data from the life insurance company.

(iv) Comment on your answers to parts (i) and (iii).

**20. CS2A September 2021 Q4**

An Actuary has graduated the mortality experience of a population aged 55 to 65 years using the following formula:

$$\mu_x = \begin{cases} ax + b\exp(cx) & \text{if } x < 65 \\ d & \text{if } x = 65 \end{cases}$$

where a, b, c and d are constants and x is age in years.

The mortality experience data and the graduated rates calculated by the Actuary are shown in the table below. All data have been collated between 1 January 2020 and 31 December 2020 inclusive on an 'age nearest birthday' basis:

| *Age x* | *Exposed-to-risk (years)* | *Observed deaths* | *Graduated rates* |
|---|---|---|---|
| 55 | 2,737 | 53 | 0.02094 |
| 56 | 2,610 | 57 | 0.02357 |
| 57 | 2,649 | 86 | 0.02636 |
| 58 | 2,611 | 77 | 0.02930 |
| 59 | 2,449 | 74 | 0.03238 |
| 60 | 2,213 | 96 | 0.03555 |
| 61 | 2,025 | 79 | 0.03880 |
| 62 | 1,969 | 68 | 0.04208 |
| 63 | 1,900 | 78 | 0.04537 |
| 64 | 1,803 | 83 | 0.04860 |
| 65 | 1,736 | 105 | *y* |



Let y be the graduated rate at age 65 and let the null hypothesis be that the graduated rates are the true rates underlying the observed data.

Determine the range of values that y needs to take so that there is insufficient evidence, at the 97.5% confidence level, to reject the null hypothesis under a chi-square goodness-of-fit test.

## 21. CS2A September 2022 Q5

An insurance company has graduated the experience of one block of its life business using the following quadratic-Gompertz formula:

$$\mu_x = \exp(a_0 + a_1 x + a_2 x^2)$$

where a0, a1 and a2 are constants and x is age in years.

The data used for this graduation exercise, together with the graduated rates, are shown in the table below. All data have been collated between 1 January 2018 and 31 December 2018 inclusive on an 'age nearest birthday' basis.

| *Age* | *Exposed to risk (years)* | *Death counts* | *Graduated rates* |
|---|---|---|---|
| 60 | 11,362 | 80 | 0.0069 |
| 61 | 11,086 | 85 | 0.0077 |
| 62 | 10,816 | 100 | 0.0085 |
| 63 | 10,530 | 105 | 0.0095 |
| 64 | 10,301 | 108 | 0.0105 |
| 65 | 10,233 | 117 | 0.0115 |
| 66 | 9,970 | 110 | 0.0126 |
| 67 | 9,708 | 131 | 0.0138 |
| 68 | 9,667 | 145 | 0.015 |
| 69 | 9,807 | 140 | 0.0163 |



(i) Perform a Chi-square test to assess the overall goodness of fit of this graduation, stating the null and alternative hypotheses.

The company would like to extend the graduation to age 70. The death count and exposed to risk at age 70 are 150 deaths and 10,000 years, respectively.

Let $\hat{\mu}_{70}$ be the graduated rate at age 70, and let the null and alternative hypotheses be the same as in part (i), except that the age range is now from 60 to 70.

(ii) Determine, using the data in the table above, the condition that μ෧ ෧70 would need to fulfil in order for there to be insufficient evidence, at the 5% significance level, to reject the null hypothesis under the cumulative deviations test.

## 22. CS2A April 2024 Q2

The following table shows mortality data and graduated mortality rates of male and female members of a pension scheme:

| *Gender* | *Age (x years)* | *Exposed-to-risk (years)* | *Observed deaths* | *Graduated rates* |
|---|---|---|---|---|
| Female | 70 | 1,007 | 153 | 0.14227 |
| Female | 71 | 978 | 166 | 0.16530 |
| Female | 72 | 1,111 | 204 | 0.19205 |
| Female | 73 | 1,500 | 326 | 0.22313 |
| Female | 74 | 1,200 | 306 | 0.25924 |
| Female | 75 | 2,001 | 599 | 0.30119 |
| Male | 70 | 908 | 338 | 0.36788 |
| Male | 71 | 998 | 439 | 0.42742 |
| Male | 72 | 1,009 | 497 | 0.49659 |
| Male | 73 | 877 | 508 | 0.57695 |
| Male | 74 | 859 | 571 | 0.67032 |

The data was graduated using the following parametric formula: $\log(\mu_x) = \alpha + \beta I_F + \delta x$

where $I_F$ is an indicator variable taking the value 1 for female members and 0 for male members.

(i) Calculate the values of α, β and δ, by considering the graduated rates at ages 70 and 71.

(ii) Comment on your results.

(iii) Perform a Chi-square test at a 5% significance level to assess the overall appropriateness of these graduated rates for both male and female members,

stating your null and alternative hypotheses.

(iv) State two limitations of the Chi-square test in this context and in each case suggest an alternative test that would address that limitation.